

Homework #1 • MATH 419 • Intro to Function Spaces

- please respect page limits.
- submit your write-up Wednesday 18 May.
- remember that the webct discussion is an open forum.
- please annotate plots well.
- refer to *Guidelines for Reports*.

A) Two Proofs (3 pages max) Problems from text, 0.8.4 (positivity part only) and 0.8.22. You should attach a copy of the rigorous statement of continuity you are quoting from as an (extra page) appendix (preferable to use same notation).

B) Orthogonalize This (3 pages max) Refer to section 0.5.3 of the text for the Gram-Schmidt orthogonalization procedure. Apply the procedure to the basis functions

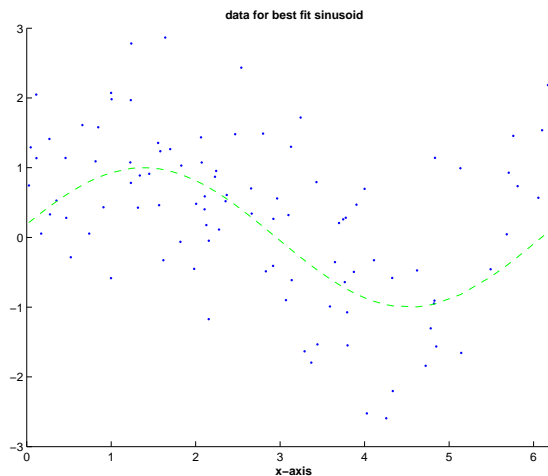
$$\{v_j(x) = \cos^j x\}_{j=0 \rightarrow 4}$$

on the interval $-\pi \leq x \leq 0$. A maple script with some example integrations can be obtained from the class webpage: *w02fourier.mw*. Include a plot of the orthonormal bases functions, and a representation of an example function of your choice. (Matlab plots preferred.)

C) Noise (2 pages max) Adapt the orthogonal projection methodology of section 0.7.1 to find a best-fit sinusoid

$$y(x) = A \cos x + B \sin x$$

to the data produced by the matlab script *w02sine_fit.m*.



bonus: How do the statistics of the (square-integrated) error depend on the sample size and the variance (amplitude) of the noise?