Name: _____   **Student Number**: _____

STAT 380: Spring 2018

Midterm Examination # 1

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Instructions: This is a closed book exam. You are permitted to use 2 sheets of notes, machine-written or hand-written. You may use both sides of the sheets and I place no limits on font size. Calculators are not permitted nor are any other electronic aids. The exam is out of 30. Please put your name and student number on the front page and on the grade sheet page at the end. You should have 10 pages; this cover page, 6 pages of questions, 2 pages of extra space, and the last page is a grade sheet. I will be marking for clarity of explanation as well as correctness. Without a clear explanation you should not expect to get more than half marks.

1. A Markov Chain has state space $\{1, 2, 3, 4, 5, 6, 7\}$ and transition matrix

$$
\begin{bmatrix}
\frac{1}{8} & 0 & \frac{3}{4} & 0 & 0 & & 0 \\[2mm]
0 & 0 & 0 & & 0 & \frac{1}{4} & 0 \\[2mm]
\frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\[2mm]
0 & 0 & 0 & 0 & \frac{1}{2} & & 0 \\[2mm]
0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\[2mm]
0 & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 \\[2mm]
\frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3}
\end{bmatrix}
$$

Fill in the 3 missing numbers in the matrix. Then identify all the communicating classes, say which states are transient, and give the period of each state. [8 marks]

2. Players A and B alternate throwing darts at a balloon, trying to break the balloon. Player A starts and has chance p_A of breaking the balloon on that throw. If Player A doesn't break the balloon then Player B throws and has chance p_B of breaking the balloon. They go on like this alternating with the chance of breaking the balloon staying at p_A for player A and p_B for player B every time. What is the chance that player A eventually breaks the balloon? [6 marks]

3. I have two routes to work. Route A costs me about $5 to drive and is busy about 2/3 of the time. Route B costs about $8 to drive and is busy about 1/3 of the time. If I drive a route one day and find it busy I switch to the other route for the next day. Describe a Markov Chain for this model and specify the transition matrix. [4 marks]

4. If I start out driving on route A what is the chance that I drive route B two days later? [4 marks]

5. In the long run how often do I drive route A? [4 marks]

6. In the long run what does it cost me to drive to work per day? [4 marks]

Extra space

Extra space

Grade Sheet

1		8	2		6
3a		4	3b		4
3c		4	3d		4

Total		**30**