STAT 380

Midterm Examination

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Instructions: This is an open book exam. You may use notes, books and a calculator. The exam is out of 25. Questions 1 and 3 are worth 5 marks each. Each of the 5 parts of question 2 is worth 3 marks. I will be marking for clarity of explanation as well as correctness. Without a clear explanation you should not expect to get more than half marks.

1. A Markov Chain has state space $\{1, 2, 3, 4\}$ and transition matrix

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

Identify all the communicating classes and say whether or not each is transient. [5 marks]

2. Each day I get a random number of pieces of voice mail. I deal with, and delete, a random number of pieces of voice mail. When the mail box gets full any further messages received are lost. Here is a simplified model. Assume that my mail box can hold two messages. Each morning I get either 1 message or 0 messages. Each evening I delete either 1 message or 0 messages. The probability that I get 1 message is p regardless of what has happened in the past. The probability that I delete 1 message is π if there is a message to delete. Let X_n be the number of messages on day n in the morning *before* any message arrives. Assume that $X_0 = 0$; day 0 is the starting day.

   (a) Write out the transition matrix of the resulting Markov Chain. [3 marks]

   (b) Suppose $p = \pi$ and compute the probability that the mailbox is empty in the morning on day 2 (before the arrival of any mail). [3 marks]

(c) Again supposing $p = \pi$, show that in the long run the fraction of days on which I lose an email is $p/3$. [3 marks]

[NOTE: This question was wrong; you should just compute the correct fraction.]

(d) Again supposing $p = \pi$, in the long run what fraction of my e-mail will be lost? [3 marks]

(e) Let T be the number of days till I lose my first e-mail and let m_k be the expected value of T given that I start in state k (for $k = 0, 1, 2$). Use first step analysis to derive a set of equations which would be solved to compute the m_k. DO NOT SOLVE THE EQUATIONS. [3 marks]

3. Imagine that buses arrive at a particular stop according to a Poisson process with rate 2 per hour. I start waiting at the stop at 1:00. Given that the second bus arrives between 2:00 and 3:00 what is the probability that the first bus arrived before 2:00? You may leave your answer as a formula—do not bother plugging the numbers into the calculator. [5 marks]

Extra space