SIMON FRASER UNIVERSITY
Department of Economics

Econ 809 | Prof. Kasa
Advanced Macroeconomic Theory | Spring 2005

PROBLEM SET 2 - Search and Matching
(Due February 24)

1. An unemployed worker samples wage offers on the following terms: Each period, with probability ϕ, $0 < \phi < 1$, she receives no offer (you can regard this as a wage offer of zero forever). With probability $1-\phi$ she receives an offer to work for w forever, where w is drawn from the cdf $F(w)$. Successive draws are independently and identically distributed over time. The worker chooses a strategy to maximize

$$E\sum_{t=0}^{\infty}\beta^t y_t$$

where $y_t = w$ if the worker is employed, and $y_t = c$ if the worker is unemployed. Assume that if a job offer is accepted, the worker stays on the job forever.

Let $v(w)$ be the expected value of $\sum_{t=0}^{\infty}\beta^t y_t$, for an unemployed worker who has offer w in hand and who behaves optimally from now on. Write down the Bellman equation for the worker's problem.

2. Each period an unemployed worker receives an offer to work forever at wage w_t, where $w_t = w$ in the first period and $w_t = \phi^t w$ after t periods on the job. Assume wages increase with tenure, ie, $\phi > 1$. Also assume the initial wage offer is drawn from a distribution $F(w)$ that is constant over time (ie, entry-level wages are stationary).

The worker's objective is to maximize $E\sum_{t=0}^{\infty}\beta^t y_t$ where $y_t = w_t$ if the worker is employed and $y_t = c$ if the worker is unemployed, where c can be interpreted as unemployment compensation. Let $v(w)$ be the optimal value of the objective function for an unemployed worker who has offer w in hand. Write down the Bellman equation for this problem. Show that, if two economies differ only in the growth rate of wages, eg, $\phi_1 > \phi_2$, then the economy with the higher wage growth has a lower reservation wage. Interpret the result. (Note: assume $\beta\phi_i < 1$, $i = 1, 2$).

3. Each period an unemployed worker receives an offer to work forever at wage w, where w is drawn from the distribution $F(w)$. Offers are i.i.d. Each worker also has another source of income, denoted by ϵ_t, which can be interpreted as financial/nonhuman wealth. Each period workers get a realization of ϵ_t, which is i.i.d., and is drawn from the distribution $G(\epsilon)$. Also assume that w_t and ϵ_t are independently distributed. A worker's objective is to maximize

$$E\sum_{t=0}^{\infty}\beta^t y_t$$

where $y_t = w + \phi\epsilon_t$ if the worker has accepted a job with wage w, and $y_t = c + \epsilon_t$ if the worker remains unemployed. To reflect the fact that an employed worker has less time to collect information on nonhuman wealth, assume that $\phi < 1$. Also assume $0 < \text{prob}[w \geq c + (1-\phi)\epsilon] < 1$.

Write down the worker's Bellman equation, and prove that the reservation wage increases with the level of nonhuman wealth. Interpret the result.

4. An economy is either in a boom (B) or recession (R), each with probability 1/2. The state of the economy is i.i.d. over time. At the beginning of each period, workers observe the state of the economy for that period. Each period an employed worker can choose to work at her last period's wage or draw a new wage. If she draws a new wage, the old wage is lost, b is received this period, and then she can start working at the new wage in the following period. During recessions new wages (for jobs starting next period) are i.i.d. draws from the cdf $F(w)$, where $F(0) = 0$ and $F(M) = 1$. During booms, workers can choose to quit and take <u>two</u> i.i.d. draws of a possible new wage (with the option of selecting the higher wage, assuming again that the job starts the following period) from the *same* cdf F that prevails during recessions. (These assumptions are meant to capture the idea that jobs are more plentiful during booms). Workers who are unemployed at the beginning of a period receive b this period and then draw either one (during recessions) or two (during booms) wage offers from the cdf F to start work the next period.

   Assume workers maximize $E_0 \sum_{t=0}^{\infty} \beta^t (1-\mu)^t I_t$, where μ is the probability that a worker dies at the end of the period, and I_t is the worker's income in period t, i.e., $I_t = w_t$ if employed and $I_t = b$ if unemployed.

   Write down the Bellman equation(s) for a previously employed worker. Compare reservation wages in booms and recessions. Do employed workers ever quit? If so, who quits and when do they quit? Interpret the results.

5. Consider a discrete-time matching model with infinitely lived, risk-neutral workers who are endowed with different skill levels. A worker of skill level i produces h_i goods each period that she is matched with a firm, where $i \in \{1, 2, \ldots N\}$ and $h_{i+1} > h_i$. Each skill type has its own (but identical) matching function $M(u_i, v_i) = A u_i^{\alpha} v_i^{1-\alpha}$, where u_i and v_i are the measures of unemployed workers and vacancies in skill market i. Firms incur a vacancy cost $c \cdot h_i$ each period a vacancy is posted (i.e., vacancy costs are proportional to the skill level). All matches are exogenously destroyed with probability $s \in (0, 1)$ at the beginning of a period. Unemployed workers receive unemployment compensation b. Finally, assume wages are determined by Nash bargaining between workers and firms, and let $\phi \in [0, 1)$ be the worker's weight in the Nash product, and assume $\phi = \alpha$.

   (a) Show analytically how the unemployment rate varies with skill level.

   (b) Assume a uniform distribution of workers across skill levels. For different benefit levels, show numerically how the aggregate steady state unemployment rate is affected by a mean-preserving spread in the skill distribution.

   (c) How do the results change if unemployment benefits are proportional to a worker's productivity?