parts of the subject. Therefore the greater part of this book is devoted to linear equations and various methods for solving them. However, Chapters 8 and 9, as well as parts of Chapter 2, are concerned with nonlinear equations. Whenever it is appropriate, we point out why nonlinear equations are, in general, more difficult, and why many of the techniques that are useful in solving linear equations cannot be applied to nonlinear equations.

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Solutions. A solution of the ordinary differential equation (8) on the interval $\alpha < t < \beta$ is a function ϕ such that $\phi', \phi'', \ldots, \phi^{(n)}$ exist and satisfy

domain

$$\phi^{(n)}(t) = f[t, \phi(t), \phi'(t), \ldots, \phi^{(n-1)}(t)] \tag{14}$$

for every t in $\alpha < t < \beta$. Unless stated otherwise, we assume that the function f of Eq. (8) is a real-valued function, and we are interested in obtaining real-valued solutions $y = \phi(t)$.

Recall that in Section 1.2 we found solutions of certain equations by a process of direct integration. For instance, we found that the equation

$$\frac{dp}{dt} = 0.5p - 450 \tag{15}$$

has the solution

$$p = 900 + ce^{t/2}, \tag{16}$$

where c is an arbitrary constant. It is often not so easy to find solutions of differential equations. However, if you find a function that you think may be a solution of a given equation, it is usually relatively easy to determine whether the function is actually a solution simply by substituting the function into the equation. For example, in this way it is easy to show that the function $y_1(t) = \cos t$ is a solution of

$$y'' + y = 0 \tag{17}$$

for all t. To confirm this, observe that $y_1'(t) = -\sin t$ and $y_1''(t) = -\cos t$; then it follows that $y_1''(t) + y_1(t) = 0$. In the same way you can easily show that $y_2(t) = \sin t$ is also a solution of Eq. (17). Of course, this does not constitute a satisfactory way to solve most differential equations because there are far too many possible functions for you to have a good chance of finding the correct one by a random choice. Nevertheless, it is important to realize that you can verify whether any proposed solution is correct by substituting it into the differential equation. For a problem of any importance this can be a very useful check and is one that you should make a habit of ~~considering~~.

Some Important Questions. Although for the equations (15) and (17) we are able to verify that certain simple functions are solutions, in general we do not have such solutions readily available. Thus a fundamental question is the following: Does an