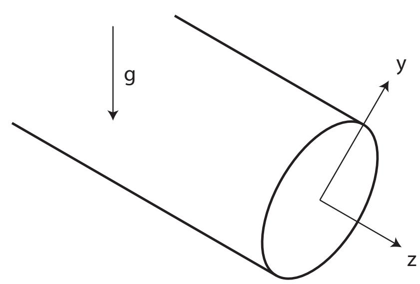
Homework #9 • MATH 462 • Last Questions

- write-ups are due Monday 29 March, **1pm** (hard deadline for delivery to grader).
- presentation is a very important aspect of these final problems.

A) Pipe Poiseuille Flow (4 pages, 15pts) Use the steady Navier-Stokes equations to calculate *gravity-driven* flow through a pipe which is inclined at an angle α from the horizontal. I recommend using cylindrical coordinates with the $\hat{z}$-axis oriented with the pipe axis – hence, gravity will be seem to be *tilted* in the equations.



Show that in the above coordinate geometry, the gravitational vector $\vec{g}$ points in the direction

$$-\cos\alpha\ (\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}) + \sin\alpha\,\hat{z}\ .$$

It is not surprising that the axial velocity is z-independent, but it turns out also to be θ-independent (like usual pipe Poiseuille flow). Thus $W(r)$ only. The pressure is also z-independent, but develops a cross-sectional distribution.

Calculate the mass flux through the pipe. Plot the cross-sectional pressure distribution.

B) Spinning Sphere (5 pages, 20 pts) Solve the problem as posed by #7.2 in Acheson. Presentation counts for 3/4 of the grade. Show that you understand both the mathematics and fluid dynamics of the problem.