Homework #0 • MATH 419 • Linearity

- please respect page limits.
- submit your write-up Wednesday 11 May (part **A** indicated otherwise).
- remember that the webct discussion is an open forum.
- refer to *Guidelines for Reports*.

A) Linearity Around Us ($\leq$1 page, due **Monday** 09 May) Discover a personal interest in linearity by researching a topic of individual choice and writing a short two-paragraph essay. The subject can be anything which raises awareness of the ubiquity of the mathematics of linearity. Creativity counts. Explain how or why you think the concepts of linearity is important to your topic (be specific). Provide at least two references; they can be either print, or web-based (please verify accuracy). You may attach one image. Be prepared to announce your topic in next Monday's lecture.

Post your essay on the webct discussion.

B) Line Plots in Matlab (1 page + 1 page plots) Matlab is a computing environment which allows both interactive use and pre-programmed scripts. Plotting is not difficult. As a first example, download *w01plot.m* from the class webpage. It is a script which reproduces the line plots for the Fourier series example

$$y(x) = \frac{\sin x}{\cosh(C) - \cos x} \tag{1}$$

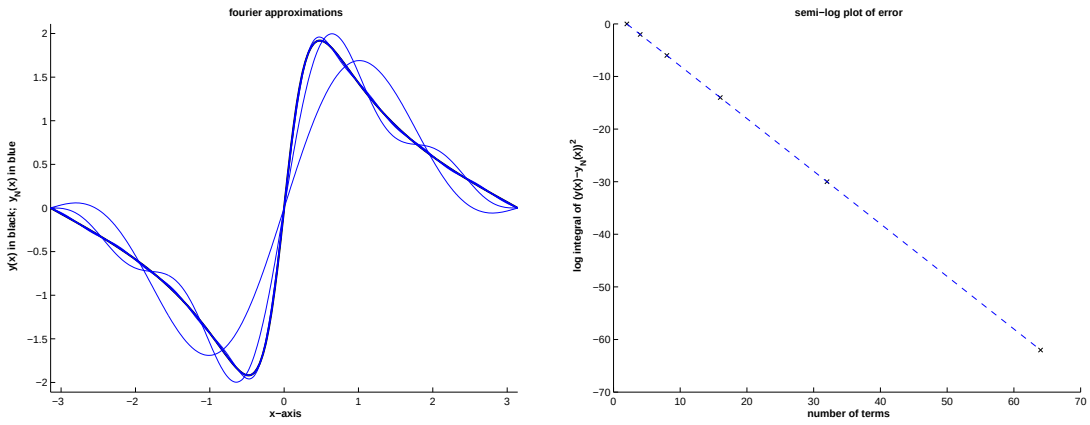
with the integrated squared-error estimated by trapezoidal rule. Play around by editing the file *w01plot.m* to see how it works. If you mess up the file, just download a new copy! Make some very minor modifications to investigate the mystery series

$$y_N(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{j=1}^{N} \frac{\cos 2jx}{4j^2 - 1} \tag{2}$$

on the interval $-\pi \leq x \leq +\pi$. Make an intelligent guess for the limiting function

$$y(x) = \lim_{N \to +\infty} y_N(x) \tag{3}$$

and use the (numerically approximated) integrated squared-error to substantiate this guess. The error $E(N)$ has a different functional decay than the example in lecture — quantify this behaviour with an appropriate plot (your computer will have to work a bit here).



C) A Few Linear Algebra Review Problems to be Named Later (2 pages) Given the orthogonal basis set for $\mathbf{R}^4$

$$\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \quad ; \quad \vec{u}_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \quad ; \quad \vec{u}_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \quad ; \quad \vec{u}_4 = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{4}$$

explain how to produce a matrix for which these are the eigenvectors corresponding to distinct eigenvalues. Produce such a matrix whose elements are integers. Try to minimize the sum of the absolute value of the matrix elements.

Give the change of basis matrices that relate representations in terms of the above basis set and the unit basis set $\{\hat{x}_j\}$.