SIMON FRASER UNIVERSITY
Department of Economics

Econ 815 — Prof. Kasa
Financial Economics I — Fall 2024

MIDTERM EXAM
(Due November 4, 6pm)

The first four questions are True, False, or Uncertain. Briefly explain. (10 points each).

1. Stocks are less risky over long holding periods.
2. According to the CAPM, stock prices follow random walks.
3. According to the CAPM, everyone has the same beliefs about asset returns.
4. According to the CAPM, it is not possible for investment managers to 'beat the market'.
5. (20 points). **Stochastic Volatility**. In class we solved the Merton problem when the 'investment opportunity set' was constant (ie., μ and σ are constants). This question asks you to consider the case where volatility is stochastic. There is strong empirical evidence to support this. Hence, now suppose the risky asset price follows the process

$$\frac{dS}{S} = \mu dt + \sigma_t dB$$

where σ_t also follows a geometric Brownian motion process

$$d\sigma_t = \sigma_t dB^\sigma$$

For simplicity, suppose dB and dB^σ are uncorrelated. Finally, continue to assume the investor has time-additive CRRA preferences

$$V(W,\sigma) = \max_{c,\pi} E_0 \int_0^\infty \frac{C^{1-\gamma}}{1-\gamma} e^{-\delta t} dt$$

subject to $dW = [(r + \pi(\mu - r))W - C]dt + \pi\sigma_t W dB$.

(a) Write down the investor's stationary HJB equation.

(b) Verify that a solution is of the form $V(W,\sigma) = f(\sigma)W^{1-\gamma}$.

(c) Derive a 2nd-order ODE for $f(\sigma)$. Under what parameter restrictions do you get an economically sensible result? (Bonus: Can you solve it? Hint: Ever heard of a Bessel function?)

(d) Is the investor's optimal portfolio still time invariant? Why or why not?

(e) Briefly discuss how your answers would change if dB and dB^σ were correlated.

6. (15 points). Part of the appeal of options is that they can be combined to form very flexible payoff profiles. A couple examples were discussed in class. Here you are asked to consider a few more. For each, illustrate the expiration date payoff and profit from the position.

(a) A *bullish vertical spread*, which is created by buying a call option with strike price K_1, and simultaneously selling a call option (on the same stock) with strike price $K_2 > K_1$. Why is it called a 'bullish' spread? (Hint: Remember that, all else equal, call options with lower strike prices are more expensive).

(b) A *strangle*, which involves buying out-of-the-money call and puts on the same underlying stock (for the same expiration date). That is, if the current stock price is S, the call has strike price $K_c > S$ and the put has strike price $K_p < S$. (Hint: This is similar to a straddle, but is cheaper, since the options are purchased out-of-the-money).

(c) A *collar*, which involves holding the underlying stock, while simultaneously buying an out-of-the-money put and selling/writing an out-of-the-money call. Why might this strategy be attractive? How does it compare to a bullish vertical spread?

7. (25 points). **Man vs. Machine**. Consider a stock which has a price that follows the following geometric Brownian motion process:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

where $\mu = .12$ and $\sigma = .20$. Suppose the current stock price is \$42, and suppose we are interested in the value of a 6-month (European) call option on this stock. Assume the risk-free rate is constant, and equal to 10%.

(a) Suppose the 'strike price' of the option is $K = 40$. Use the Black-Scholes formula derived in class to compute the value of the option. (Hint 1: Note that the time unit here is a year, so that for a 6-month option we have $T - t = 0.5$. Hint 2: Is μ a relevant parameter? Why, or why not?).

(b) Now suppose you trust computers more than math. Write a simple program (using the software of your choice) to numerically calculate the value of the option. Do you get the same answer as in part (a)?