NEW YORK UNIVERSITY
Department of Economics

Econ-GA 2021 — Prof. Kasa

Financial Economics I — Fall 2018

PROBLEM SET 4
(Due November 12)

Each of the following questions is worth 10 points.

1. Part of the appeal of options is that they can be combined to form very flexible payoff profiles. A couple of examples were discussed in class. Here you are asked to consider a few more. For each, illustrate the expiration date payoff and profit from the position.

   (a) A *bullish vertical spread*, which is created by buying a call option with strike price K_1, and simultaneously selling a call option (on the same stock) with strike price $K_2 > K_1$. Why is it called a 'bullish' spread? (Hint: Remember that, all else equal, call options with lower strike prices are more expensive).

   (b) A *strangle*, which involves buying out-of-the-money call and puts on the same underlying stock (for the same expiration date). That is, if the current stock price is S, the call has strike price $K_c > S$ and the put has strike price $K_p < S$. (Hint: This is similar to a straddle, but is cheaper, since the options are purchased out-of-the-money).

   (c) A *collar*, which involves holding the underlying stock, while simultaneously buying an out-of-the-money put and selling/writing an out-of-the-money call. Why might this strategy be attractive? How does it compare to a bullish vertical spread?

2. **Man vs. Machine**. Consider a stock which has a price that follows a geometric Brownian motion:
$$\frac{dS}{S} = \mu dt + \sigma dB$$
where $\mu = .12$ and $\sigma = .20$. Suppose the current stock price is \$42, and suppose we are interested in the value of a 6-month (European) call option on this stock. Assume the risk-free rate is constant, and equal to 10%.

   (a) Suppose the strike price of the option is $K = 40$. Use the Black-Scholes formula to compute the value of the option. (Hint: Note that the time unit here is a year, so that for a 6-month option $T - t = 0.5$).

   (b) Now suppose you trust computers more than math. Write a simple program to numerically calculate the value of the option. Do you get the same answer as in part (a)?

3. **Knockout Options**. In class we focused on 'plain vanilla' options. In practice, many so-called 'exotic' options are traded. One example is a 'knockout option'. These options terminate worthless if the price of the underlying asset hits a threshold. For calls they are called 'down and out' options, because the option is terminated if the stock price falls to a certain level. In general, their values must be computed numerically, but let's consider an example that can be computed analytically. In particular, consider a *perpetual* down and out option on a non-dividend paying stock (i.e., as long as the stock price remains above the threshold, it never expires). In this case, the Black-Scholes PDE becomes an ODE.

(a) Write down the ODE that characterizes the value of the perpetual knockout option.

(b) Show that the general form of the solution is

$$C(S) = a_1 S + a_2 S^{-\gamma}$$

where (a_1, a_2) are constants, and $\gamma = 2r/\sigma^2$.

(c) What are the relevant boundary conditions at $C(L)$ and $C(\infty)$? Use these boundary conditions to to solve for (a_1, a_2).

(d) Note that the value of a perpetual call is just S (if you can buy the stock whenever you want, with no knockout, then the price of that option is simply the current stock price). Given this, provide an economic interpretation of your formula.