Name: _____   Student Number _____

STAT 380

Final Examination

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Instructions: This is an open book exam. You may use notes, books and a calculator. The exam is out of 50. I will be marking for clarity of explanation as well as correctness.

1a	1b	1c	1d	2a	2b	2c	3a	3b	4	Total

1. Each week I rent a video: either a drama (D), action flick (A) or comedy (C). If I like a movie then I rent the same kind next week otherwise I toss a coin to decide which of the other two types to rent. I like about 1/4 of the dramas, 1/2 of the comedies and 3/4 of the action flicks.

   Your answer to the following questions must set down clearly what assumptions you are making to answer the question. Answers with inadequate explanations will get low marks.

   [15 marks: 3 marks for part a, 4 marks for each other part — 1 for the answer and 3 for the explanation.]

   (a) Describe clearly a model for this system.

(b) In the long run on what fraction of weeks do I rent each kind of movie?

(c) In the long run what fraction of the movies I rent do I like?

(d) Suppose that the first movie I rent is a drama. What is the expected time till I watch my first comedy?

Extra space for Q1

2. A sample begins with n atoms of A. Independently, each atom of type A waits an exponentially distributed amount of time with rate λ then emits a particle to become an atom of B. In turn each atom of B waits an exponentially distributed amount of time with rate μ before turning into an atom of C (atoms of C never change).

(a) What is the density of the time at which the last atom of A changes to B? [2 marks]

(b) Let $A(t)$ be the number of atoms of A at time t and $B(t)$ be the number of atoms of B at time t. Then $X(t) = (A(t), B(t))$ is a continuous time Markov Chain. For $n = 6$ one possible state of X is $(3,3)$. Identify those states to which transitions from this state are possible and give the corresponding entries in the matrix $\mathbf{R} = \mathbf{P}'(0)$ and in the transition matrix $\mathbf{P}$, of the skeleton chain. [8 marks]

(c) Now let $n = 2$. Use the skeleton chain to compute the probability that the first atom of A to become an atom of B goes on to become an atom of C before the other atom transforms to B. [6 marks: 3 for giving the complete matrix $\mathbf{P}$, 3 for development of answer.]

3. An insurance company receives claims at the times of a Poisson Process with rate λ. Each claim has an exponential distribution with rate μ independently of everything else.

(a) Suppose $\lambda = 5$ and that $\mu = 1/4$. During a certain period of two time units the company receives 7 claims valued over 8. Given this information what is the probability it also receives exactly 9 claims valued under 8? [Answer: 1 mark, explanation 5 marks.]

(b) In another period of 3 time units a total of 7 claims are received. What is the probability that all are valued under 8? [Answer: 1 mark, explanation 4 marks.]

4. Suppose that $P(t)$ is the price of a certain stock at time t measured in years. Assume that

$$\log_{10}\{P(t)/10\}$$

is a standard Brownian motion. (If you can't cope with base 10 logs you may do the question assuming instead the same thing but with ordinary logarithms in the formula.) What is the probability that during the first year the stock's price goes higher than 1000 and then ends up the year below 10?   [8 marks: 2 for answer and 6 for explanation]

Extra Space