STAT 380

Midterm Examination 2

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Instructions: This is an open book exam. You may use notes, books and a
   calculator. The exam is out of 20. Each part of question 2 is worth 4 marks.
   Question 1b) is worth 5 and 1a) is worth 3. Be as clear as possible about
   what you are doing but if you can only give an intuitive explanation for your
   calculations please do give it.

1. An experiment is performed to compare two brands of light bulbs. At
   time 0 one light bulb of brand X and one of brand Y are turned on.
   Let X and Y be the corresponding lifetimes. Assume that the light bulbs
   have exponentially distributed lifetimes and that the mean lifetime is 1000
   hours for brand X and 2000 hours for brand Y. Let A be the event that
   the brand X light bulb lasts at least twice as long as the brand Y bulb.

   (a) Show that $P(A|Y) = \exp(-Y/500)$.

   (b) Use the results of a) to compute $P(A)$.
NOTE: If you cannot do b) using a) but can do it some other way you can still earn up to 4/5 for b).

2. Now assume that at time 0 one brand X light bulb and 3 brand Y light
   bulbs were started and that whenever a light bulb burns out it is replaced
   instantly by a new bulb of the same kind. The experiment continues for
   2000 hours.

   (a) What is the probability that exactly 2 brand X bulbs have been
       replaced (either a number or a formula with numbers plugged in will
       do)?

   (b) If $N(s)$ is the number of light bulbs of either kind replaced by time
       s what is the distribution of $N(1000)$?

   (c) Given that 5 light bulbs of either kind have been replaced at the end
       of the experiment (i.e. at 2000 hours) what is the probability that
       exactly 2 of them were of brand X?