Name: ____________________ **Student Number**: ____________________

STAT 380: Spring 2016

Midterm Examination

Richard Lockhart 17 February 2016

Instructions: This is a closed book exam. You are permitted to use 2 sheets of notes, machine-written or hand-written. You may use both sides of the sheets and I place no limits on font size. Calculators are not permitted nor are any other electronic aids. The exam is out of 25. Please put your name on each page. You should have 8 pages; the first page is a grade sheet and the last is extra space. I will be marking for clarity of explanation as well as correctness. Without a clear explanation you should not expect to get more than half marks.

1		5						
2a		4	2b		3	2c		3
2d		4	2e		3	2f		3

Total		**25**

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1. A Markov Chain has state space $\{1, 2, 3, 4, 5, 6\}$ and transition matrix

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Identify all the communicating classes, say whether or not each is transient, and give the period of each state. [5 marks]

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2. There is an experimental design strategy called play-the-winner. A simplified version goes like this. Imagine two players, A and B, play a game. On each turn of the game one player 'serves' and can either score a point on that turn or not. If the player who served scores a point she serves again. If not, no point is scored and the other player begins to serve. Suppose that when A serves she scores a point with probability p_A and that when B serves she scores a point with probability p_B.

   (a) Define a suitable Markov chain to analyse this system. [4 marks]

   (b) Write out the transition matrix of the resulting Markov Chain. [3 marks]

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(c) Assume that a fair coin is tossed to see who serves first. What is the probability that A serves on the third turn? [3 marks]

(d) In the long run on what fraction of the turns does A serve? [4 marks]

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(e) Also in the long run what is the average number of points scored per turn (by either player)? [3 marks]

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(f) Let M_n be the number of times that the serve changes in the first n turns. (For clarity if the player serving on turn n does not score a point that counts as a change of serve in the first n trials.) Let $\mu_{A,n} = \mathrm{E}(M_n)$ given that A serves first. Let $\mu_{B,n}$ be the same expected value given that B serves first. Use first step analysis to derive equations for $\mu_{A,n}$ and $\mu_{B,n}$ in terms of $\mu_{A,n-1}$ and $\mu_{B,n-1}$. Find the values for $n = 1$. Do not solve the set of equations. [3 marks]

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Extra space