Discontinuity-induced bifurcations in TCP/RED communication algorithms

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Roadmap

- Introduction
- TCP/RED congestion control algorithms: an overview
- Discrete-time dynamical models of TCP Reno with RED:
  - modeling assumptions
  - S-TCP/RED models with:
    - one state variable
    - two state variables
  - model validation and modifications
  - comparison of TCP/RED models
- Bifurcation and chaos phenomena in TCP/RED:
  - discontinuity-induced bifurcations
- Conclusion and references

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Motivation

- Modeling TCP Reno with RED is important to:
  - examine the interactions between TCP and RED
  - understand and predict the dynamical network behavior
  - analyze the impact of system parameters
  - investigate bifurcations and complex behavior

TCP: Transmission Control Protocol

RED: Random Early Detection Gateways for Congestion Avoidance

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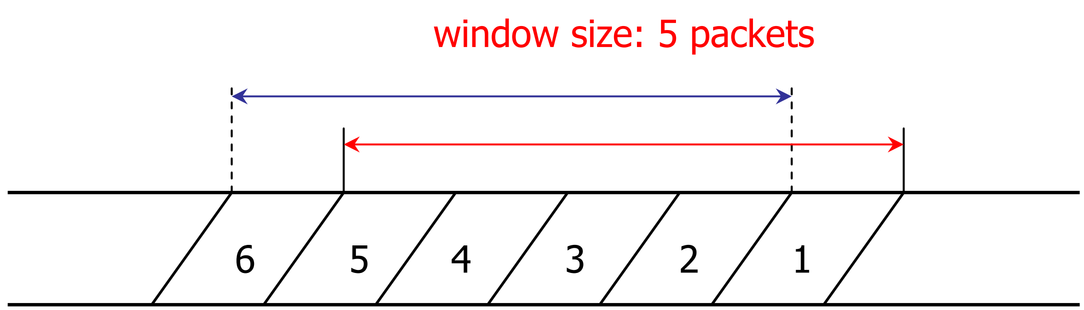
TCP

- TCP: Transmission Control Protocol
- Fourth layer of the OSI model
- Connection oriented, reliable, and byte-stream service
- Employs window based flow and congestion control algorithms

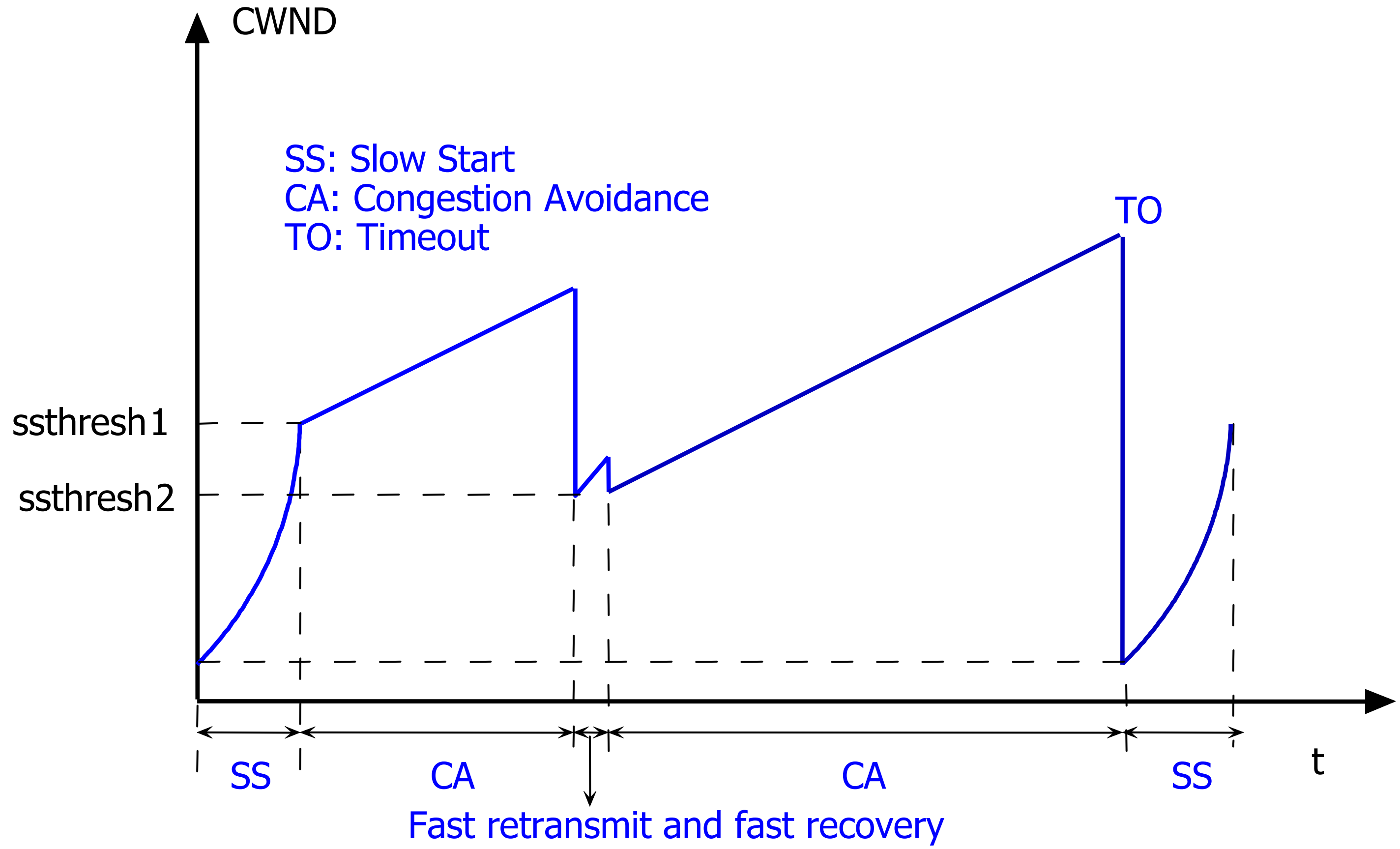


OSI: Open System Interconnection reference model

TCP

- Several flavors of TCP:
  - Tahoe: 4.3 BSD Tahoe (~ 1988)
    - slow start, congestion avoidance, and fast retransmit (RFC 793, RFC 2001)
  - Reno: 4.3 BSD Reno (~ 1990)
    - slow start, congestion avoidance, fast retransmit, and fast recovery (RFC 2001, RFC 2581)
  - NewReno (~ 1996)
    - new fast recovery algorithm (RFC 2582)
  - SACK (~ 1996, RFC 2018)

TCP Reno


CWND
SS: Slow Start
CA: Congestion Avoidance
TO: Timeout
TO
ssthresh1
ssthresh2
t
SS
CA
CA
SS
Fast retransmit and fast recovery

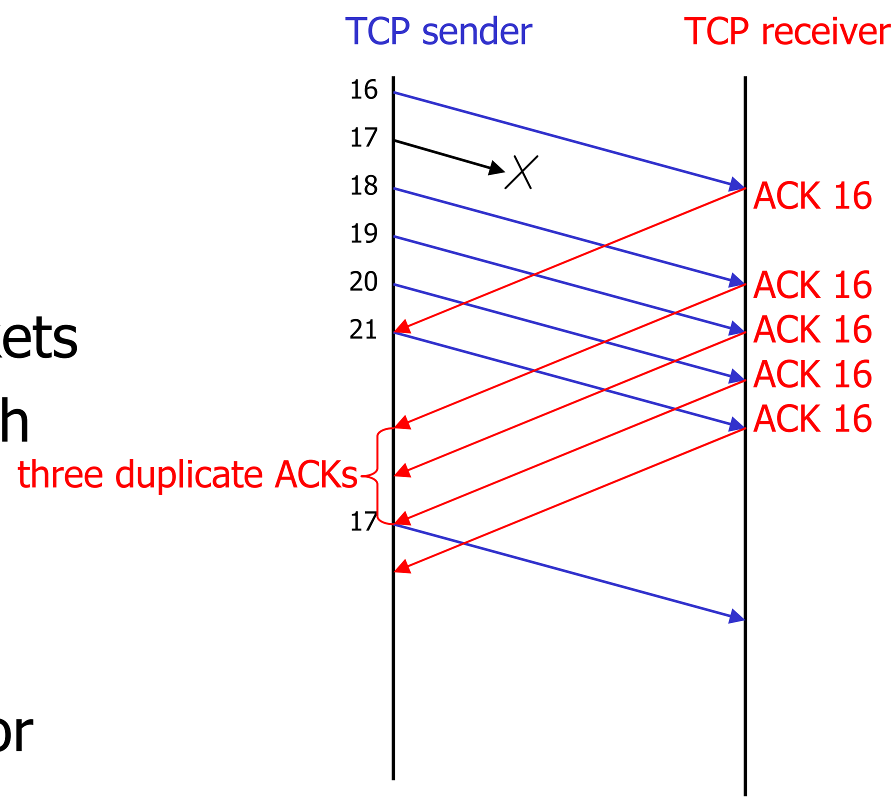
TCP Reno: slow start and congestion avoidance

- Slow start:
  - $cwnd = IW$ (1 or 2 packets)
  - when $cwnd < ssthresh$
    $cwnd = cwnd + 1$ for each received ACK
- Congestion avoidance:
  - when $cwnd > ssthresh$
    $cwnd = cwnd + 1/cwnd$ for each ACK

cwnd : congestion window size
IW : initial window size
ssthresh : slow start threshold
ACK : acknowledgement
RTT : round trip time

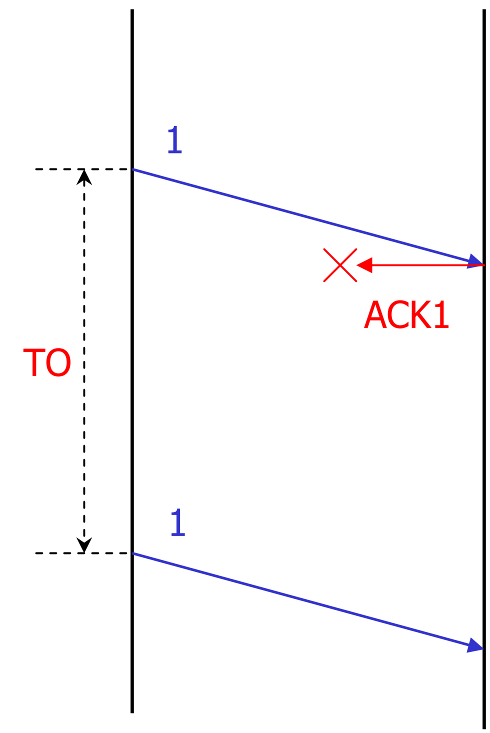
TCP Reno:
fast retransmit and fast recovery

- three duplicate *ACKs* are received
- retransmit the packet
- *ssthresh* = *cwnd*/2,
  cwnd = *ssthresh* + 3 packets
- *cwnd* = *cwnd* + 1, for each additional duplicate ACK
- transmit the new data, if *cwnd* allows
- *cwnd* = *ssthresh*, if ACK for new data is received



TCP Reno: timeout

- TCP maintains a retransmission timer
- The duration of the timer is called retransmission timeout
- Timeout occurs when the ACK for the delivered data is not received before the retransmission timer expires
- TCP sender retransmits the lost packet
- *ssthresh* = *cwnd*/2
  cwnd = 1 or 2 packets



AQM: Active Queue Management

- AQM (RFC 2309):
  - reduces bursty packet drops in routers
  - provides lower-delay interactive service
  - avoids the “lock-out” problem
  - reacts to the incipient congestion before buffers overflow
- AQM algorithms:
  - RED (RFC 2309)
  - ARED, CHOKe, BLUE, …

RED

- Random Early Detection Gateways for Congestion Avoidance
  - Proposed by S. Floyd and V. Jacobson, LBN, 1993:

    S. Floyd and V. Jacobson, “Random early detection gateways for congestion avoidance,” *IEEE/ACM Trans. Networking*, vol. 1, no. 4, pp. 397–413, Aug. 1993.
- Main concept:
  - drop packets before the queue becomes full

RED variables and parameters

- Main variables and parameters:
  - average queue size: $\bar{q}_{k+1}$
  - instantaneous queue size: q_{k+1}
  - drop probability: p_{k+1}
  - queue weight: w_q
  - maximum drop probability: $p_{\max}$
  - queue thresholds: $q_{\min}$ and $q_{\max}$

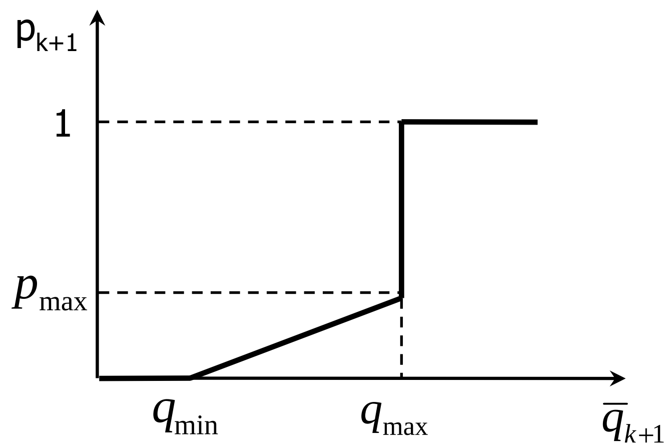
RED algorithm

Calculate:

- average queue size for each packet arrival

$$\overline{q}_{k+1} = (1 - w_q) \cdot \overline{q}_k + w_q \cdot q_{k+1}$$

- drop probability



RED algorithm: drop probability

- if $(q_{\min} < \bar{q} < q_{\max})$

$$p_b = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}} \qquad p_a = \frac{p_b}{1 - count \times p_b}$$

count: number of packets that arrived since the last packet drop

- else if $(\bar{q} > q_{\max})$

$$p_a = 1$$

- else $(\bar{q} < q_{\min})$

$$p_a = 0$$

- Mark or drop the arriving packet with probability p_a

RED algorithm: drop probability

- if$(q_{\min} < \bar{q}_{k+1} < q_{\max})$

$$p_{k+1} = \frac{\bar{q}_{k+1} - q_{\min}}{q_{\max} - q_{\min}} p_{\max}$$

- else if $(\bar{q}_{k+1} \geq q_{\max})$

$$p_{k+1} = 1$$

- else $(\bar{q}_{k+1} \leq q_{\min})$

$$p_{k+1} = 0$$

- mark or drop the arriving packet with probability p_{k+1}

Simulation tool: ns-2

- ns-2 is a discrete event network simulator
  http://www.isi.edu/nsnam/ns
- Supports simulation of TCP, routing, and multicast protocols over wired and wireless networks
- We used ns-2 to validate the proposed S-TCP/RED model

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Modeling methodology

- Categories of TCP models:
  - averaged and discrete-time models
  - short-lived and long-lived TCP connections
- S-TCP/RED models:
  - discrete-time model with a long-lived connection
- State variables:
  - window size (TCP)
  - average queue size (RED)

S-TCP/RED model

- Key properties of the proposed S-TCP/RED models:
  - slow start, congestion avoidance, fast retransmit, and fast recovery (simplified)
  - Timeout:

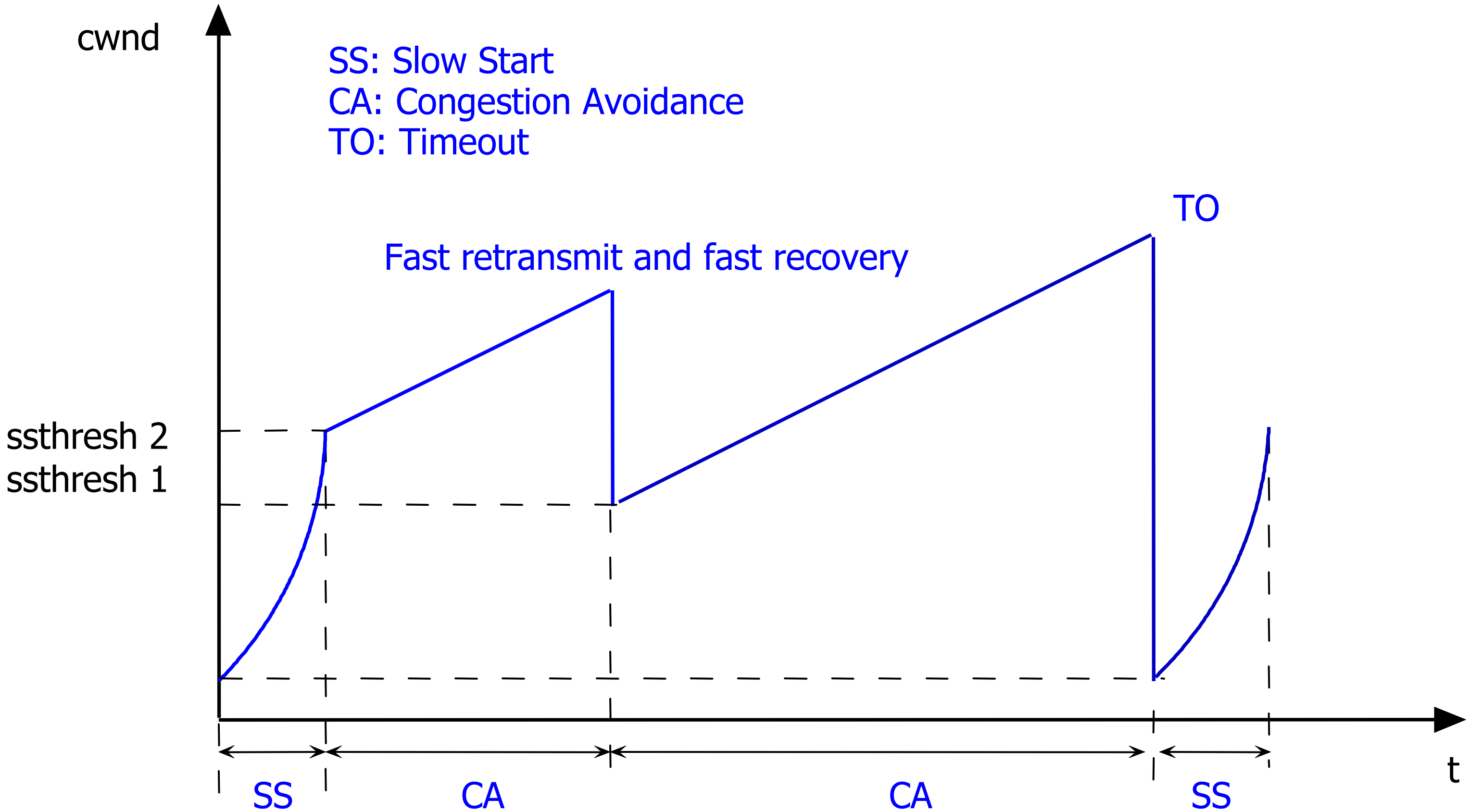
    J. Padhye, V. Firoiu, and D. F. Towsley, “Modeling TCP Reno performance: a simple model and its empirical validation,” *IEEE/ACM Trans. Networking*, vol. 8, no. 2, pp. 133–145, Apr. 2000.
  - Captures the basic RED algorithm

Assumptions

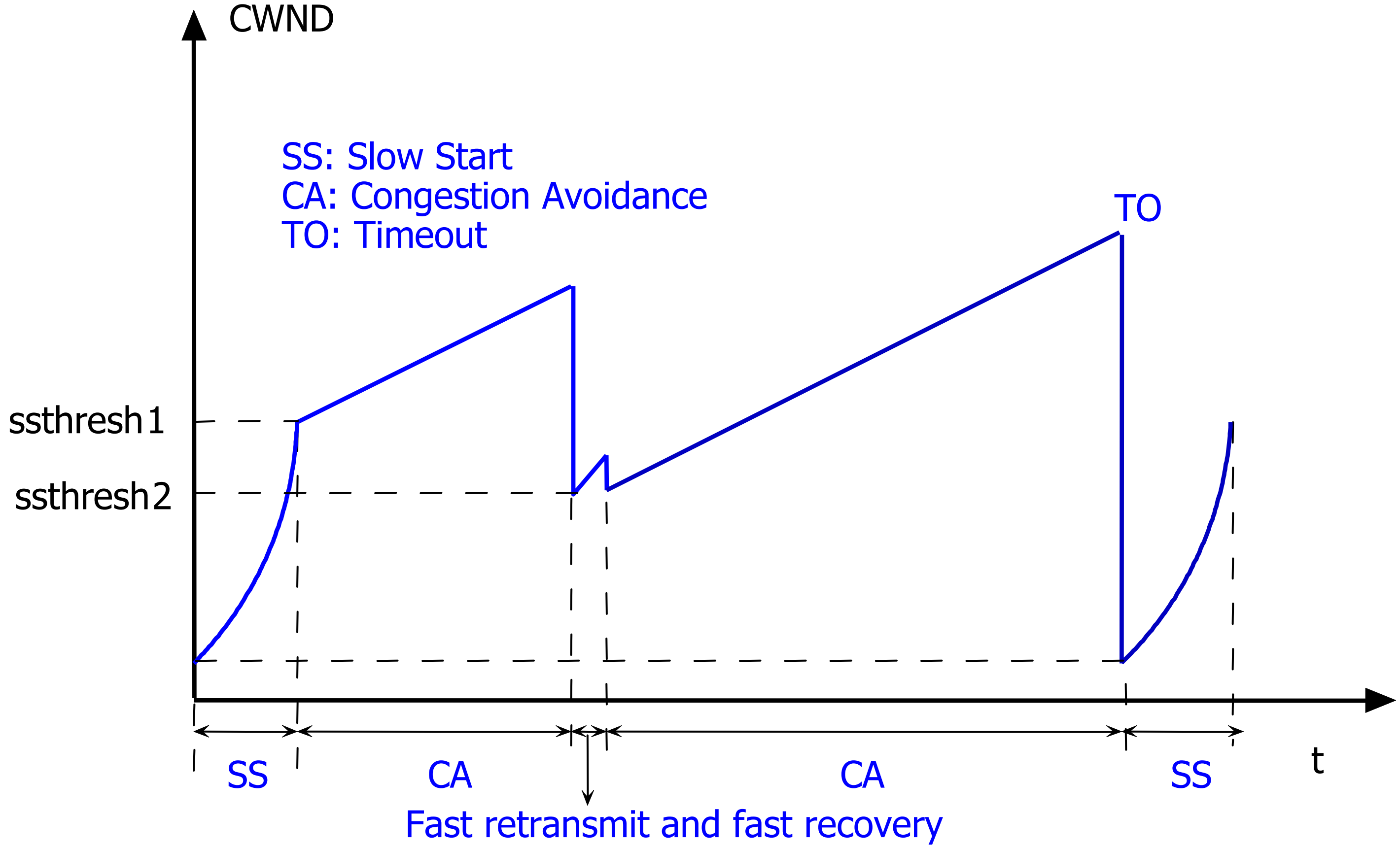
- Long-lived TCP connection
- Constant propagation delay between the source and the destination
- Constant packet size
- ACK packets are never lost
- Timeout occurs only due to packet loss
- The system is sampled at the end of every RTT interval

TCP/RED model simplifications

- Simplified fast recovery



TCP Reno: fast recovery



S-TCP/RED model simplifications

- TO = 5 RTT

  V. Firoiu and M. Borden, "A study of active queue management for congestion control," in *Proc. of IEEE INFOCOM 2000*, vol. 3, pp. 1435–1444, Tel-Aviv, Israel, Mar. 2000.

- RED: parameter count is not used

$$\text{if } (q_{\min} < \bar{q} < q_{\max})$$

$$p_b = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}}$$

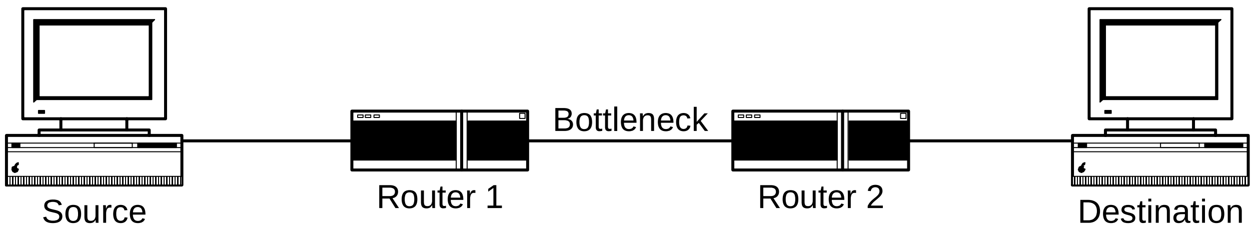
$$p_a = \frac{p_b}{1 - count \times p_b}$$

$$\xrightarrow{p_a = p_b}$$

$$\text{if } (q_{\min} < \bar{q} < q_{\max})$$

$$p_a = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}}$$

Network topology



- Components: one source, two routers, and one destination
- The link between routers 1 and 2 is the only bottleneck
- RED algorithm is deployed in router 1

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TCP/RED models

- S-models: discrete nonlinear dynamical models of TCP Reno with RED
- State variables:
  - window size
  - average queue size
- The proposed TCP/RED models are:
  - simple and intuitively derived
  - able to capture detailed dynamical behavior of TCP/RED systems
  - have been verified via ns-2 simulations

S-TCP/RED: parameters and variables

- Variables:

  w : window size　　$\overline{q}$: average queue size

  p : drop probability　　q : instantaneous queue size

- Parameters:

  q_{max}: maximum queue threshold

  q_{min}: minimum queue threshold

  p_{max}: maximum drop probability

  w_q : queue weight　　d : propagation delay

  M : packet size　　C : link capacity

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Simplified S-TCP/RED model

- M-model, a discrete nonlinear dynamical model of TCP Reno with RED:

  P. Ranjan, E. H. Abed, and R. J. La, "Nonlinear instabilities in TCP-RED," in *Proc. IEEE INFOCOM 2002*, New York, NY, USA, June 2002, vol. 1, pp. 249–258 and *IEEE/ACM Trans. on Networking*, vol. 12, no. 6, pp. 1079–1092, Dec. 2004.

Simplified S-TCP/RED model: one state variable

- Variables:
  - $\overline{q}_{k+1}$: average queue size in round k+1
  - $\overline{q}_k$: average queue size in round k
  - w_q: queue weight in RED
  - N: number of TCP connections
  - K: constant $= \sqrt{3/2}$
  - p_k: drop probability in round k
  - C: capacity of the link between the two routers
  - d: round-trip propagation delay
  - M: packet size
  - *rwnd*: receiver's advertised window size

Simplified S-TCP/RED model: packet lost

- Drop probability: $p_k \neq 0$

$$q_{k+1} = q_k + B(p_k) \cdot RTT_{k+1} \cdot N - \frac{C \cdot RTT_{k+1}}{M}$$

$$= q_k + \frac{K}{\sqrt{p_k} \cdot RTT_{k+1}} \cdot RTT_{k+1} \cdot N - \frac{C}{M}(d + \frac{q_k \cdot M}{C})$$

$$= \frac{K \cdot N}{\sqrt{p_k}} - \frac{C \cdot d}{M}$$

where:

$B(p_k)$	: TCP sending rate
$B(p_k) \cdot RTT_{k+1} \cdot N$	: the number of incoming packets
$C \cdot \frac{RTT_{k+1}}{M}$	: the number of outgoing packets

Simplified S-TCP/RED model: packet lost

The average queue size is:

$$\bar{q}_{k+1} = (1 - w_q) \cdot \bar{q}_k + w_q \cdot q_{k+1}$$

hence

$$\bar{q}_{k+1} = (1 - w_q) \cdot \bar{q}_k + w_q \cdot \max\left(\frac{N \cdot K}{\sqrt{p_k}} - \frac{C \cdot d}{M}, 0\right)$$

Simplified S-TCP/RED model: no loss

- Drop probability: $p_k = 0$

$$q_{k+1} = q_k + B(p_k)\cdot RTT_{k+1}\cdot N - \frac{C\cdot RTT_{k+1}}{M}$$

$$= q_k + \frac{rwnd}{RTT_{k+1}}\cdot RTT_{k+1}\cdot N - \frac{C}{M}(d + \frac{q_k\cdot M}{C})$$

$$= rwnd\cdot N - \frac{C\cdot d}{M}$$

The average queue size is:

$$\bar{q}_{k+1} = (1-w_q)\cdot\bar{q}_k + w_q\cdot q_{k+1}$$

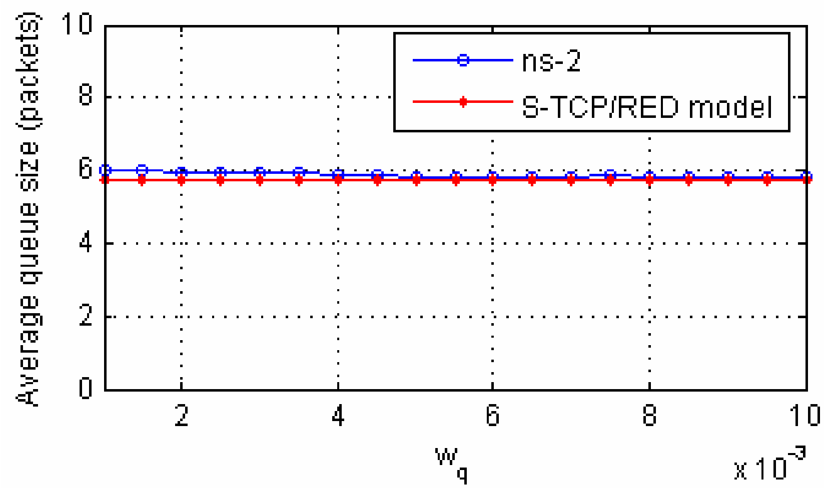
hence

$$\bar{q}_{k+1} = (1-w_q)\cdot\bar{q}_k + w_q\cdot(rwnd\cdot N - \frac{C\cdot d}{M})$$

Simplified S-TCP/RED model

- Dynamical model of TCP/RED:

$$\bar{q}_{k+1} = \begin{cases} (1-w_q)\cdot\bar{q}_k + w_q\cdot\max(\frac{N\cdot K}{\sqrt{p_k}} - \frac{C\cdot d}{M}, 0) \ \ if\ \ p_k \neq 0 \\ (1-w_q)\cdot\bar{q}_k + w_q\cdot(rwnd\cdot N - \frac{C\cdot d}{M}) \ \ if\ \ p_k = 0 \end{cases}$$



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TCP/RED model: state variable and parameters

- q_{k+1}: instantaneous queue size in round k+1
- $\overline{q}_{k+1}$: average queue size in round k+1
- W_{k+1}: current TCP window size in round k+1
- w_q: queue weight in RED
- p_k: drop probability in round k
- RTT_{k+1}: round-trip time at k+1
- C: capacity of the link between the two routers
- M: packet size
- d: round-trip propagation delay
- ssthesh: slow start threshold
- rwnd: receiver's advertised window size

S-TCP/RED: discrete-time model for TCP Reno with RED

- Calculate the average queue size:

$$q_{k+1} = q_k + W_{k+1} - \frac{C}{M}\left(d + \frac{q_k \cdot M}{C}\right)$$

$$= W_{k+1} - \frac{C \cdot d}{M} \qquad (1)$$

$$\overline{q}_{k+1} = (1 - w_q) \cdot \overline{q}_k + w_q \cdot q_{k+1} \qquad (2)$$

  - the average queue size is updated after each packet arrival
  - $\overline{q}_{k+1}$ is updated W_{k+1} times in k+1-th round

From (1) and (2):

$$\overline{q}_{k+1} = (1 - w_q)^{W_{k+1}} \cdot \overline{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

S-TCP/RED model: drop probability

- Calculate the drop probability:

$$p_{k+1} = \begin{cases} 0 & \textit{if } \bar{q}_{k+1} \le q_{\min} \\ 1 & \textit{if } \bar{q}_{k+1} \ge q_{\max} \\ \dfrac{\bar{q}_{k+1} - q_{\min}}{q_{\max} - q_{\min}} p_{\max} & \textit{otherwise} \end{cases}$$

S-TCP/RED model: three cases

- No packet lost:
  - slow start
  - congestion avoidance
- Single packet lost:
  - fast retransmit
  - fast recovery
- At least two packets lost:
  - timeout

S-TCP/RED model: no packet loss

- number of lost packets: $p_k W_k < 0.5$
- window size:

$$W_{k+1} = \begin{cases} \min(2W_k, ssthresh) & if\ W_k < ssthresh \\ \min(W_k + 1, rwnd) & if\ W_k \geq ssthresh \end{cases}$$

- where:
  - W_{k+1}: window size in round k+1
  - ssthesh: slow start threshold
  - rwnd: receiver's advertised window size

S-TCP/RED model: no packet loss

- current queue size:

$$q_{k+1} = q_k + W_{k+1} - C \cdot \frac{RTT_{k+1}}{M}$$

$$= q_k + W_{k+1} - \frac{C}{M}(d + \frac{q_k M}{C})$$

$$= W_{k+1} - \frac{C \cdot d}{M}$$

- where:
  - RTT_{k+1}: round-trip time at k+1
  - C: capacity of the link between the two routers
  - M: packet size
  - d: round-trip propagation delay

S-TCP/RED model: no packet loss

- average queue size:

$$\bar{q}_{k+1} = (1 - w_q) \cdot \bar{q}_k + w_q \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

- hence:

$$\bar{q}_{k+1} = (1 - w_q)^{W_{k+1}} \bar{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

S-TCP/RED model: no packet loss

- number of lost packets: $p_k W_k < 0.5$

- window size:

$$W_{k+1} = \begin{cases} \min(2W_k, ssthresh) & if\ W_k < ssthresh \\ \min(W_k + 1, rwnd) & if\ W_k \geq ssthresh \end{cases}$$

- average queue size:

$$\overline{q}_{k+1} = (1-w_q)^{W_{k+1}} \overline{q}_k + (1-(1-w_q)^{W_{k+1}}) \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

S-TCP/RED model: one packet loss

- number of lost packets: $0.5 \le p_k W_k < 1.5$
- window size: $W_{k+1} = \frac{1}{2} W_k$
- average queue size:

$$\overline{q}_{k+1} = (1 - w_q)^{W_{k+1}} \overline{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

S-TCP/RED model: two packet losses

- number of lost packets: $p_k W_k \geq 1.5$
- window size: $W_{k+1} = 0$
- average queue size: $\overline{q}_{k+1} = \overline{q}_k$

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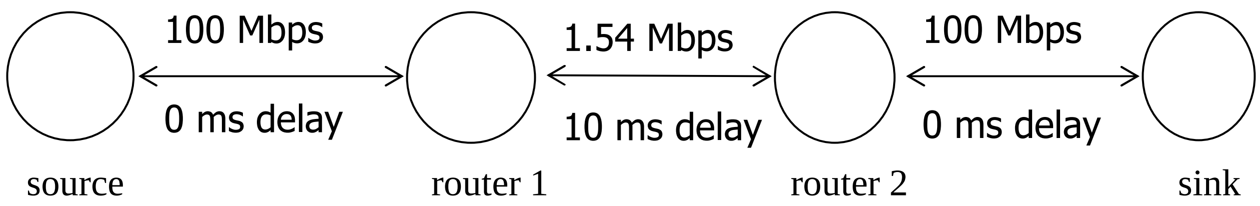
Simulation scenario



- source to router1:
  - link capacity: 100 Mbps with 0 ms delay
- router 1 to router 2: the only bottleneck in the network
  - link capacity: 1.54 Mbps with 10 ms delay
- router 2 to sink:
  - link capacity: 100 Mbps with 0 ms delay

RED: default parameters

- RED parameters:

  S. Floyd, "RED: Discussions of Setting Parameters," Nov. 1997: http://www.icir.org/floyd/REDparameters.txt

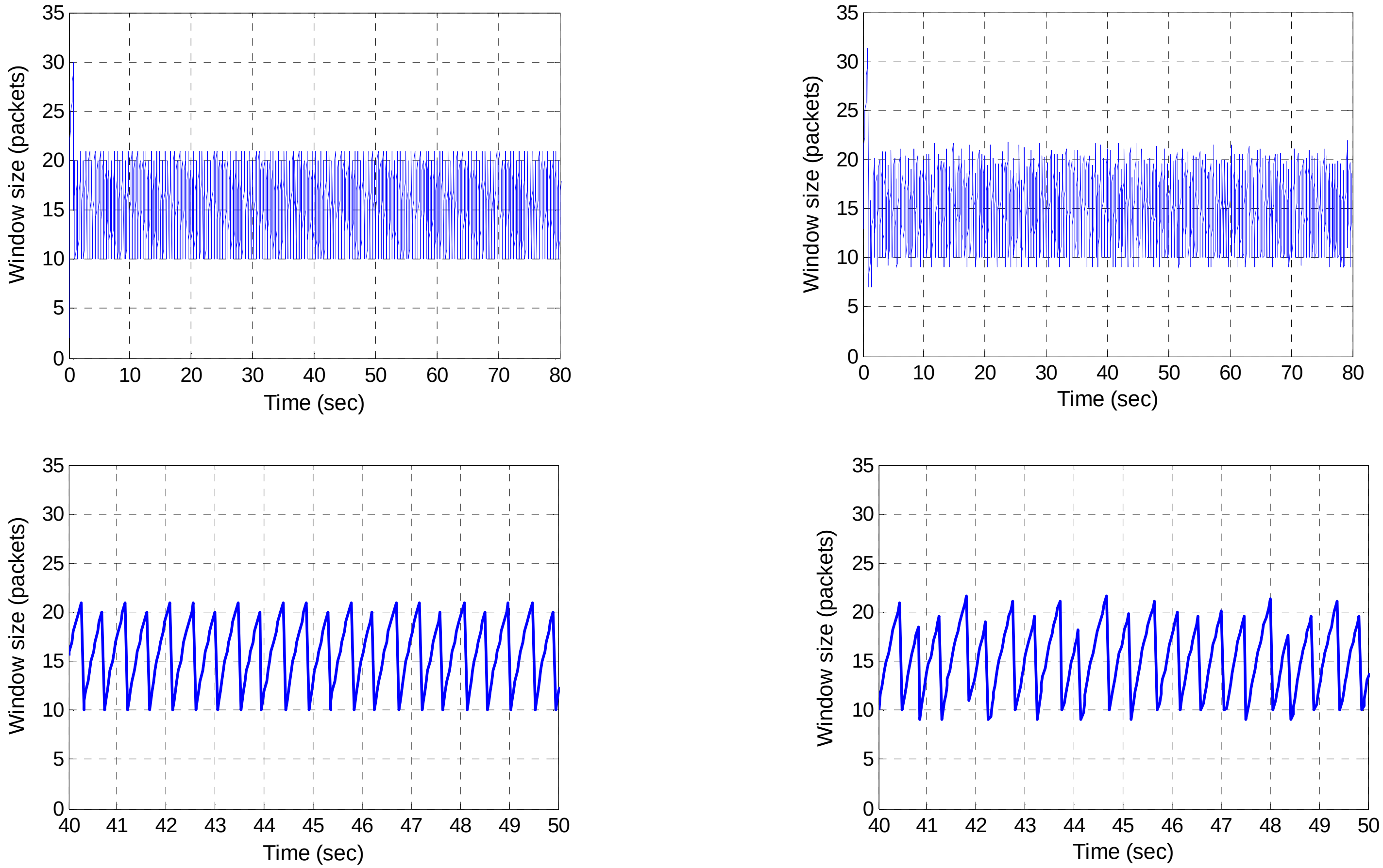
Queue weight (w_q)	0.002
Maximum drop probability (p_{max})	0.1
Minimum queue threshold (q_{min})	5 (packets)
Maximum queue threshold (q_{max})	15 (packets)
Packet size (M)	4,000 (bytes)

S-TCP/RED model validation

- Waveforms of the state variables with default parameters:
  - window size
  - average queue size

- Validation for various values of the system parameters:
  - queue weight: w_q
  - maximum drop probability: p_{max}
  - queue thresholds: q_{min} and q_{max} , $q_{max}/q_{min} = 3$

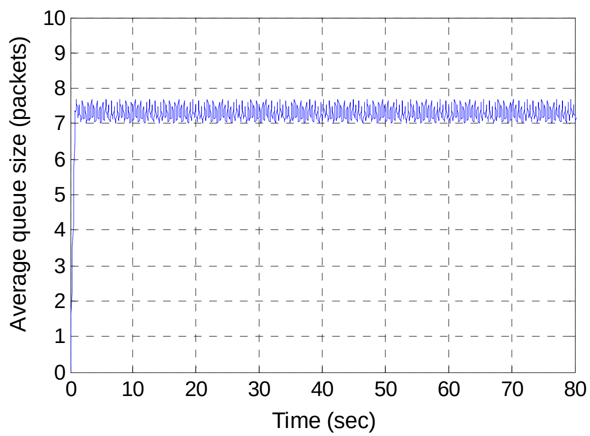
Window size: waveforms

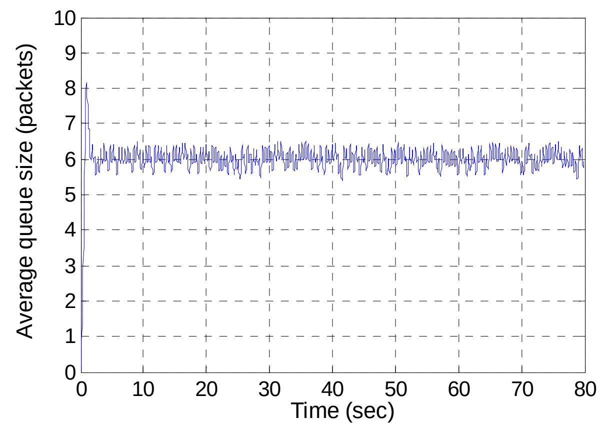


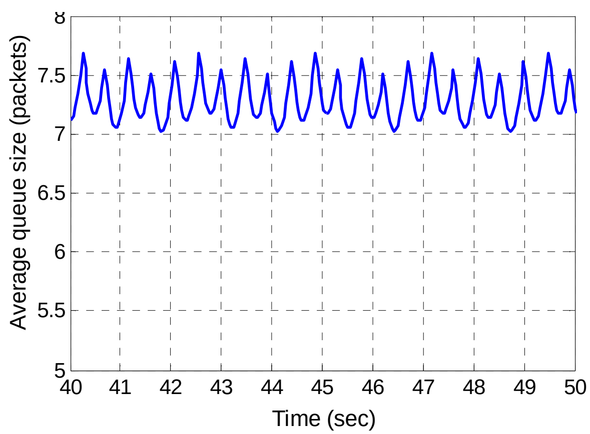
S-TCP/RED model

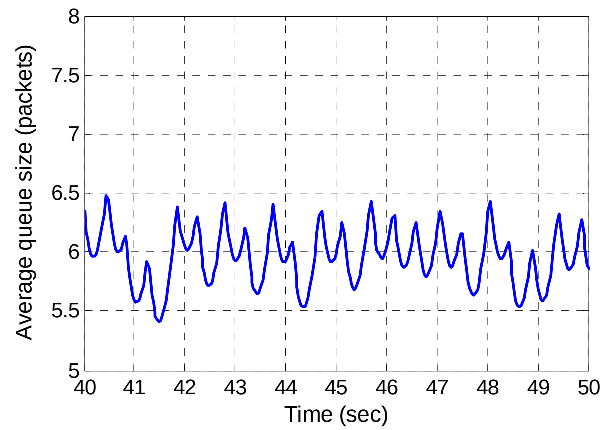
ns-2

Average queue size: waveforms







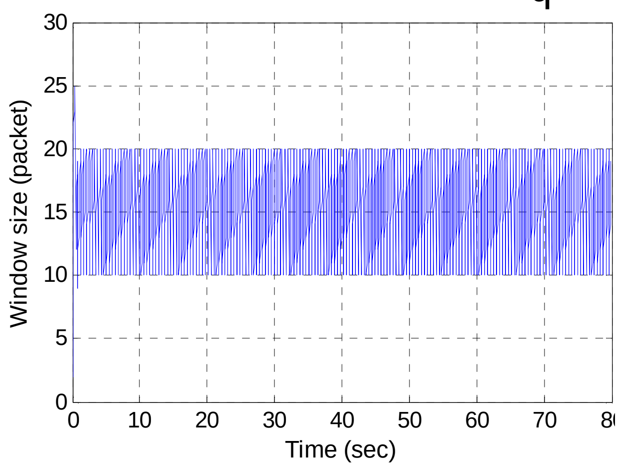


S-TCP/RED model

ns-2

Model validation: w_q

- w_q =[0.001, 0.01], with other parameters default
  - window size: w_q = 0.006



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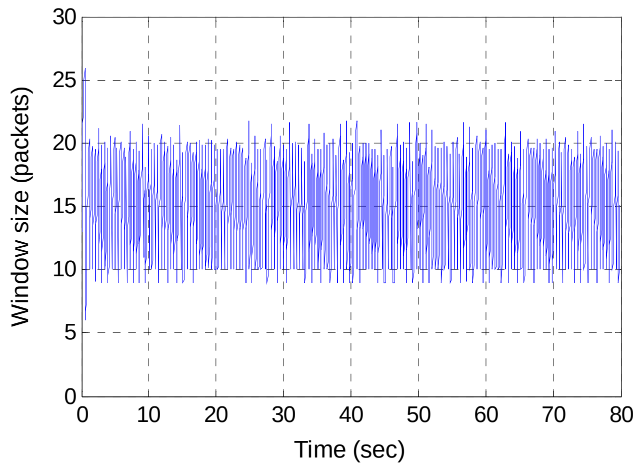
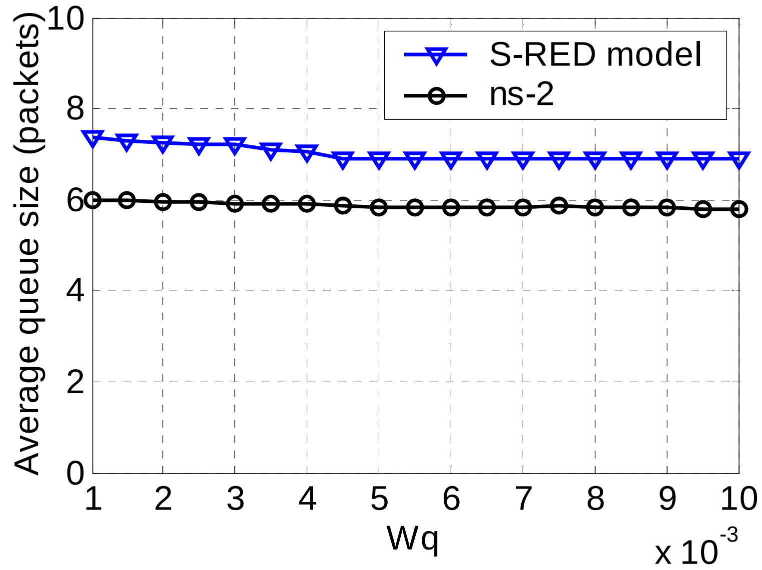


ns-2

Model validation: w_q

- average queue size during steady state:



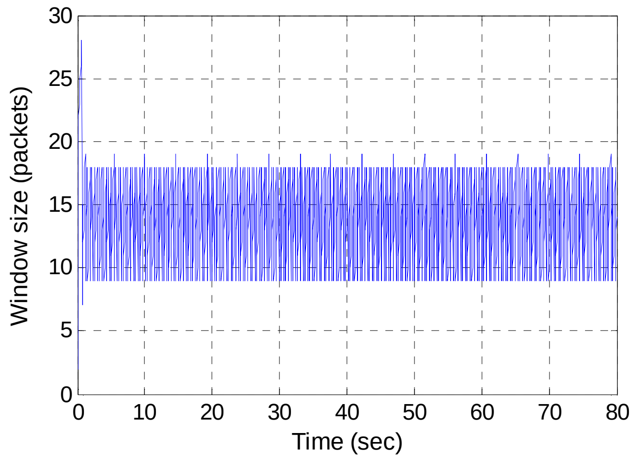
Model validation: w_q

- Comparison of system variables:

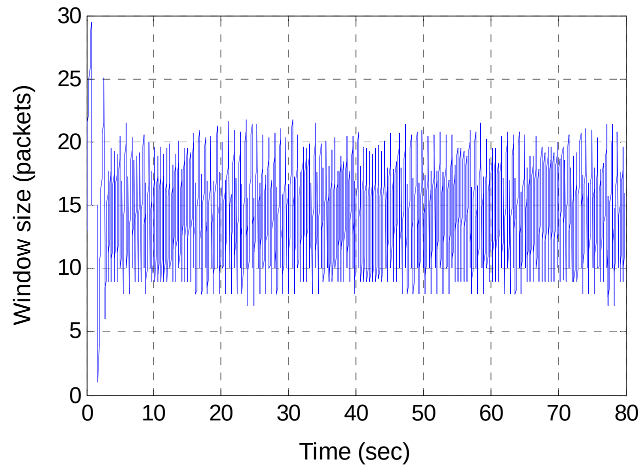
Parameters	Average RTT (msec)			Sending rate (packets/sec)			Drop rate (%)		
weight (w_q)	S-RED model	ns-2	Δ (%)	S-RED model	ns-2	Δ (%)	S-RED model	ns-2	Δ (%)
0.001	40.3	36.1	11.63	384.99	384.71	0.073	0.55	0.54	1.29
0.002	39.9	36.0	10.83	384.98	384.77	0.056	0.56	0.55	2.56
0.004	39.4	36.2	8.80	385.11	384.79	0.083	0.59	0.56	6.12
0.006	39.0	35.8	8.93	385.08	384.73	0.093	0.60	0.56	7.91
0.008	39.0	35.8	8.90	385.10	384.68	0.109	0.61	0.55	11.11
0.010	38.9	35.7	8.96	385.02	384.70	0.083	0.61	0.55	11.72

Model validation: p_{max}

- p_{max} = [0.05, 0.95], with other parameters default
  - window size: waveforms, p_{max} = 0.5



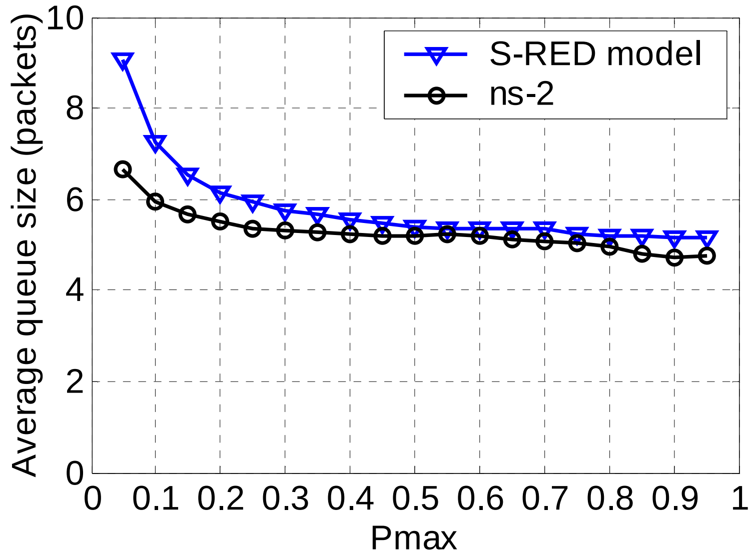
S-TCP/RED



ns-2

Model validation: p_{max}

- average queue size during steady state:



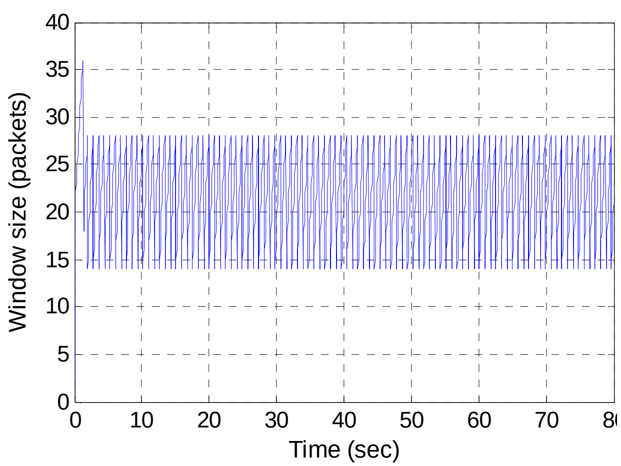
Model validation: p_{max}

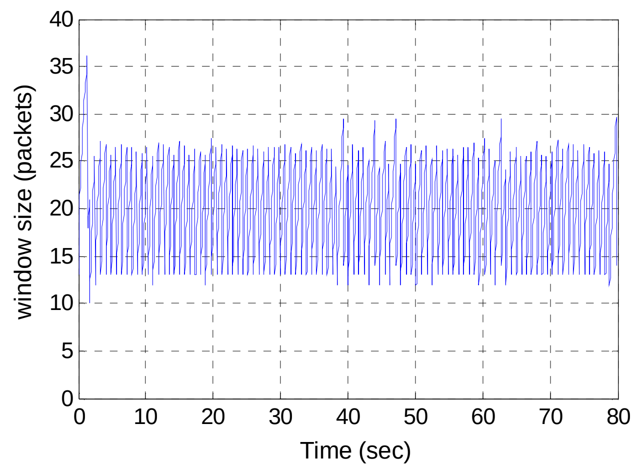
- Comparison of system variables:

	Average RTT (msec)			Sending rate (packets/sec)			Drop rate (%)		
p_{max}	S-RED model	ns-2	Δ (%)	S-RED model	ns-2	Δ (%)	S-RED model	ns-2	Δ (%)
0.05	44.3	38.1	16.27	385.13	384.70	0.11	0.45	0.51	-11.76
0.10	39.9	36.0	10.83	384.98	384.77	0.06	0.56	0.55	2.56
0.25	36.5	34.5	5.80	384.93	384.73	0.05	0.65	0.59	11.28
0.50	35.3	34.0	3.80	384.98	379.37	1.48	0.73	0.61	19.09
0.75	34.8	35.1	-0.85	384.63	357.55	7.60	0.74	0.65	14.37

Model validation: q_{min} and q_{max}

- q_{min}= [1, 20] packets, q_{max}/q_{min} = 3, with other parameters default
  - window size: waveforms, q_{min} = 10 packets



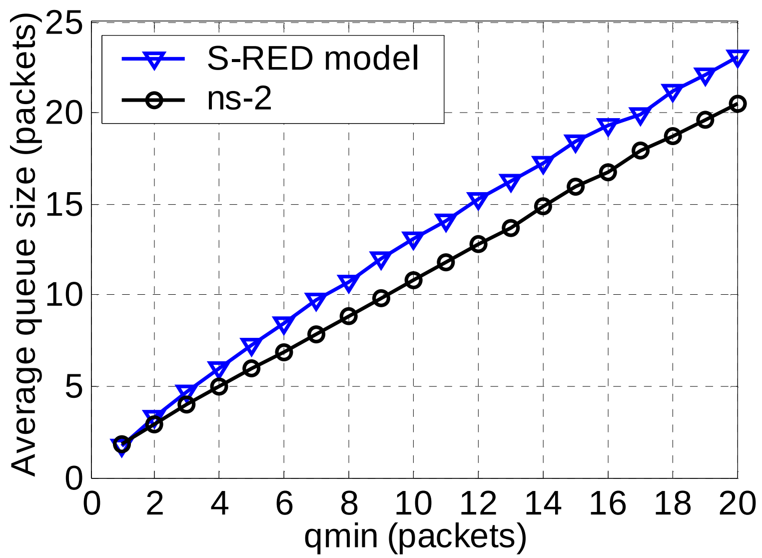


S-TCP/RED

ns-2

Model validation: q_{min} and q_{max}

- average queue size during steady state:



Model validation: q_{min} and q_{max}

- Comparison of system variables:

	Average RTT (msec)			Sending rate (packets/sec)			Drop rate (%)		
q_{min} (packets)	S-RED model	ns-2	Δ (%)	S-RED model	ns-2	Δ (%)	S-RED model	ns-2	Δ (%)
3	33.4	31.1	7.4	383.22	382.44	0.20	0.78	0.71	10.01
5	39.9	36.0	10.83	384.98	384.77	0.06	0.56	0.55	2.56
10	54.7	48.1	13.72	385.10	384.85	0.06	0.31	0.33	-6.34
15	67.7	60.3	12.27	385.06	384.83	0.06	0.20	0.22	-10.71
20	79.1	73.0	8.36	385.30	384.95	0.09	0.15	0.16	-5.66

S-TCP/RED: model evaluation

- Waveforms of the window size:
  - match the ns-2 simulation results
- The average queue size:
  - mismatch, but similar trend
- System variables RTT, sending rate, and drop rate:
  - reasonable agreement with ns-2 simulation results, depending on the system parameters

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S-TCP/RED: modification

- The difference in the average queue size between S-TCP/RED model and ns-2 is due to the simplification of p:

if $(q_{\min} < \bar{q} < q_{\max})$

$$p_b = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}}$$

$$p_a = \frac{p_b}{1 - count \times p_b}$$

$\xrightarrow{p_a = p_b}$

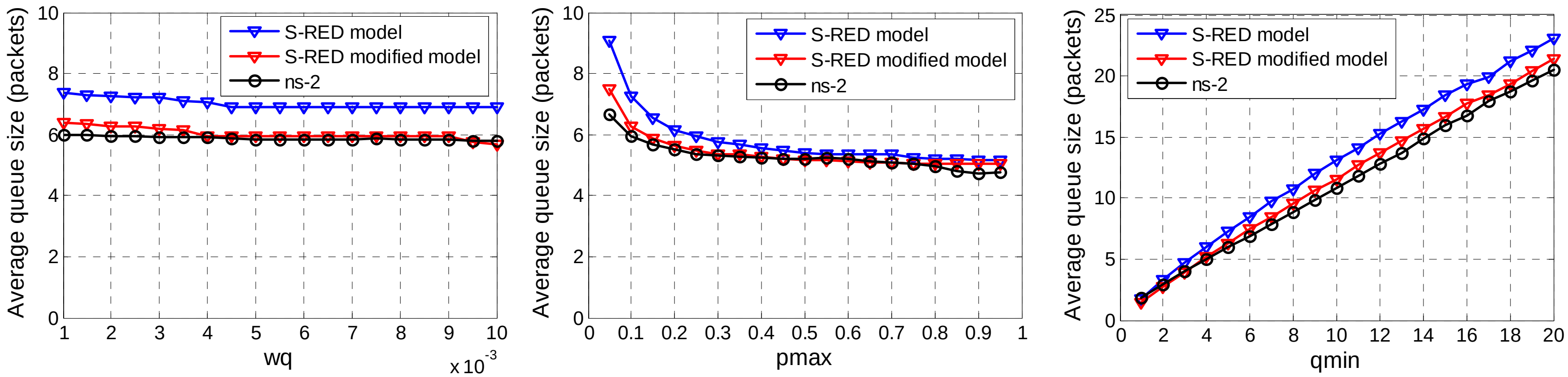
if $(q_{\min} < \bar{q} < q_{\max})$

$$p_a = p_{\max} \times \frac{\bar{q} - q_{\min}}{q_{\max} - q_{\min}}$$

- Modification of p_a: $p_a = \alpha \cdot p_b \quad (\alpha > 1)$

Modified S-TCP/RED model

- Modification: $\alpha = 1.8$



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Comparison: S-TCP/RED vs. M-model

- M-model:

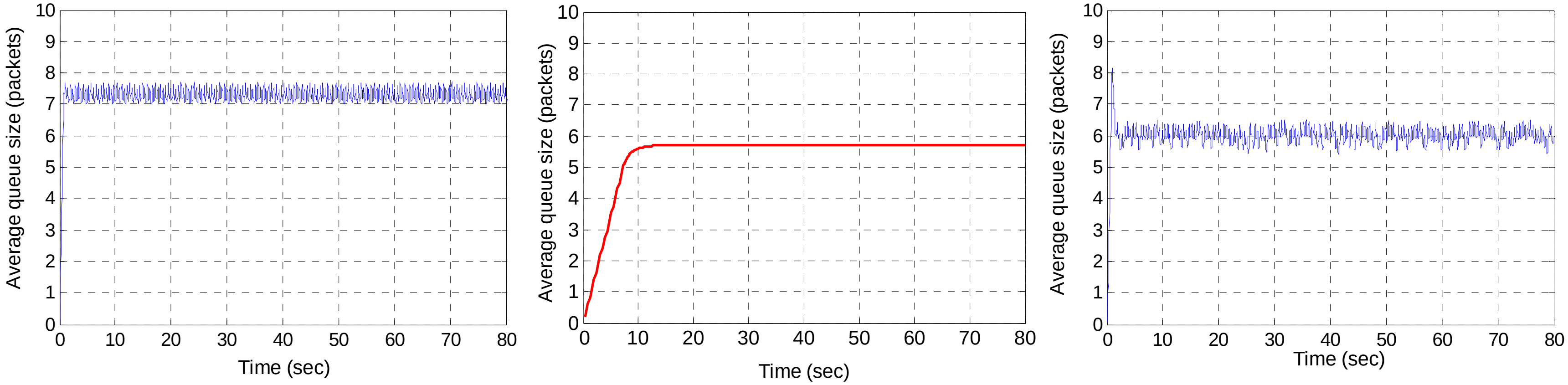
  A discrete nonlinear dynamical model of TCP Reno with RED proposed by a research group from University of Maryland:

  P. Ranjan, E. H. Abed, and R. J. La, "Nonlinear instabilities in TCP-RED," in *Proc. IEEE INFOCOM 2002*, New York, NY, USA, June 2002, vol. 1, pp. 249–258 and *IEEE/ACM Trans. on Networking*, vol. 12, no. 6, pp. 1079–1092, Dec. 2004.

- One state variable: average queue size

Model comparison: default parameters

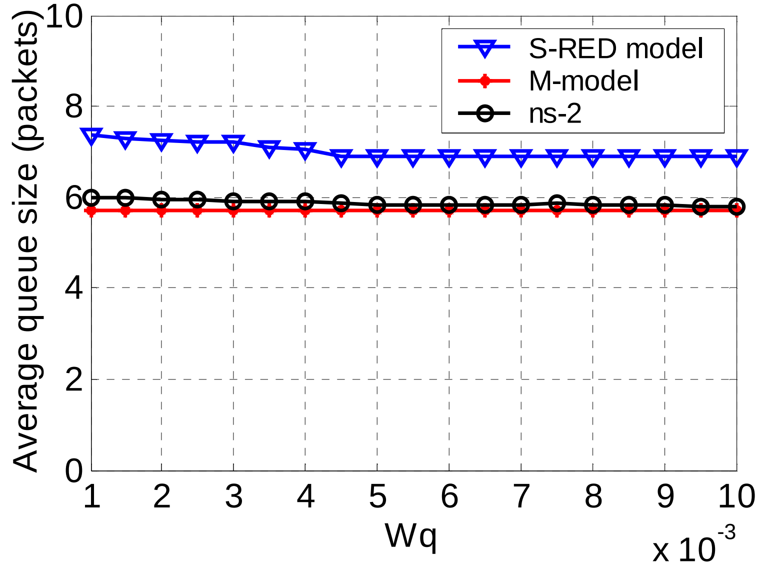
- The waveform of the average queue size with default RED parameters:



S-TCP/RED M-model ns-2

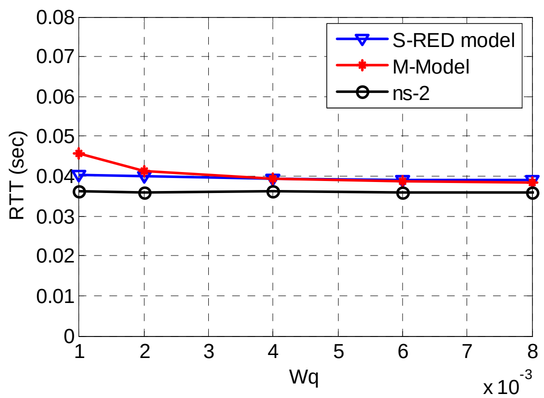
Model comparisons: w_q

- w_q = [0.001, 0.01], with other parameters default
  - average queue size during steady state:

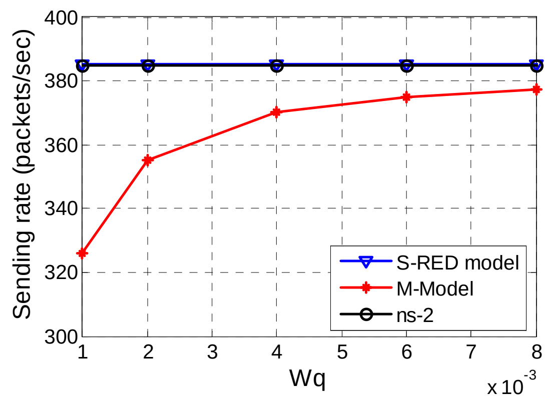


Model comparisons: w_q

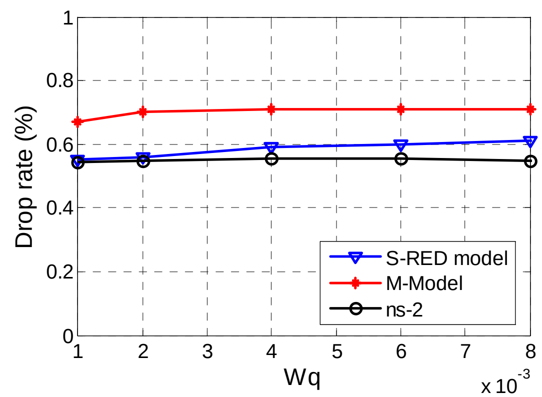
- system variables:



RTT



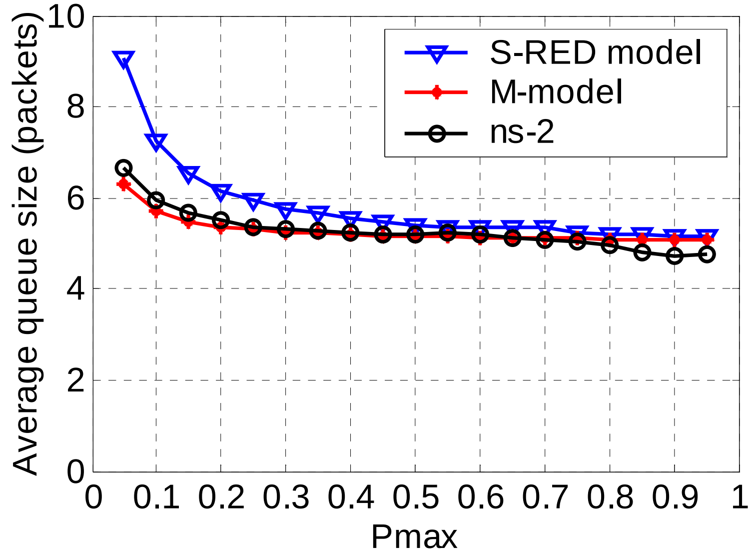
sending rate



drop rate

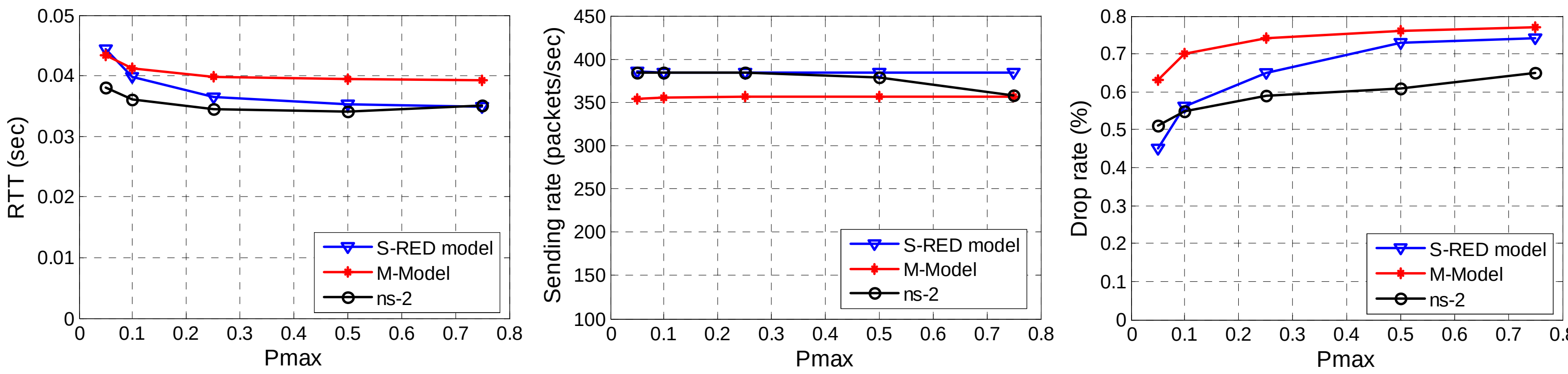
Model comparisons: p_{max}

- p_{max}= [0.05, 0.95], with other parameters default
  - average queue size during steady state:



Model comparisons: p_{max}

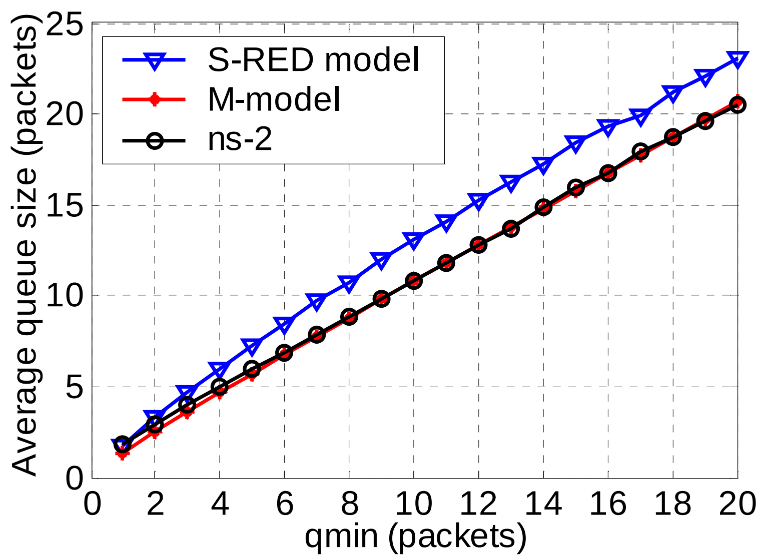
- system variables:



RTT sending rate drop rate

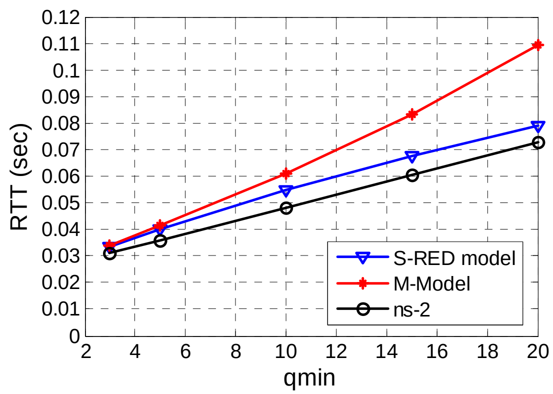
Model comparisons: q_{min} and q_{max}

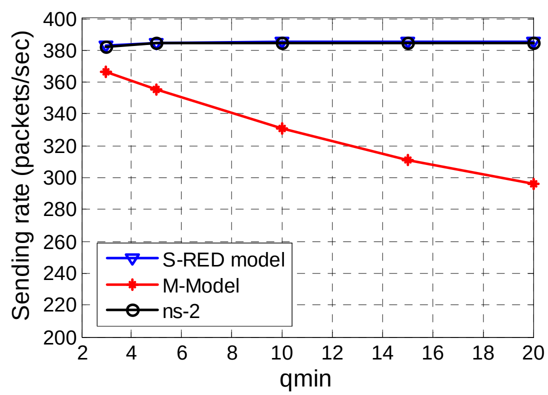
- q_{min}= [1, 20] packets, q_{max}/q_{min} =3, with other parameters default
  - average queue size during steady state:

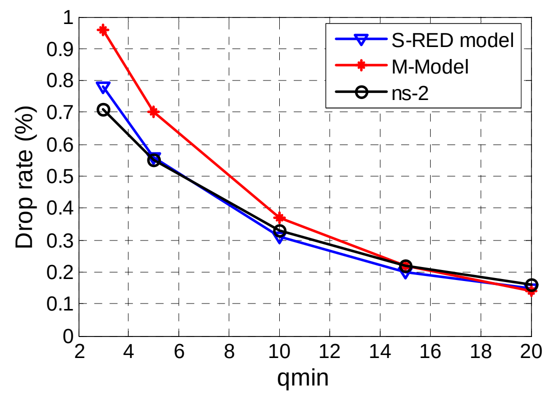


Model comparisons: q_{min} and q_{max}

- system variables:







RTT sending rate drop rate

Model comparison: summary

- S-TCP/RED model captures dynamical details of TCP/RED
- RTT, sending rate, and drop rate: S-TCP/RED model, in general, matches the ns-2 simulation results better than the M-model
- M-model: average queue size
  - constant during steady-state
  - matches better the ns-2 simulation results

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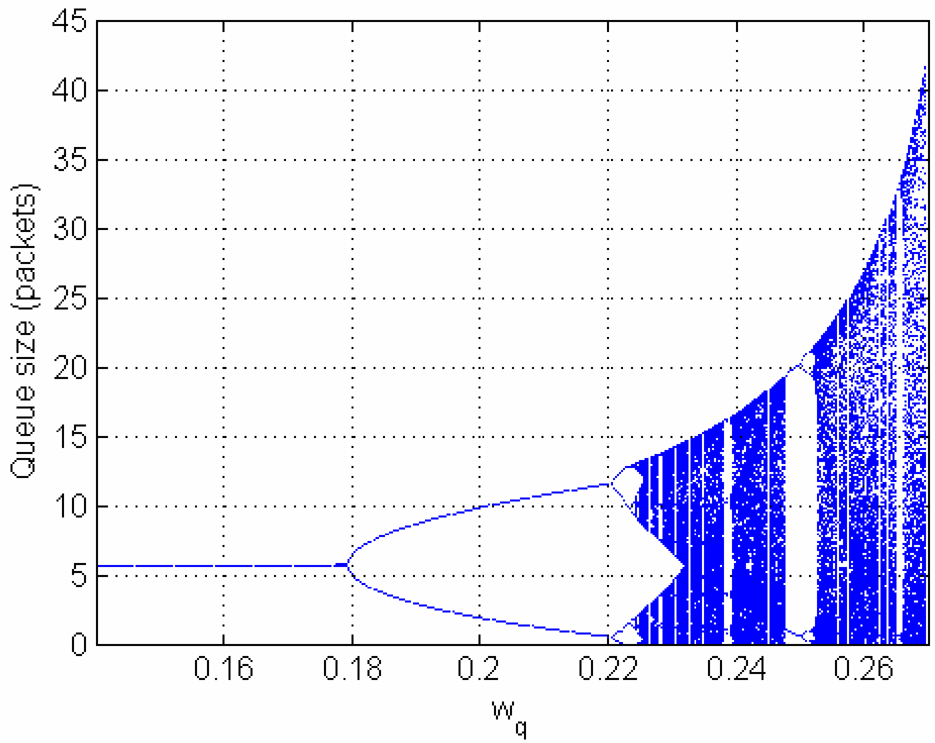
TCP/RED: bifurcation and chaos

- Bifurcation diagrams for various values of the system parameters:
  - queue weight: w_q
  - maximum drop probability: p_{max}
  - queue thresholds: q_{min} and q_{max} ($q_{max}/q_{min} = 3$)
  - round-trip propagation delay: d

Queue weight (w_q)	0.002
Maximum drop probability (p_{max})	0.1
Minimum queue threshold (q_{min})	5 (packets)
Maximum queue threshold (q_{max})	15 (packets)
Packet size (M)	4,000 (bytes)

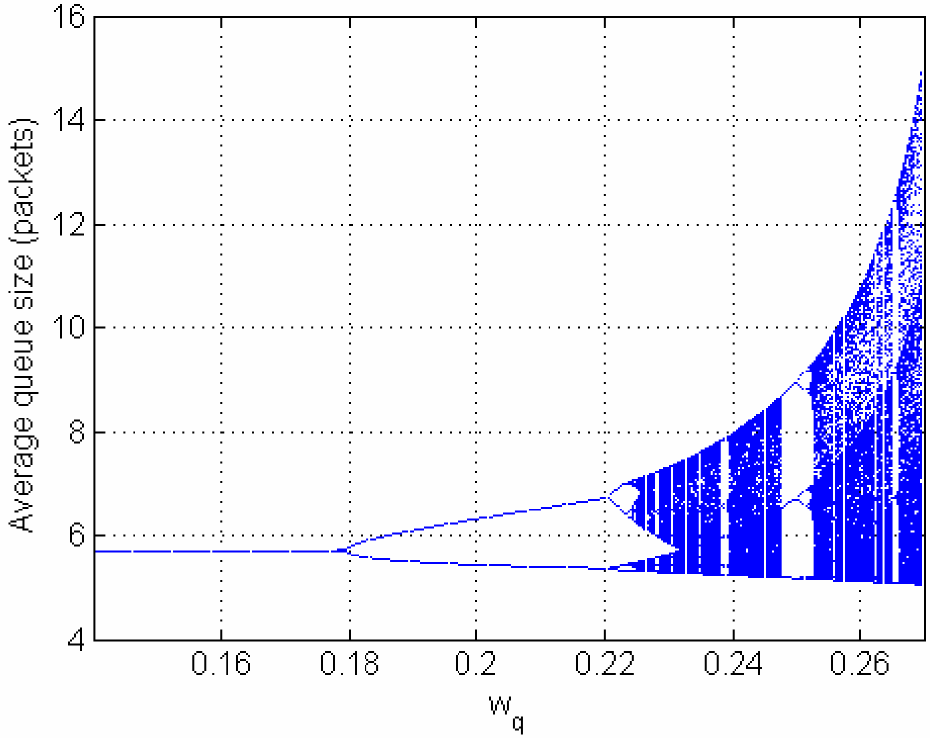
Queue size vs. w_q

- $p_{max} = 0.1$, $q_{min} = 5$, $q_{max} = 15$



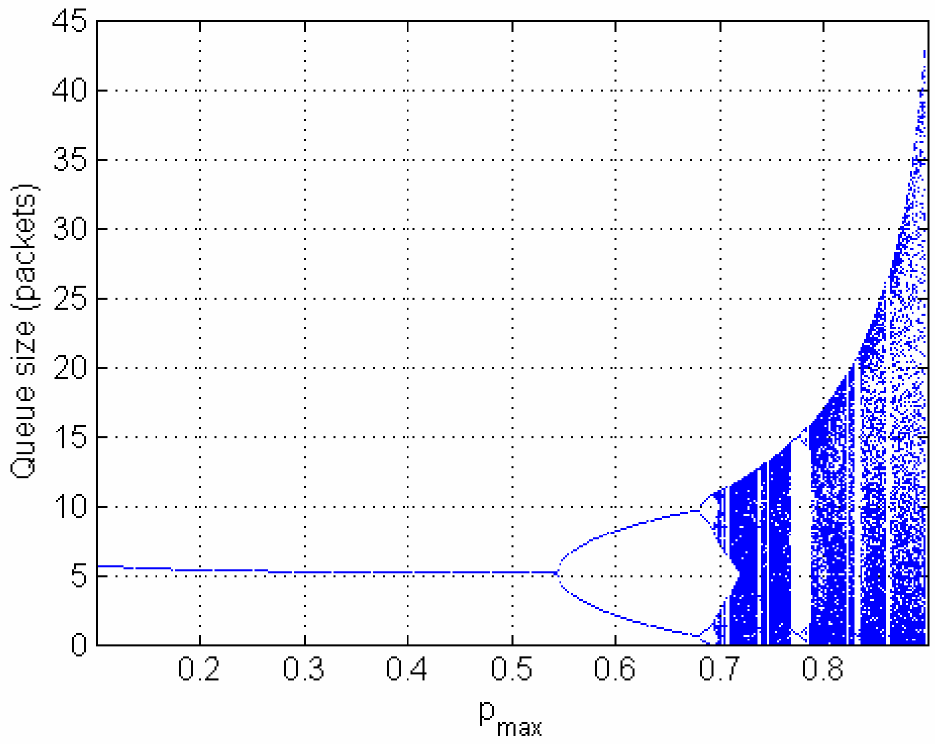
Average queue size vs. w_q

- $p_{max} = 0.1$, $q_{min} = 5$, $q_{max} = 15$



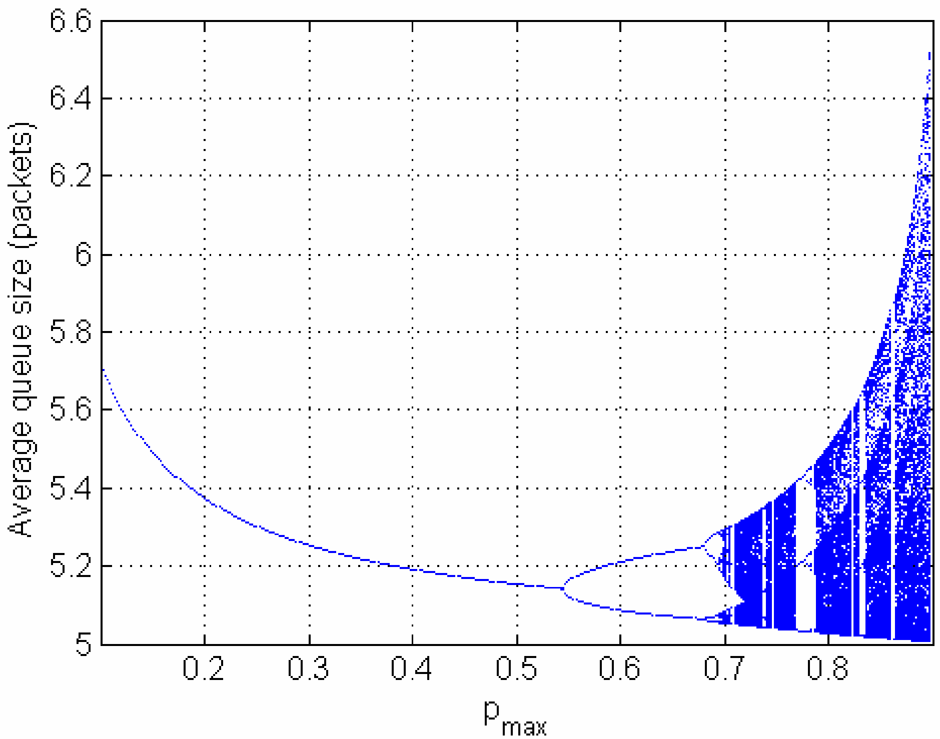
Queue size vs. p_{max}

- $w_{qx} = 0.04$, $q_{min} = 5$, $q_{max} = 15$



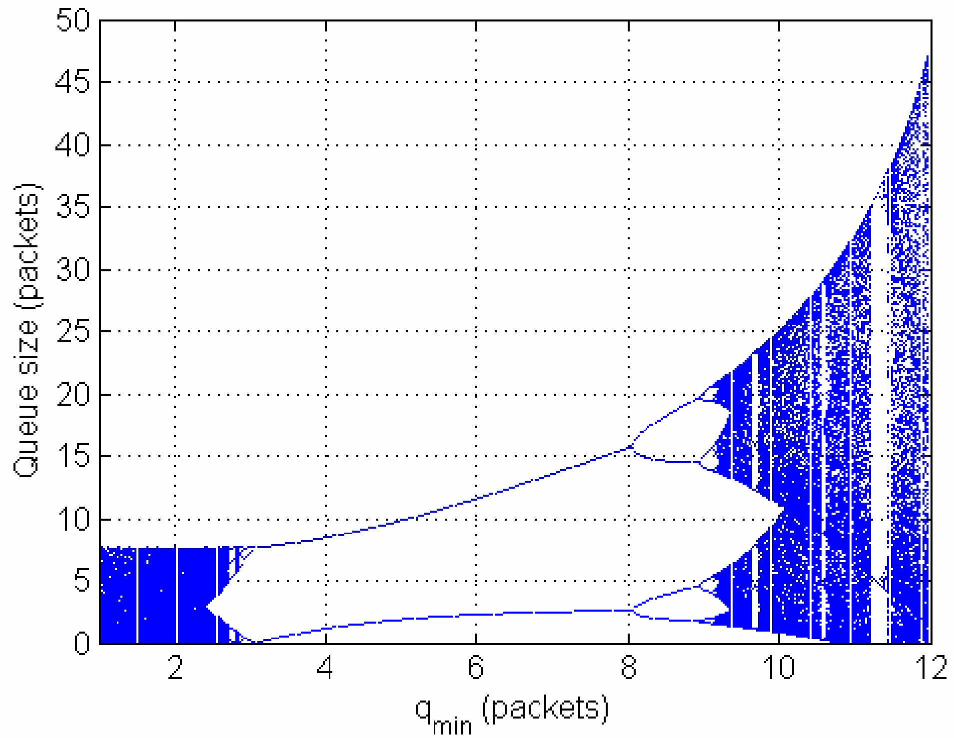
Average queue size vs. p_{max}

- $w_q = 0.04$, $q_{min} = 5$, $q_{max} = 15$



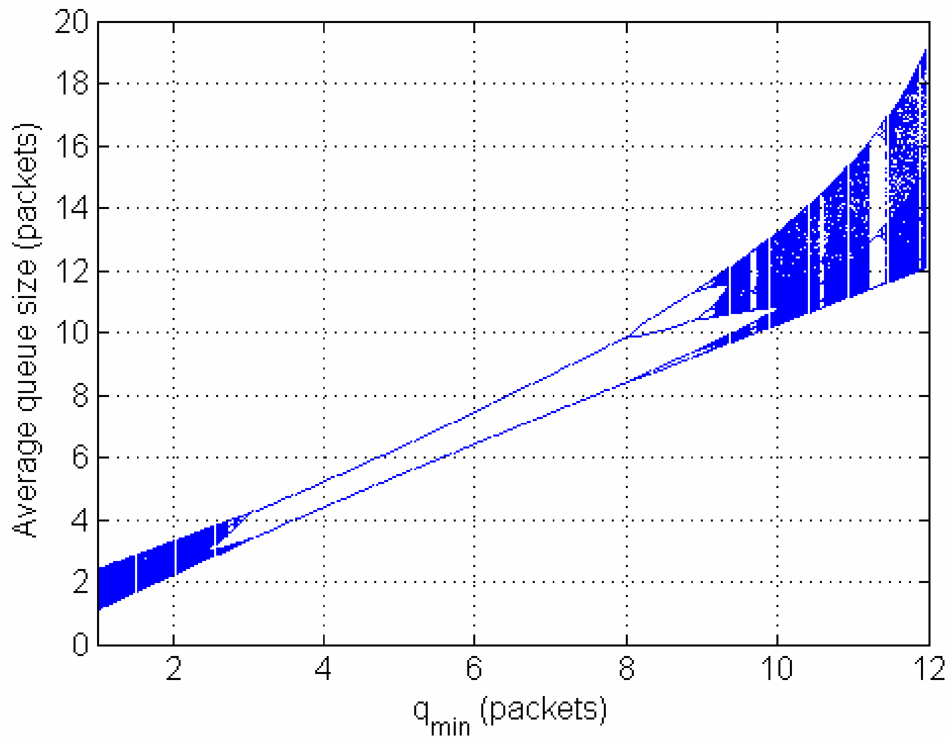
Queue size vs. q_{min}/q_{max}

- $w_q = 0.2$, $p_{max} = 0.1$, $q_{max} = 3 \times q_{min}$



Average queue size vs. q_{min}/q_{max}

- $w_q = 0.2$, $p_{max} = 0.1$, $q_{max} = 3 \times q_{min}$



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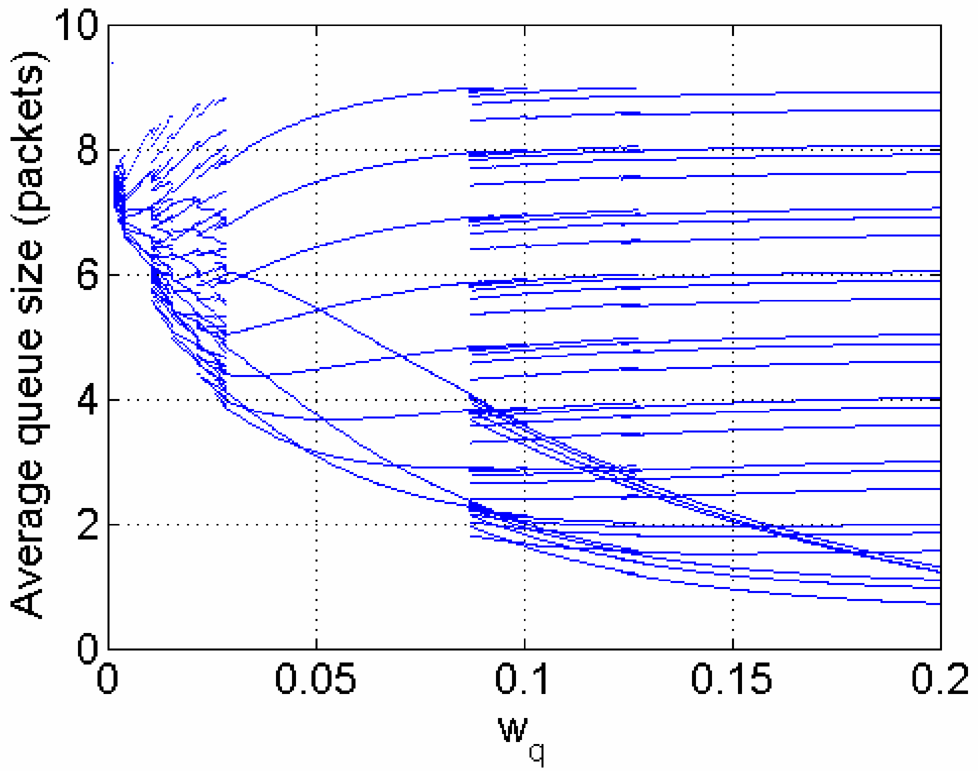
  - maximum drop probability: p_{max}
  - queue thresholds: q_{min} and q_{max} (q_{max}/q_{min} = 3)
  - round-trip propagation delay: d

Queue weight (w_q)	0.002
Maximum drop probability (p_{max})	0.1
Minimum queue threshold (q_{min})	5 (packets)
Maximum queue threshold (q_{max})	15 (packets)
Packet size (M)	4,000 (bytes)

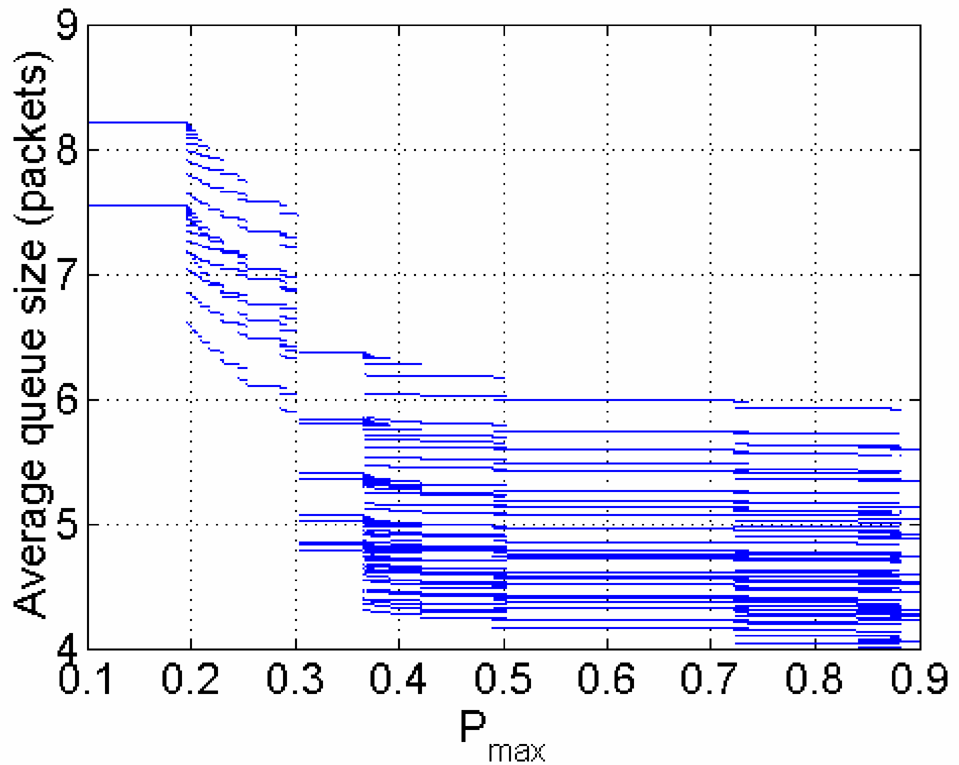
Average queue size vs. w_q

- $p_{max} = 0.1$, $q_{min} = 5$, $q_{max} = 15$, and sstresh = 80



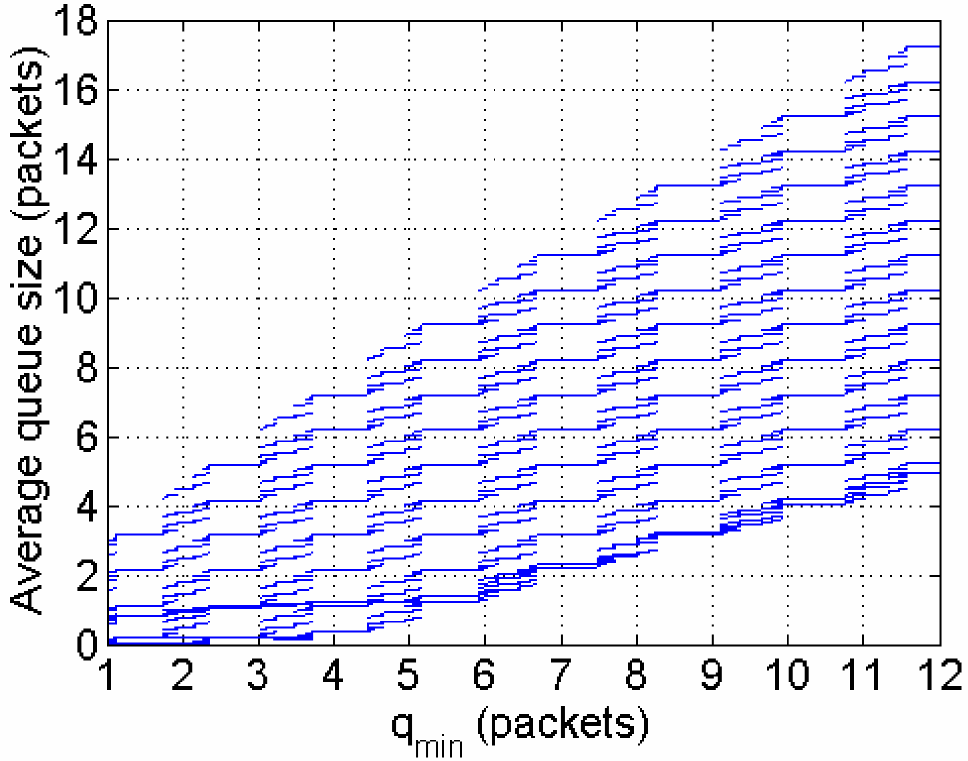
Average queue size vs. p_{max}

- $w_q = 0.01$, $q_{min} = 5$, $q_{max} = 15$, and ssthresh = 20



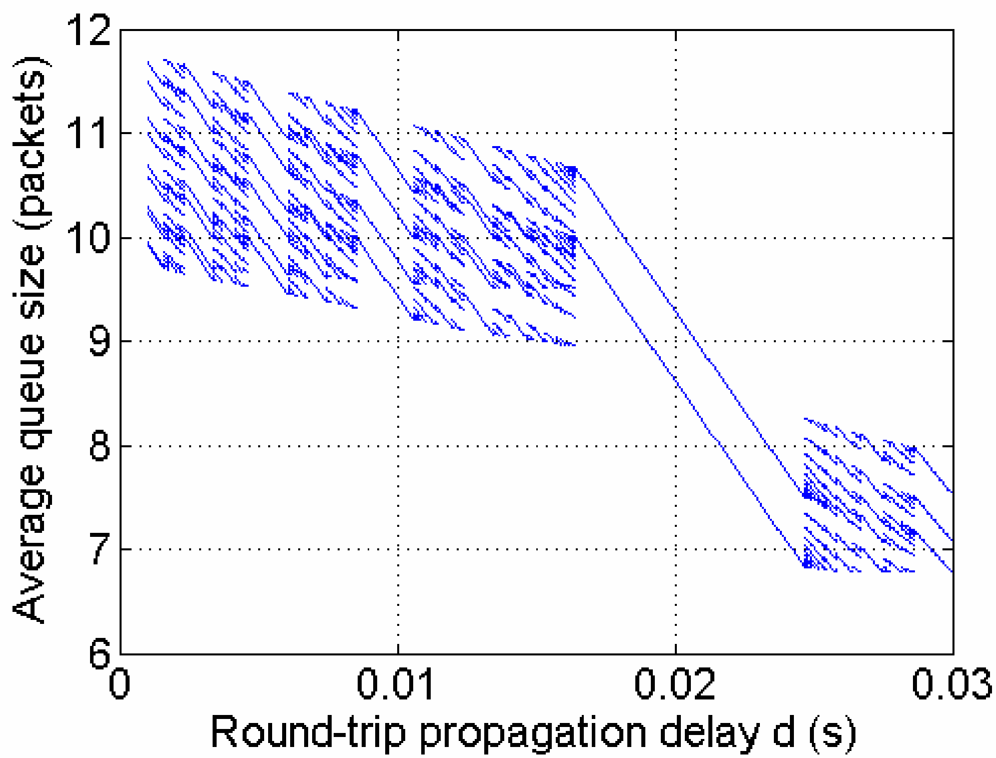
Average queue size vs. q_{min}/q_{max}

- $w_q = 0.01$, $p_{max} = 0.1$, $q_{max} = 3\ q_{min}$, and ssthresh = 20



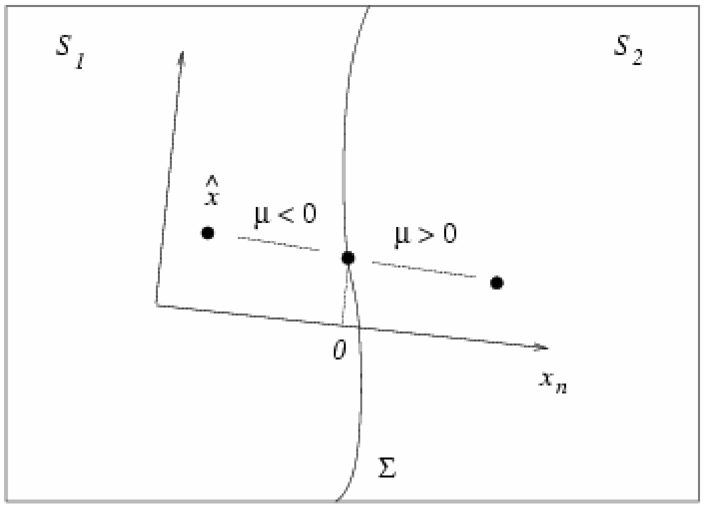
Average queue size vs. d

- $w_q = 0.01$, $p_{max} = 0.1$, $q_{min} = 5$, $q_{max} = 15$, and ssthresh = 20



An analytical explanation

- Nonsmooth systems may exhibit discontinuity-induced bifurcations: a class of bifurcations unique to their nonsmooth nature
- These phenomena occur when a fixed point, cycle, or aperiodic attractor interacts nontrivially with one of the phase space boundaries where the system is discontinuous



Discontinuity-induced bifurcations: classification

- Standard:
  - SN (smooth saddle-node)
  - PD (smooth period-doubling)
- C-bifurcations or DIBs
  - PWS maps: border collisions of fixed points
  - PWS flows: discontinuous bifurcations of equilibriums
  - Grazing bifurcations of periodic orbits
  - Sliding bifurcations

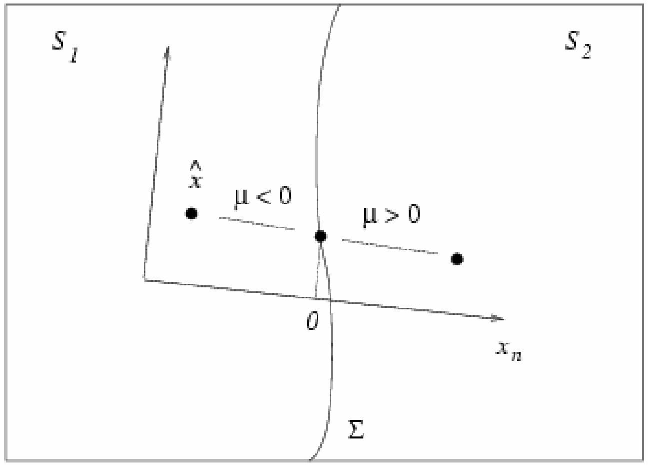
Border collisions in PWS maps

- Consider a map of the form:

$$x_{k+1} = \begin{cases} F_1(x_k, p), & H(x_k) < 0 \\ F_2(x_k, p), & H(x_k) > 0 \end{cases}$$

- A fixed point is undergoing a border-collision bifurcation at $p=0$ if:

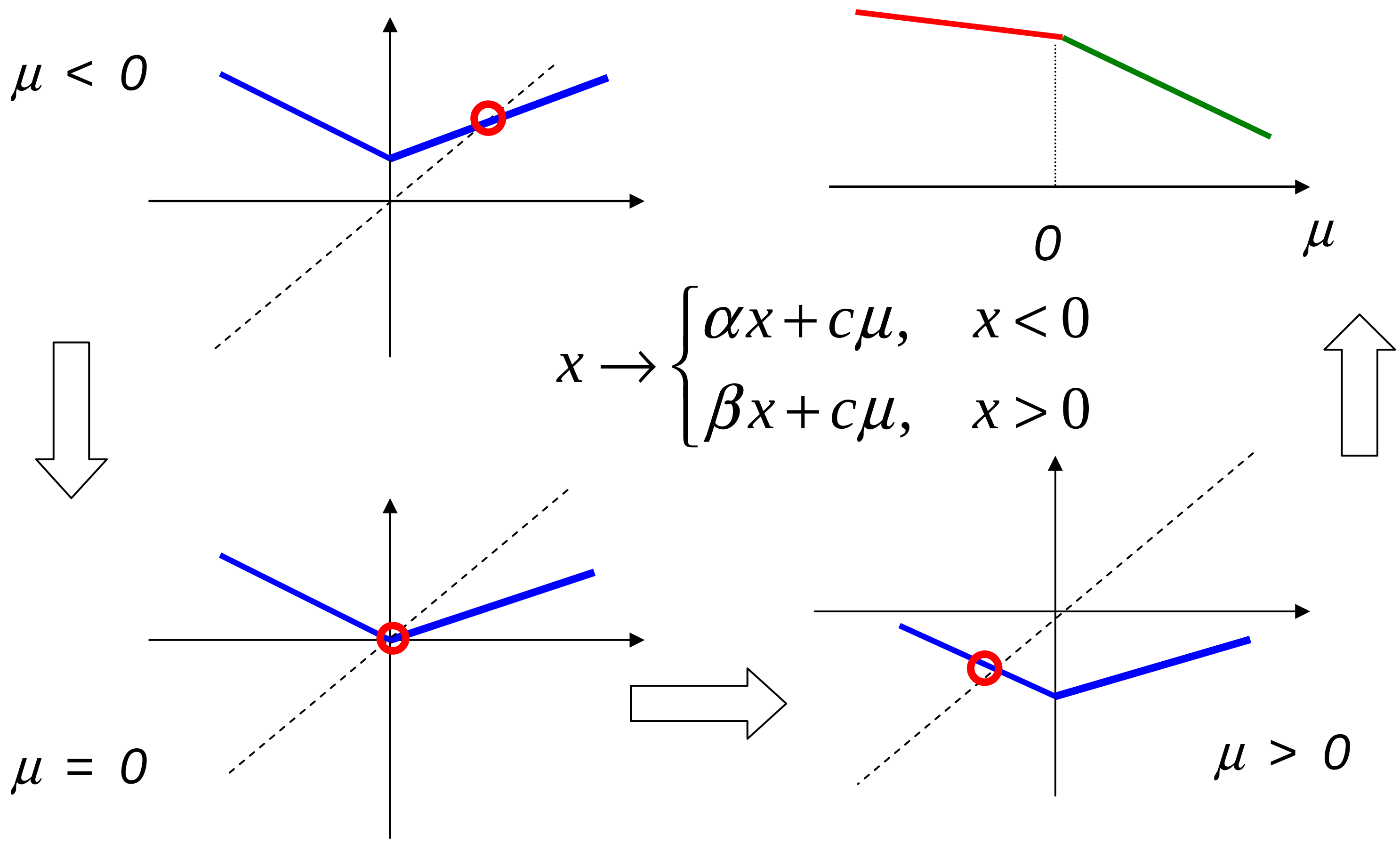
- $\mu \in (-\varepsilon, 0) \Rightarrow x^* \in S_1$
- $\mu \in (0, \varepsilon) \Rightarrow x^* \in S_2$
- $\mu = 0 \Rightarrow x^* \in \Sigma$
- $DF_1 \neq DF_2$ on Σ



Classifying border collisions

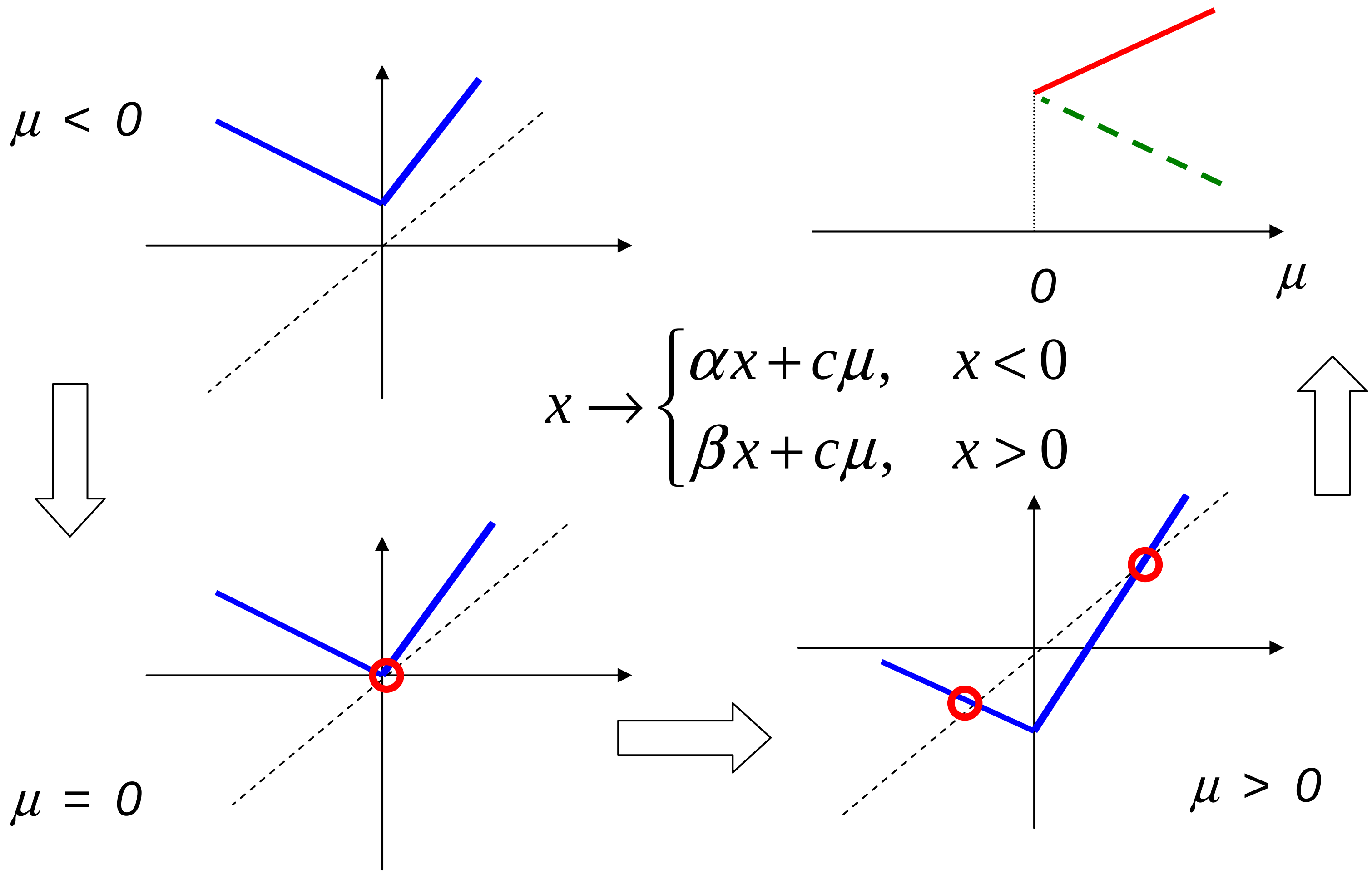
- Several scenarios are possible when a border-collision occurs
- They can be classified by observing the map eigenvalues on both sides of the boundary
- The phenomenon can be illustrated by a very simple 1D map where the eigenvalues are the slopes of the map on both sides of the boundary

Persistence



$$x \to \begin{cases} \alpha x + c\mu, & x < 0 \\ \beta x + c\mu, & x > 0 \end{cases}$$

Non-smooth saddle-node



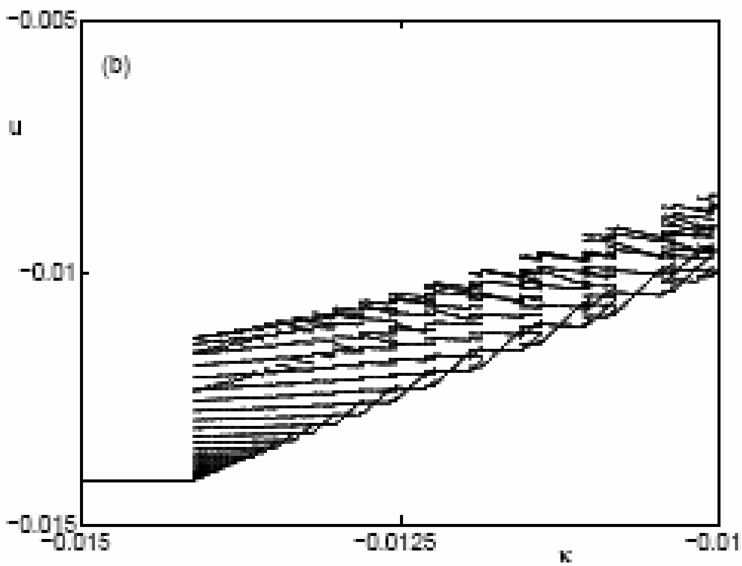
$$x \to \begin{cases} \alpha x + c\mu, & x < 0 \\ \beta x + c\mu, & x > 0 \end{cases}$$

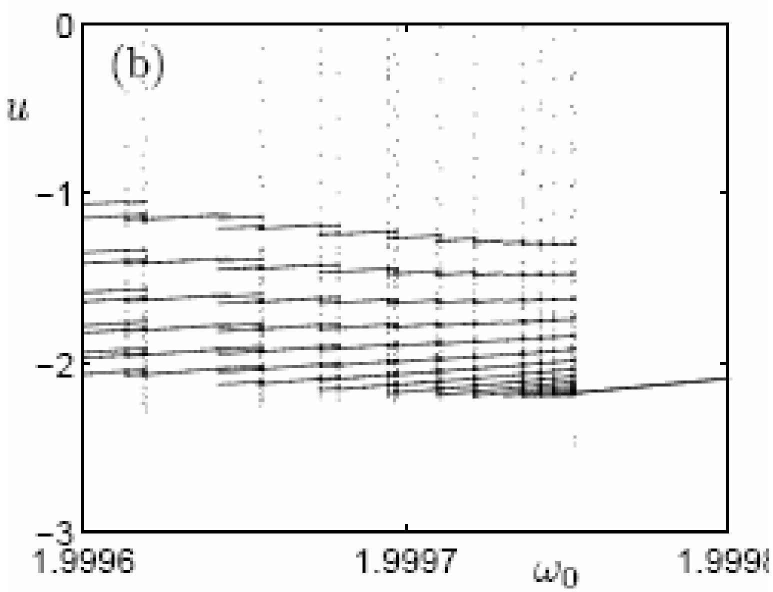
Border-collisions in the TCP/RED model

- The analysis has focussed mostly on continuous maps
- Recently proposed: further bifurcations are possible when the map is piecewise with a gap
- Complete classification method is available only for the one-dimensional case
- The TCP/RED case is a 2D map with a gap: its dynamics resemble closely those observed in very different systems: the impact oscillator considered by Budd and Piiroinen, 2006
- They might be explained in terms of border-collision bifurcations of 2D discontinuous maps

Numerical evidence

Cascades of corner-impact bifurcations in a forced impact oscillator show a striking resemblance to the phenomena detected in the TCP/RED model. They were explained in terms of border-collisions of local maps with a gap.





C. J. Budd and P. Piiroinen, "Corner bifurcations in nonsmoothly forced impact oscillators," to appear in *Physica D*, 2005.

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Conclusions

- We developed two discrete-time models for TCP Reno with RED
- S-TCP/RED models include:
  - slow start, congestion avoidance, fast retransmit, timeout, elements of fast recovery, and RED
- Proposed models were validated by comparing their performance to ns-2 simulations
- They capture the main features of the dynamical behavior of TCP/RED communication algorithms

Conclusions

- S-TCP/RED models were used to study bifurcations and chaos in TPC/RED systems with a single connection
- Bifurcations diagrams were characterized by period-adding cascades and devil staircases
- The observed behavior can be explained in terms of a novel class of piecewise-smooth maps with a gap

References: TCP and RED

[1] M. Allman, V. Paxson, and W. Steven, "TCP congestion control," *IETF Request for Comments (RFC) 2581*, Apr. 1999.

[2] B. Barden et al., "Recommendations on queue management and congestion avoidance in the Internet," *IETF Request for Comments (RFC) 2309*, Apr. 1998.

[3] R. Braden, "Requirements for Internet hosts: communication layers," *IETF Request for Comments (RFC) 1122*, Oct. 1989.

[4] C. Casetti, M. Gerla, S. Lee, S. Mascolo, and M. Sanadidi, "TCP with faster recovery," in *Proc. MILCOM 2000*, Los Angeles, CA, USA, Oct. 2000, vol. 1, pp. 320–324.

[5] K. Fall and S. Floyd, "Simulation-based comparison of Tahoe, Reno, and SACK TCP," *ACM Communication Review*, vol. 26, no. 3, pp. 5–21, July 1996.

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References: TCP and RED

[8] J. Padhye and S. Floyd, "On inferring TCP behavior," in *Proc. ACM SIGCOMM 2001*, San Diego, CA, USA, Aug. 2001, pp. 287–298.

[9] V. Paxson and M. Allman, "Computing TCP's retransmission timer," *IETF Request for Comments (RFC) 2988*, Nov. 2000.

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