Homework #1 • MATH 314 • Linear Algebra Review

- please respect page limits. Have each problem start at the top of a new page.
- submit your write-up into the Math314 box by 4pm, Friday 16 January.
- remember that webct is an open forum for discussion.
- please acknowledge collaborations & assistance from colleagues.

- read the *Guideline* for assignments as posted on the class webpage.
- for this assignment, clearly written descriptions of your calculational steps are essential.

A) Linear Combinations (2 pages max, 10pts) Given the 3-component vectors

$$\vec{w}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad ; \quad \vec{w}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad ; \quad \vec{w}_3 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad ; \quad \vec{b} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} \quad ; \quad \vec{f} = \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

express the vector $\vec{b}$ as a linear combination of the vector set $\mathcal{S} = \{\vec{w}_j\}_{j=1\to 3}$. Give as many one sentence reasons as you can that explains why your representation is unique.

Finally, explain why the vector set $\mathcal{S}_f = \{\vec{w}_1, \vec{w}_2, \vec{f}\}$ is not linearly independent.

B) A Real Symmetric Matrix (3 pages max, 10pts) Given the 3×3 $\mathbb{R}$-valued matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

use Gaussian elimination (choose an efficient strategy) to solve the system of linear equations

$$\mathbf{A}\,\vec{x} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} = \vec{b}$$

for the 3-component vector $\vec{x}$.

Give all of the eigenvalues and eigenvectors of the matrix $\mathbf{A}$. You need only present the detailed derivation for one of the eigenvectors.

Show that the set of eigenvectors are a mutually-orthogonal set. (Is this a surprise? Give a reference for your answer.) Redo the above linear solve for $\vec{x}$ using the projection argument for an orthogonal basis set.

C) Complex-Valued Linear Algebra (2 pages max, 10pts) Find all of the eigenvalues and eigenvectors for the $\mathbb{C}$-valued matrix

$$\mathbf{B} = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix}$$

and show that the projection argument can be used to give the solution to the linear system

$$\mathbf{B}\,\vec{y} = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} .$$