Homework #6 • MATH 462 • Surface Waves

- submit your write-up Wednesday 13 March.
- thank you for acknowledging collaborations & assistance from colleagues.

A) Finite Depth Fluid (3 pages, 10pts) Give a complete discussion for the derivation of the travelling wave solution to the linearized surface wave equations with a bottom boundary (located at $y = -H$). Summarize clearly the formulas for the PDE solutions and the wavespeed. Note that the ratio $c(k, H)/c(k, \infty)$ is only a function of one variable, make a plot and explain what it tells. Quality of presentation will be a significant part of the grade for this problem.

B) Kelvin-Helmholtz Instability (3 pages, 10pts) It was shown in lecture that an interface separating two counterflowing fluids undergoes the *Kelvin-Helmholtz instability*. For the PDE model equations (with BCs), a special wave solution was constructed using a complex-valued representation of the solution (that e^{ikx} stuff). Add together the complex conjugates of the *unstable* wave to produce the real-valued representation. Calculate how the phase between the surface values of ϕ^0, ϕ^1 and η changes with the shear velocity U.

Modify the matlab plotting routine *code09.m* to include the upper fluid. Explain using just one plot at a fixed time why the flow results in a growing wave? Present the analogous plot for the stable wave solution.