CMPT 308 - Computability and Complexity: Problem Set 2

1. **TM design** For each of the following languages over the alphabet $\{0, 1\}$, write a single-tape Turing machine program that decides that language. First, explain your TM algorithm at a high level using English. Then provide a complete description of the TM program that can be run on the TM simulator linked off the course webpage. You need to provide a text file with the description of your TM that can be loaded into the TM simulator (by "copy-paste" into the appropriate window for a new program), and then run on any input. To get full marks, you need to provide a file that can be run by the TA on the TM simulator! (Please email your file to the TA.)

   (a) $L = \{w \mid w \in \{0, 1\}^*$ has an equal number of 0s and 1s$\}$.

   (b) $L = \{ww \mid w \in \{0, 1\}^*\}$ (i.e., all strings that are the concatenation of two copies of the same string).

2. **Closure properties** Show that the collection of *decidable* languages is closed under the operations of (a) concatenation, and (b) intersection.

3. **Enumerators** An *enumerator* is a TM with an extra write-only tape (think of it as a printer attached to the TM). During its computation, an enumerator may print out some number of strings (maybe an infinite number, if the enumerator TM runs forever). Show that a language L is decidable if and only if some enumerator TM exists that prints out all the strings in L in the lexicographic order.

4. Let C be a language. Prove that C is semi-decidable if and only if there exists a decidable language D such that $C = \{x \mid \exists y \langle x, y\rangle \in D\}$.

5. Consider the language

   $O = \{\langle M\rangle \mid$ TM M accepts every string of *odd* length, but no string of *even* length$\}$.

   (a) Is O decidable?

   (b) Is O semi-decidable?

   (c) Is its complement $\bar{O}$ semi-decidable?

   Justify your answers.