Homework #5 • MATH 462 • More Potential Flow

- submit your write-up noon, Thursday 23 February.
- in-class midterm reminder: Wednesday 01 March.

***) Memory Sheet** (1/2 page, pts on midterm) For reference during the midterm, Acheson appendices A1-A6 will be attached to the exam. You will also be allowed to prepare a memory sheet of formulas & ideas on the 1/2-page of blue paper handed out in lecture. Memory sheets are to be submitted with the midterm, and will be given credit under the following guidelines:

- no microfilm (reasonable size writing please), 1/2-page single-sided.
- no derivations, only basic formulas & ideas.

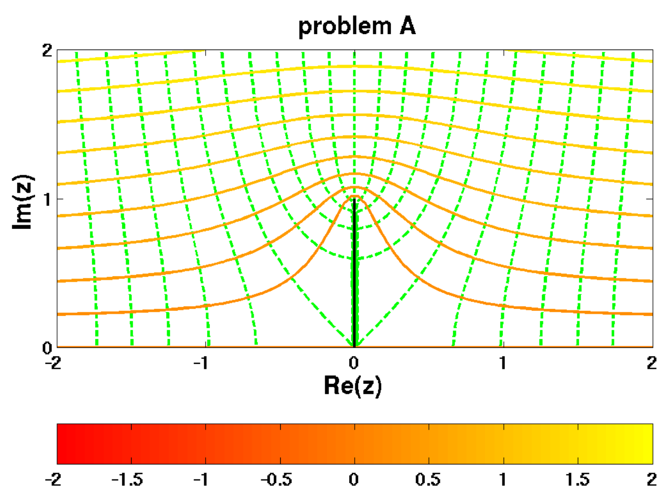
A) Flow past the Wall (2 pages, 10pts) Consider a potential flow as defined by the conformal map $M(z) = \sqrt{z^2 - 1}$ of the z-plane flow where streamlines are lines of constant imaginary part. Consider only the mapping of the upper half z-plane to the upper half M-plane — this also uniquely defines the branch of the square root.

i) Determine the image of the Re(z)-axis.

ii) Defining $M = m + in$, find an equation of the curve in the (m, n)-plane which is the image of the streamline in the z-plane identified by $\text{Im}(z) = \bar{y}$.

iii) Then, find an equation of the curve in the (m, n)-plane which is the image of the contour of velocity potential identified by $\text{Re}(z) = \bar{x}$.

iv) Show by an explicit calculation that these two image curves are orthogonal in the (m, n)-plane.



B) Outflow (3 pages, 10pts) Consider the potential flow as defined by the complex potential

$$\Phi(z) = Uz + \frac{Q}{2\pi} \ln z$$

for $z \neq 0$. This flow can be plotted for $U = 1, Q = 2$ using *w06plate.m*. Calculate the volume flux (per unit height in z) emanating from the origin in three different ways.

i) by an integration involving the radial velocity on circles $r = a$,

ii) by an integration of the complex potential on arbitrary closed contours enclosing the origin exactly once, and

iii) by an explicit identification of the separating streamlines (first, express the streamlines using the polar form $z = re^{i\theta}$).

Calculate the limiting gap (Δy) between the separating streamlines as $\mathrm{Re}(z) \to \infty$ in two different ways:

iv) by the streamline expression from **iii)**, and

v) by a deduction involving the volume flux.