Homework #7 • MATH 462 • Surface Waves & Sound Waves

- submit your write-up Thursday 16 March.

A) Finite Depth Fluid (3 pages + plot, 10pts) Give a complete discussion for the derivation of the travelling wave solution to the linearized surface wave equations with a bottom boundary (located at $y = -H$). Summarize clearly the formulas for the PDE solutions and the wavespeed. Note that the ratio $c(k, H)/c(k, \infty)$ is only a function of one variable, make a plot and explain what it tells. Quality of presentation will be a significant part of the grade for this problem.

B) Music of the Sphere? (3 pages, 10pts) This problem is based on #3.13 in Acheson, and requires reading of section 3.6 of the chapter on waves.

i) Derive the linear wave equation for a spherically symmetric wave from the Euler equations.

ii) For a spherical shell of radius L, derive the eigenvalue relation for the natural (temporal) frequencies, ω, of the interior standing waves

$$\tan \frac{\omega L}{c} = \frac{\omega L}{c} \ .$$

Explain why the boundary conditions of bounded density at the origin and zero velocity at $r = L$ are reasonable choices.

iii) Numerically calculate the first three eigenfrequencies (as multiples of c/L), and comment upon whether or not you think this *music of the sphere* would be a truly harmonious sound.