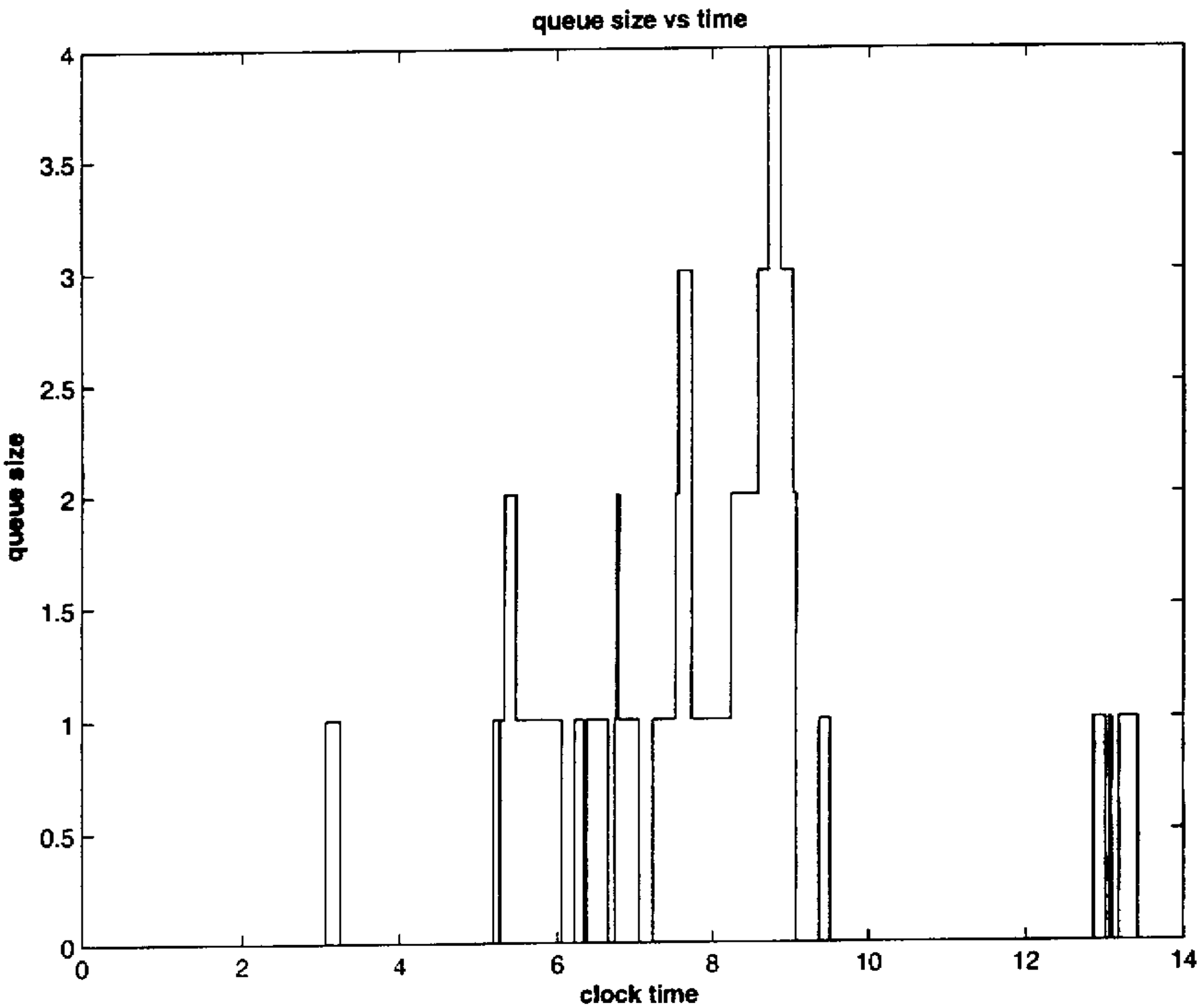
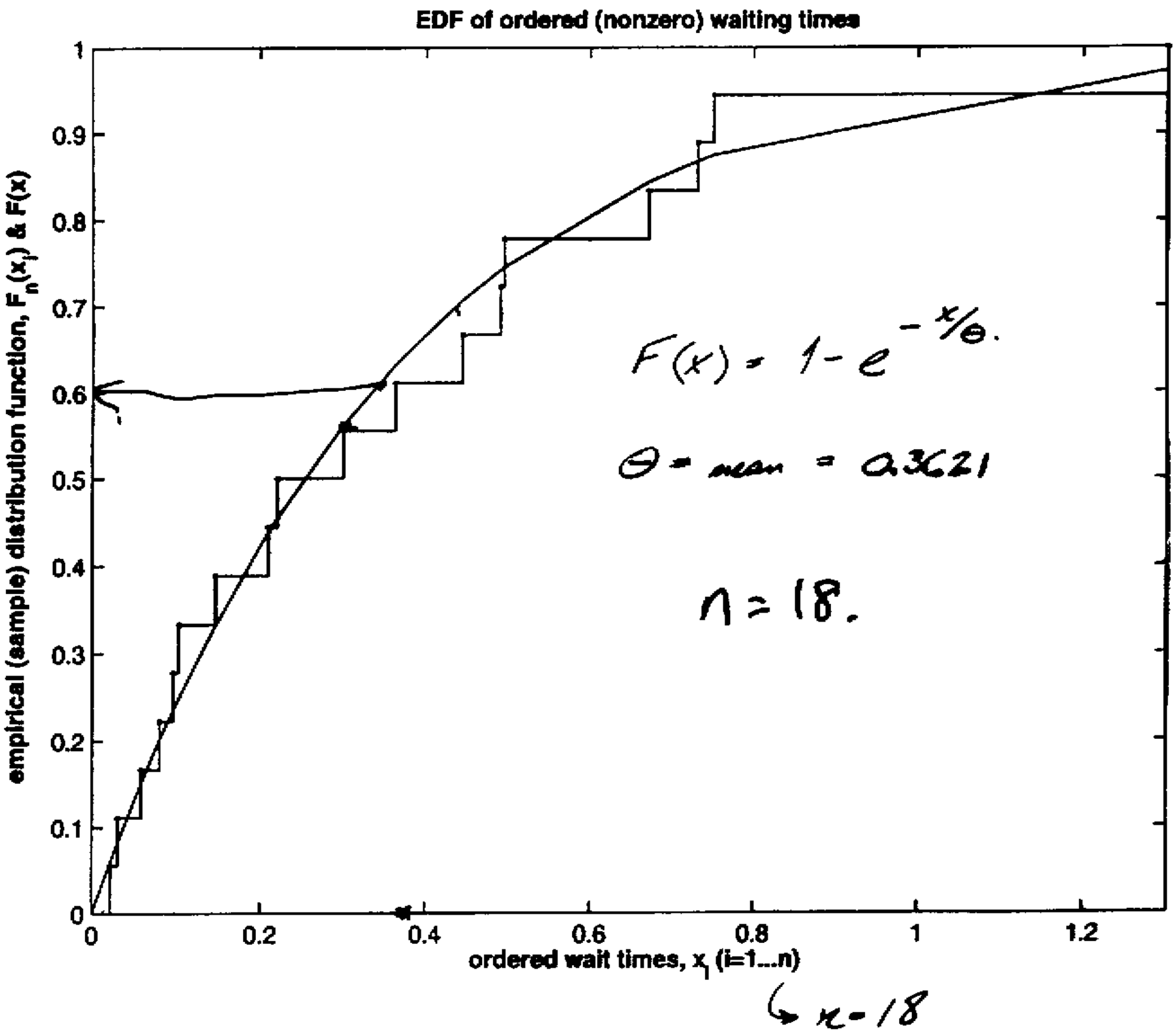
LECTURE 9M
MURIKI
M.NCM 202

Sample 1.



Ex 1. 1.


EDF of ordered (nonzero) waiting times
empirical (sample) distribution function, $F_n(x_i)$ & $F(x)$
ordered wait times, x_i (i=1...n)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
$F(x) = 1 - e^{-x/\theta}$
θ = mean = 0.3621
n = 18.
x = 18

1. EDF TESTS FOR DISTRIBUTIONS

1. Given random sample $x_1, x_2, \cdots, x_n$, to test the parent distribution is $F(x)$.

2. Empirical (Sample) Distribution Function

$$F_n(x) = \frac{\text{no. of } x_i \leq x}{n}$$

3. EDF tests compare $F_n(x)$ with $F(x)$

for exponential dist. $F(x) = 1 - e^{-\mu x}$ (mean $= 1/\mu$)

4. Kolmogorov-Smirnov test statistic:

$$\sup_x |F_n(x) - F(x)|$$

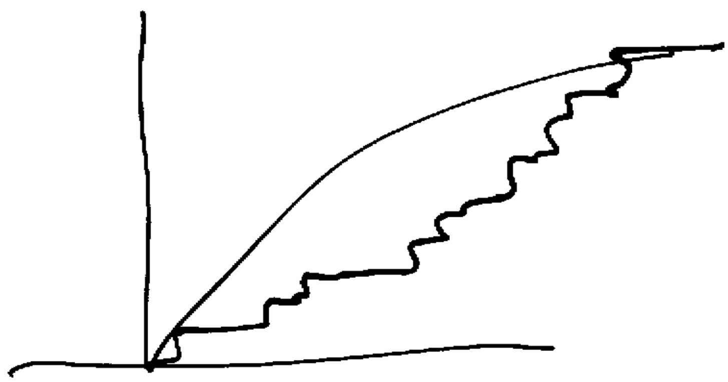
5. Cramér-von Mises family:

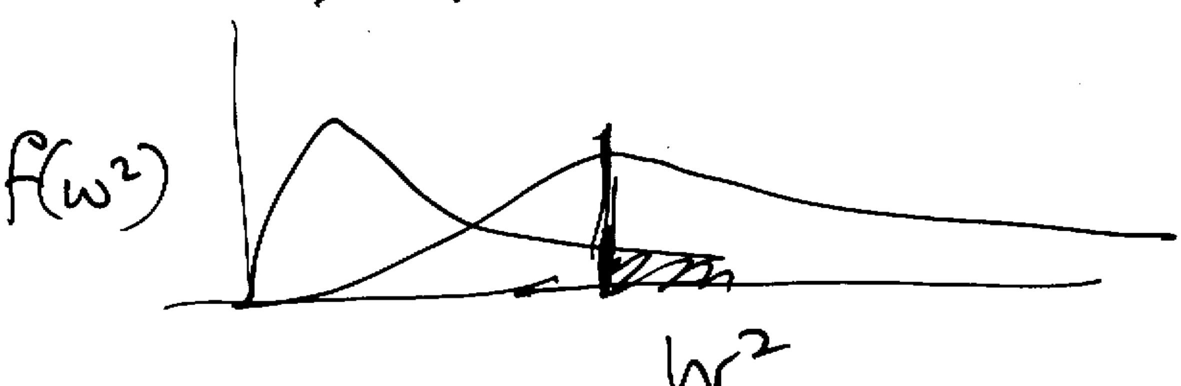
   $W_n^{*2} = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \Psi(x) dF(x)$

   when $\Psi(x) = 1$: Cramér-von Mises W_n^2

   when $\Psi(x) = [F(x)(1 - F(x))]^{-1}$: Anderson-Darling A_n^2

6. Much is known about these tests: tables exist and they have good power.





Actual Practice

1. Transform your x-values by:

$$z_i = F(x_i) = 1 - \exp\{-x_i/\theta\}$$

with θ known, or

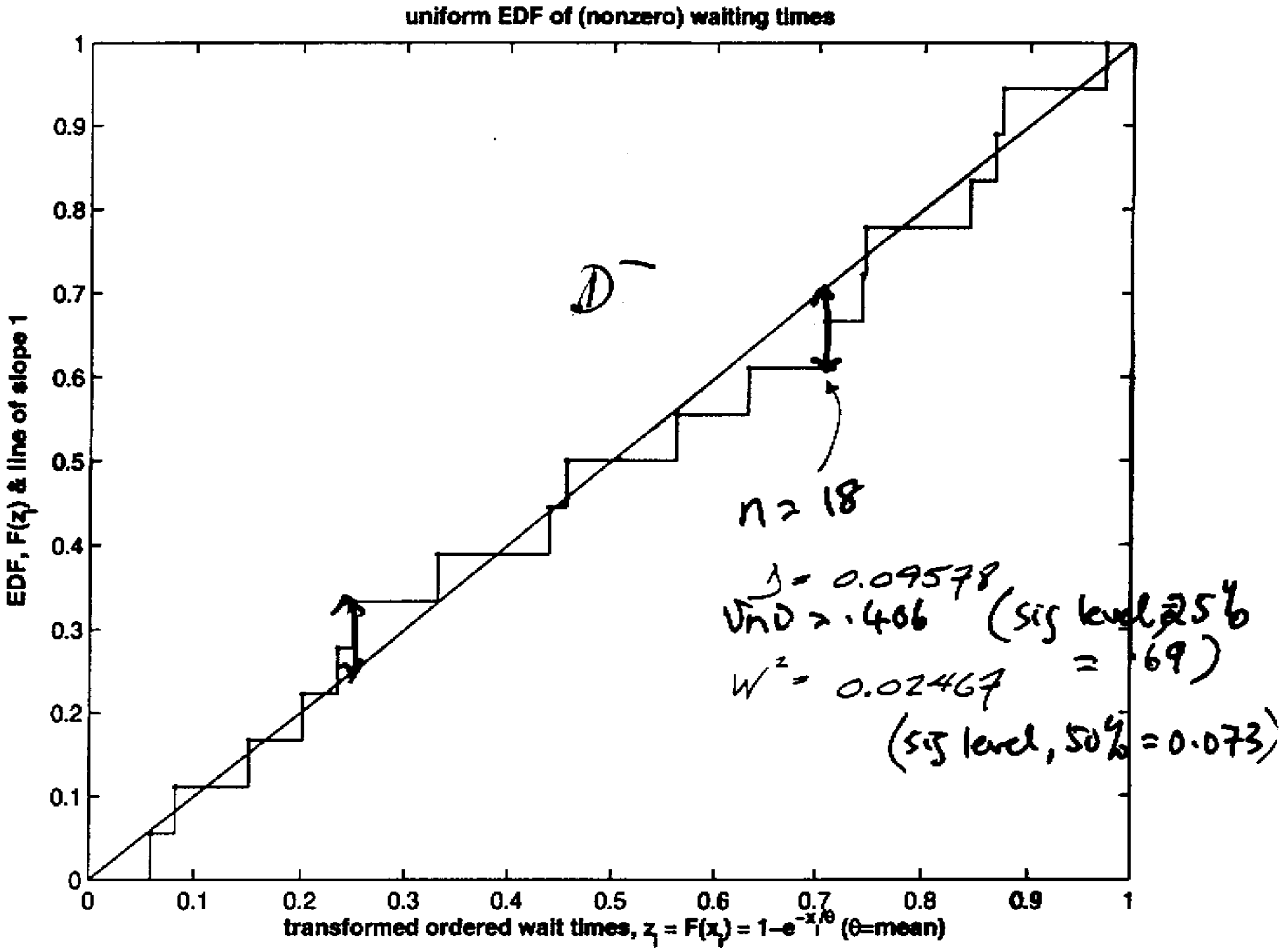
$$z_i = 1 - \exp\{-x_i/\overline{x}\}$$

with θ unknown.

2. The z_i will be between 0 and 1.

3. Make the EDF plot of the order statistics $z_{(i)}$.

Ex 1. 2


uniform EDF of (nonzero) waiting times
EDF, F(z) & line of slope 1
transformed ordered wait times, $z_i = F(x_i) = 1-e^{-x_i/\theta}$ (θ=mean)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
D^-
n = 18
D = 0.09578
$\sqrt{n}D$ = .406 (sig level 25% = .69)
W^2 = 0.02467
(sig level, 50% = 0.073)

4. Calculate the statistics by

$$D^{\mp} = \max_i \left| z_{(i)} - \frac{i-1}{n} \right|$$

$$D^{+} = \max_i \left| \frac{i}{n} - z_{(i)} \right|$$

then $$D = \max\left(D^{+}, D^{-}\right)$$

and

$$W^2 = \sum_{i=1}^{n} \left(z_{(i)} - \frac{2i-1}{2n} \right)^2 + 1/(12n)$$

5

$F(x) = 1 - \exp[-x]$

Correlation Tests

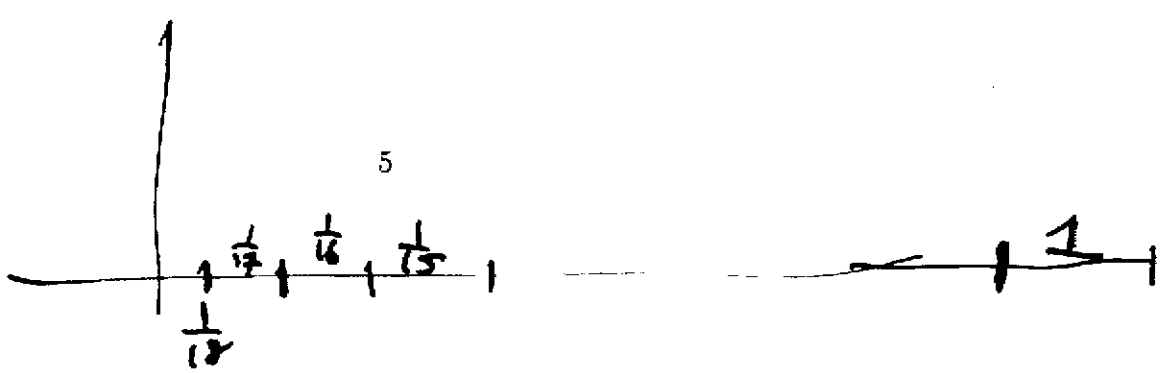
1. The expected values of order statistics from Exp dist^n with mean 1

$$\omega_{(1)}, \omega_{(2)}, \cdots, \omega_{(n)}$$

from an Exp(1) distribution are

$$m_1 = \frac{1}{n} = E(\omega_{(1)})$$
$$m_2 = \frac{1}{n} + \frac{1}{n-1}, = E(\omega_{(2)})$$
$$m_3 = = \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2}$$
$$\vdots$$

2. If your sample is from Exp(θ), the expected values of the order statistics will be $\theta \cdot m_1, \theta \cdot m_2, \cdots$ etc.



3. So: Plot your order statistics

$$\cancel{X}_{(1)}, \cancel{X}_{(2)}, \cdots, \cancel{X}_{(n)}$$

x x x

against

$$m_1, m_2, \cdots, m_n$$

The plot should look like a straight line through origin, slope θ.

4. Measure of "straight line fit": correlation coefficient

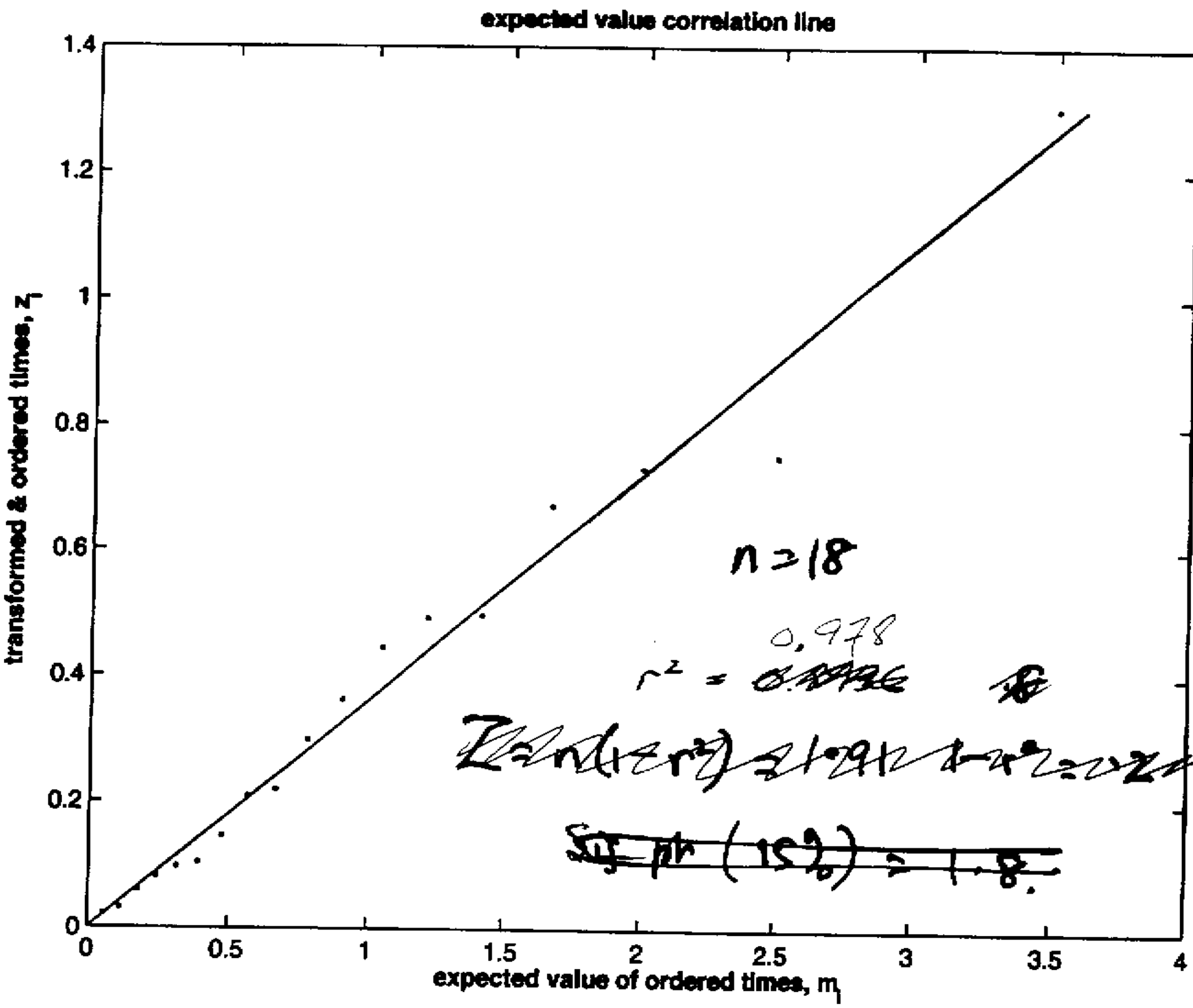
$$r = \frac{\Sigma_{i=1}^{n}(m_i - \overline{m})(x_{(i)} - \overline{x})}{\sqrt{\Sigma_{i=1}^{n}(m_i - \overline{m})^2 \, \Sigma_{i=1}^{n}(x_{(i)} - \overline{x})^2}}$$

PAGE 154 of text.

5. Actual test statistic $Z = n(1 - r^2)$ because $r^2 \to 1$ as $n \to \infty$ when the null hypothesis is true.

6. Tables in Stephens (1986b).

Ex 1. 3



$n = 18$

$r^2 = \cancel{0.996}$ 0.978

~~$Z = n(1-r^2) = 1.9$~~ [illegible]

~~Sig pt (15%) = 1.8.~~

Z