Reflections #1 • Math 495 / Phys 484 • Dynamics in the Phase Plane

- write ups should present ideas & results.
- plots should be fully annotated so that the reader can reproduce it. (Include the equation, parameters, IVs, etc.)
- due date is Monday 22 January.
- remember that webct is an open forum for discussion.
- please acknowledge collaborations & assistance from colleagues – these are encouraged.
- treat the assignment questions as case studies for investigation. The specific questions are there to direct your thinking, but you should take a larger perspective of the case study.

A) Introduction to Computing ODE Solutions: (2 pages + annotated plots) Modify the *pperiod.m* script to compute solutions, $u(t;r)$ of the Van der Pol equation (Manneville, page 48 with $g = 1$)

$$u'' - (r - u^2)\, u' + u = 0$$

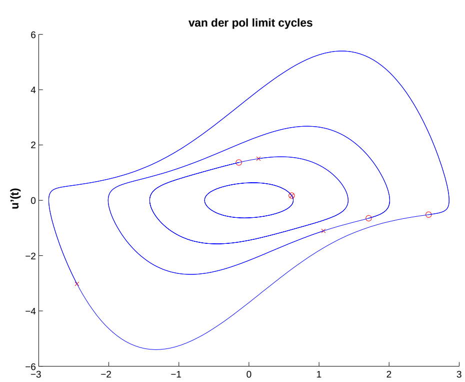
where r is a real-valued parameter. You should benchmark the correctness of your script – one way is to reproduce some of the figures on page 49.

This ODE exhibits *limit cycle* behaviour for some values of the parameter r — that is, for large t the solutions limit to a periodic orbit. In light of this fact, design a computation to produce a plot of

$$\max |\tilde{u}(t;r)| \quad \text{versus} \quad r$$

where $\tilde{u}(t;r)$ denotes periodic-in-t solutions for the parameter r. For this plot, consider steady solutions as a class of periodic solutions with zero period.

Include in your presentation, a brief discussion of the numerical errors that are inherent in your computation. Explain how the controls of your computation allow for increasing convergence to the true continuous result.



B) Divergence of Phase Space Area: Exercise 2.5.1 from Manneville (page 61).

C) Energy Method: Exercise 2.5.2 from Manneville (page 62-63).

***) The Pendulum Separatrix** (optional) Express the separatrix solutions of the nonlinear pendulum in terms of elementary functions. The infinite-time nature of the trajectory follows immediately.