Homework #5 • MATH 462 • More Potential Flow

- submit your write-up Friday 20 February.
- in-class midterm reminder: Monday 01 March.

***) Memory Sheet** (1/2 pages, pts on midterm) For reference during the midterm, Acheson appendices A1-A6 will be attached to the exam. You will also be allowed to prepare a memory sheet of formulas & ideas on the 1/2-page of beige paper handed out in lecture. Memory sheets are to be submitted with the midterm, and will be given credit under the following rules:

- no microfilm (reasonable size writing please), 1/2-page single-sided.
- no derivations, only basic formulas & ideas.

A) Flow over the Wall (2 pages, 10pts) Consider a potential flow as defined by the complex potential $\Phi(z) = \sqrt{z^2+1}$ in the upper-half z-plane, $\mathrm{Im}(z) > 0$. Manipulate the complex-valued relation

$$\phi + i\psi = \Phi(z) = \sqrt{z^2+1} = \sqrt{(x+iy)^2+1}$$

to obtain a (real-valued) function for $y(x,\psi)$. State the domain and ranges of all variables, so that the branch corresponds to left-to-right flow. Explain how to use this result to determine the flow streamlines (matlab's contourplot will fail here unless you know how to deal with the multi-valuedness). Devise a similar formula for plotting the velocity potential (make sure it always gives the proper sign).

Extra: Produce a Matlab plot showing equi-spaced values of streamfunction and velocity potential (without using the *contour* function). Indicate areas of fast/slow flow and high/low pressure.

As a comparison, the potential flow plotter *w06plate.m* gives the contourplot technique which illustrates the branch cut fix. This flow hints at the basic principles that explain how an atomizer works.

B) Outflow (2 pages, 10pts) Consider the potential flow as defined by the complex potential

$$\Phi(z) = Uz + \frac{Q}{2\pi}\ln z$$

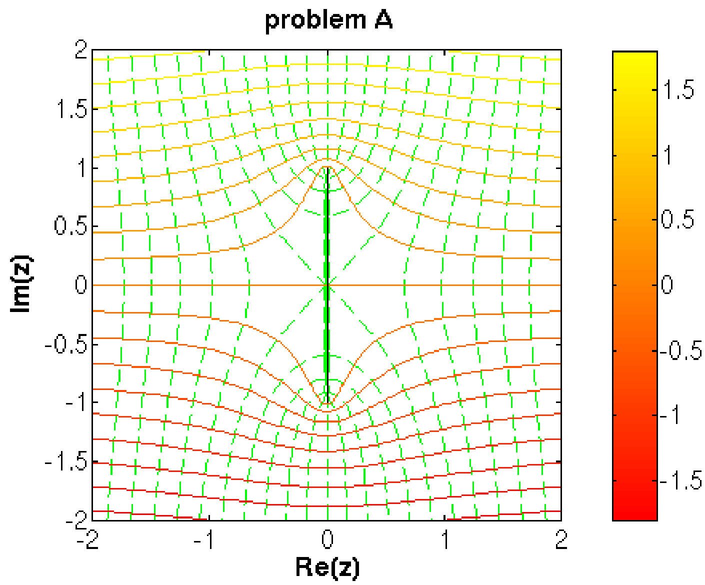
for $z \neq 0$. This flow can be plotted for $U = 1, Q = 2$ using *w06plate.m*. Calculate the volume flux emanating from the origin in two different ways: by integrating the flux due to the radial velocity, and by integrating the complex potential.

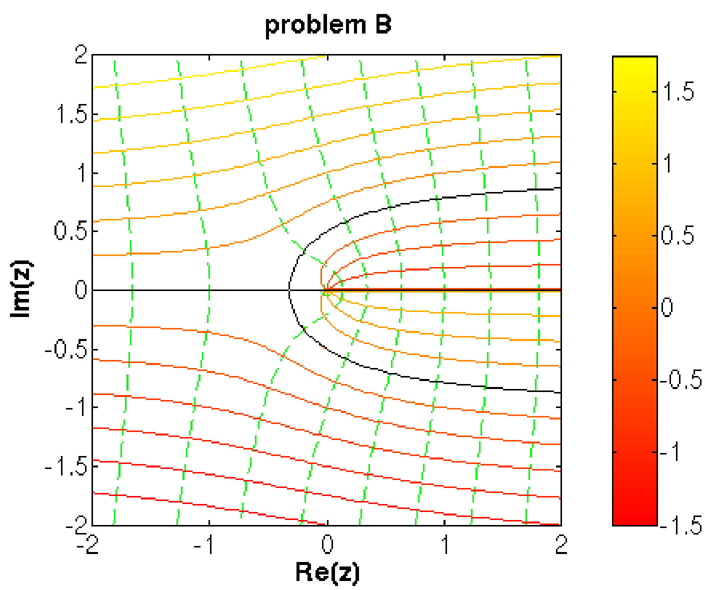
Then, find the streamlines for the superposition flow

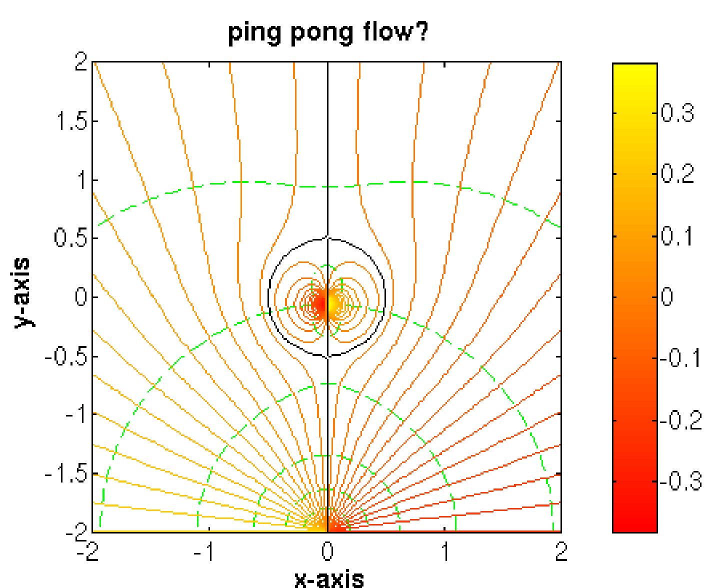
$$\Phi(z) = Uz + \frac{M}{2\pi}\ln z$$

beginning from a polar coordinate ($z = re^{i\theta}$) version of the method used in part **A)**. What is the limiting gap between the separating streamlines as $\mathrm{Re}(z) \to \infty$? By what logic can this separation be deduced directly from the volume flux concept?

***) Extra extra:** What is the potential flow suggested by the ping pong plot?


problem A
Im(z)
Re(z)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2
1.5
1
0.5
0
-0.5
-1
-1.5


problem B
Im(z)
Re(z)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2
1.5
1
0.5
0
-0.5
-1
-1.5


ping pong flow?
y-axis
x-axis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2
0.3
0.2
0.1
0
-0.1
-0.2
-0.3