Theory and Applications of Complex Networks: Advances and Challenges

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***Abstract*—Over the last decade, complex networks have emerged to be a promising research field in the area of circuits and systems. This mini-review paper introduces the special session that deals with theory and applications of complex networks and provides brief review of their advances and challenges. The paper further promotes some important research topics in the field with emphasis on the multidisciplinary research interests.**

I. Introduction

Complex networks are ubiquitous, as shown by many real-world examples [1]–[5]. They have non-trivial topological characteristics that traditional networks do not possess [1], [5]. Typical examples include the Internet, world wide web, wireless communication networks, power grids, and social, economical and biological networks. The study of complex networks has emerged to be a promising research field stimulated by the empirical investigations of genuine complex structures and configurations of large-scale engineering systems such as the Internet [6] and power grids [7].

In the late 1950s, Erdös and Rényi laid the foundation of the now-classic random graph theory [8]. The theory and model have prompted extensive research investigations over more than five decades. Even though many real-world complex networks are neither completely regular nor completely random [9], until the last decade using the random graph model was the only rigorous and reliable approach to the analysis of complex networks. The widespread use of supercomputers, the availability of large databases capturing various structures of real-world networks, and the discoveries of small-world [10], [11] and scale-free networks [12] have changed the traditional views of complex networks [13], [14].

Distinct from the uniform distributions of random graphs and small-world models, the degree distributions of scale-free networks proved to be heterogeneous and follow a power-law. It has been increasingly recognized over the last decade that network science is highly relevant to practical engineering. In the current information-rich era, technology supports a variety of richly connected networks thus making our world highly interconnected via complex networks. Information technology also generates big data and makes the world even more complex and fragile. Over the last decade, numerous new tools and techniques have been developed to deal with various large-scale complex networks. The investigation of complex networks has become an active research field in the area of circuits and systems and is attracting increasing attention and interest from the engineering community. In addition to theoretical studies, a number of new practical applications have been identified.

In this paper, we briefly review the main advances and challenges in the area of complex networks by presenting some state-of-the-art findings. The paper is organized as follows. Brief summaries of recent advances in the area of theory and applications of complex networks are given in Section II and Section III, respectively. Some challenges and opportunities are further discussed in Section IV. Concluding remarks are given in Section V.

II. Recent Theoretical Advances

Numerous theoretical advances related to complex networks in the field of circuits and systems have been reported over the last decade. An exhaustive literature overview of these theoretical advances is beyond the scope of this paper. Hence, only some recent advances in fields related to our own research interests are presented, such as the network controllability, observability, and pinning control.

A. Controllability and Observability of Complex Networks

Controllability and observability are two fundamental concepts in modern control theory. Controllability provides basic mathematical tools for guiding engineered and natural systems towards a desired state while observability determines the system's initial state that generated the system evolution. A theoretical framework for controlling and observing large-scale complex networks, especially in case of directed and nonlinear structures, remains an open research problem.

A dynamical system is controllable if it could be driven from any initial state to any desired final state within finite time via a suitable control input. The notion of controllability has been discussed also for directed nonlinear networks [15]–[17]. Unlike a linear dynamical system, it is difficult to provide a precise mathematical definition with rigorous criteria for the controllability of large-scale directed and nonlinear networks. The essential difficulty is attributed to at least two independent factors [17]: i) interactions among the components of the network architecture and ii) dynamical rules that capture the time-dependent interactions among the components.

Over the last decade, efforts have been made to control complex networks such as a small network of biological circuits [18], large synchronized networks [19]–[21], and biological networks [22]. These are only a few examples within the circuits and systems context.

To investigate the controllability of a complex network, a typical approach is to apply pinning control [23], [24]. A controlled undirected network may be described as:

$$\frac{dx_i}{dt} = f(x_i) - \sigma \sum_{j=1}^{N} a_{ij} h(x_j) - \sigma \sum_{k=1}^{n} \delta(i-k) \kappa_i (x_i - s), \quad (1)$$

where $i = 1, 2, .., N$. In this model, $x_i \in R^n$ is the state vector of vertex i, $f(\cdot)$ is a nonlinear function satisfying the Lipschitz condition, $A = [a_{ij}]$ is the coupling matrix, $h(\cdot)$ is the component-coupled function, $\delta(i-k)$ is the delta function equals 1 if the kth vertex is being controlled (being "pined") but otherwise equals 0, and $u_i = \kappa_i(x_i - s)$ is a controller with a constant control gain κ_i and target state vector $s \in R^n$, $i = 1, 2, ..., N$. Using the master stability function approach [25], [26], the concept of pinning-controllability of the undirected network (1) is introduced and analyzed [24], giving controllability criteria in terms of the spectral properties of an extended network topology.

Most real-world complex networks are directed. Controlling a directed network is substantially different from controlling an undirected one. As an example, consider two connected vertices A and B where each vertex is a dynamical system to be controlled. If the connection is undirected, then a controller may be placed in either A or B. However, if the connection is directed from A pointing to B, then the controller should be placed in A rather that B since otherwise vertex A will not be affected by the controller. This observation led the authors [17] to introduce the concepts of driver (vertex A) and redundant vertex (vertex B). Consequently, a main task is to identify a minimum number of drivers in a given directed network so that the entire network is controllable in the conventional sense of being able to reach any target state from any initial state of the entire network under the designed controllers.

Very few, if any, results have been reported regarding the observability of complex networks. For linear systems, observability is a dual concept to controllability and every criterion regarding the controllability may be directly "translated" to one for the observability. For nonlinear systems, they are unrelated and, hence, should be considered separately using different theory and methods.

In the past, structural controllability and observability of directly-connected networks of dynamical systems have been studied [15], [16]. They provide useful ideas and tools for the current investigations of controllability and observability of directed networks of dynamical systems.

B. *Pinning Control of Complex Networks*

Most real-world complex networks consist of a large number of dynamical system vertices. Hence, it is impossible to control every vertex of such a network even if the number of vertices is not large. An intuitive approach to reducing the number of controllers is to control only a small fraction of the network vertices. This is referred to as "pinning control" for complex networks.

An early attempt of the pinning control strategy was to suppress spatiotemporal chaos via pinning control based on numerical experiments [27], [28]. The approach was made rigorous with a systematic formulation and design approach for complex networks including small-world and scale-free networks [5], [29]. The controllability of general complex networks under pinning control may also be investigated using an extended system approach [24].

Even though numerous results were reported over the last decade regarding the pinning control strategy for complex networks, the question which vertex or vertices to pin in order to realize a specific control objective, such as network synchronization or consensus, remains unanswered. An effort was made [30] to explore two fundamental issues in pinning control of complex networks with a fixed network structure: i) the number of vertices that should be pinned in a network with fixed coupling strength in order to achieve network synchronization; and ii) the coupling strength that should be applied to a fixed number of pined vertices to realize network synchronization. Estimation of the number of pined vertices and the magnitudes of the coupling strengths were also given.

Another effort was made [31] to further investigate three challenging problems in pinning control of complex networks using a basic mathematical model: i) What type of pinning schemes may be chosen for a given complex network to realize synchronization? ii) What type of controllers may be designed to ensure that the network reaches synchronization? iii) How large should the coupling strength be used in a given complex network to achieve synchronization? Some weak conditions were derived for the controlled undirected network to reach global synchronization [31].

III. Recent Applications

Thanks to the availability of super-computing and big data emanating from large-scale real-world complex networks, it is possible to obtain the underlying description of various complex networks and infer their structures and functions. Over the last decade, numerous applications based on new understandings of complex networks have been reported [32].

Due to the space limitation, only two recent applications of complex networks are considered: systems biology and the Internet.

A. *Applications in Systems Biology*

Most real-world biological networks have a large number of network nodes and complex topological structures. Systems biology is the study of systems of biological components, which may be molecules, cells, organisms, or entire species.

Over the last two decades, the rapid advances of complex networks have greatly promoted the development of systems biology.

Many biological networks consist of simple building blocks called network motifs, which were uncovered to be one of the shared global statistical features in ecology, neurobiology, biochemistry, and bioengineering [22]. The feed-forward loops (FFLs) are typical network motifs. Over the last decade, analyzing their structures, functions, and noise characteristics have received increasing attention [33]–[35].

It may be very difficult to obtain significant insights into biological functions by simply considering the connection architecture of single gene network or by its decomposition into simple structural motifs. Hence, network motif structures cannot completely determine biological functions since these motifs occur less frequently in the complex biochemical networks [22]. Furthermore, most network motifs are often embedded in various large biological systems that may have different inputs to motifs. For example, the bi-fan motif may exhibit a wide range of dynamical responses depending on the inputs [33]–[35].

It has been accepted that there are various inherent relationships between network structure and its dynamical function. The investigation of motif structures is the first step in revealing the inherent mechanism how small network motifs may form the overall complex networks. Recent explorations involve the oscillatory mechanisms in a merged artificial genetic regulatory network [35] and the global relative parameter sensitivities of FFLs in genetic networks modeled by Hill kinetics [22].

B. Applications in Engineering Networks

Most large-scale engineering networks are typically complex, such as the Internet. The theory of complex networks proved useful in improving the structures and functions of various engineering networks.

Analyzing the Internet topology using randomly generated graphs, where routers are represented by vertices and transmission lines by edges, has been widely replaced by mining data that capture information about Internet Autonomous Systems (ASes)and by exploring properties of associated graphs on the AS-level. The Route Views [36] and RIPE [37] datasets collected from Border Gateway Protocols (BGP) routing tables have been extensively used by the research community [38]–[40]. The discovery of power-laws and spectral properties of the Internet topology indicated a complex underlying network infrastructure.

Analysis of the collected datasets indicated that the Internet topology is characterized by the presence of various power-laws observed when considering a node degree vs. node rank, a node degree frequency vs. degree, and a number of nodes within a number of hops vs. number of hops [38], [40]. The power-law connectivity distribution of the node degree k of the network implies that the probability distribution function is of the form $P(k) \propto k^{-\gamma}$. The smaller the parameter γ, the more the network becomes heterogeneous in its connectivity distribution and, accordingly, the average network distance decreases. A more complete AS-level representation of the Internet topology revealed that these extended maps have heavy tailed or highly variable degree distributions and only the distribution tails have the power-law property [39], [41]. It has been observed that the power-law exponents associated with Internet topology have not substantially changed over the years in spite of the Internet exponential growth [42], [43]. Power-laws also appear in the eigenvalues of the adjacency matrix and the normalized Laplacian matrix vs. the order of the eigenvalues. They also show invariance regardless of the growth of the Internet.

While various power-law exponents associated with the Internet topology have remained similar over the years, indicating that the power-laws do not capture every property of a graph and are only one measure used to characterize the Internet, spectral analysis of both the adjacency matrix and the normalized Laplacian matrix of the associated graphs revealed new historical trends in the clustering of AS nodes and their connectivity [6]. The eigenvectors corresponding to the largest eigenvalues of the normalized Laplacian matrix have been used to identify clusters of AS nodes with certain characteristics [42]. Spectral analysis was employed to analyze the Route Views and RIPE datasets in order to find distinct clustering features of the Internet AS nodes [44]. For example, the connectivity graphs of these datasets indicate visible changes in the clustering of AS nodes and the AS connectivity over the period of five years [43]. Clusters of AS nodes may be also identified based on the eigenvectors corresponding to the second smallest and the largest eigenvalue of the adjacency matrix and the normalized Laplacian matrix [45]. The connectivity and clustering properties of the Internet topology may be further analyzed by examining element values of the corresponding eigenvectors.

IV. Opportunities and Challenges

Over the last few decades, complex networks such as the Internet have changed the way we live, work, and play. They have also changed the notions of democracy, education, healthcare, entertainment, and commerce [14]. While complex networks bring various opportunities, they also bring challenges such as a large volume of redundant information and various security concerns. To meet such challenges, new mathematical tools and frameworks to model, analyze, understand, and predict the structures and functions of various complex networks are needed in hope to make them more secure, accessible, predictable, and reliable.

Many issues in the field of network engineering deal with the design, utilization, control, and protection of complex networks and offer a large number of opportunities for more thorough investigations. For example, the complete controllability and observability of large-scale directed and weighted complex networks of nonlinear dynamical systems of higher dimensions open many questions to researchers in various fields of circuits and systems. Although they are central issues in many interesting and important real-world applications that involve complex networks, little is known about how to address these issues because of the absence of general theory and effective techniques for quantitative investigation of various complex networks.

V. Conclusions

In this paper, we have briefly reviewed the main advances and challenges in the theory and applications of complex networks by presenting some current state-of-the-art views from the perspective of our own research interests. The aim of this mini-review is to further promote this important research topic by emphasizing the multidisciplinary research interests within the circuits and systems areas.

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