Practice Questions for Chapter 17

Note: Not for grading.

1. A coin is tossed three times. Let the random variable X denote the number of heads in the tosses less the number of tails.
   a. Find and graph the pdf of X.
   b. Find and graph the cdf of X.
   c. Determine the mean and variance of X.

2. The following table gives the joint probability distribution between employment status and university graduation status among those either employed or looking for work (unemployed) in the working age population in Canada.

	***Unemployed Y*=0**	***Employed Y*=1**
***Non-University grads X*=0**	0.045	0.709
***University grads X*=1**	0.005	0.241

   a. Calculate the mean of Y.
   b. Demonstrate the Law of Iterated Expectation explicitly for $E(Y)$.
   c. Are university graduation status and employment status independent?
   d. What is the unemployment rate for university graduates? The unemployment rate is defined as the fraction of the labour force that is unemployed.

3. Suppose that two dice are tossed one time. Let X denote the number of 2's that appear and Y the number of 3's.
   a. Find the joint pdf for X and Y.
   b. Suppose a third random variable, Z, is defined, where $Z = X + Y$. Find the pdf of Z.

4. X and Y are discrete random variables with the following joint probability distribution:

		Y				
		14	**22**	**30**	**40**	**65**
X	**1**	0.02	0.05	0.10	0.03	0.01
	5	0.17	0.15	0.05	0.02	0.01
	8	0.02	0.03	0.15	0.10	0.09

   a. Calculate the marginal distribution of Y.
   b. Calculate the mean and variance of Y.
   c. Calculate the conditional distribution of Y given X.
   d. Calculate the conditional mean and conditional variance of Y given $X = 8$.
   e. Show the Law of Iterated Expectation for $E(Y)$.
   f. Determine whether X and Y are statistically independent.
   g. Calculate the covariance and correlation between X and Y.

5. Show the following property holds for any random variables X, Y and V and any constants *a* and *b*:

$$Cov(aX + bV, Y) = a\sigma_{XY} + b\sigma_{VY}.$$

6. Show the following property holds for any random variables X and Y and any constants *a* and *b*:

$$Var(aX + bY) = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\sigma_{XY}.$$

7. Using Table B-7 on page 595, find the following probabilities. Provide a graph shading in the relevant area with your answer.
   a. If $Y \sim N(1,4)$ find $\Pr(Y \leq 3)$.
   b. If $Y \sim N(50,25)$ find $\Pr(40 \leq Y \leq 52)$.
   c. If $Y \sim N(5,2)$ find $\Pr(6 \leq Y \leq 8)$.

8. Using Table B-1 on page 585, approximate the following probabilities. Provide a graph shading in the relevant area with your answer.
   a. If $Y \sim t_{15}$ find $\Pr(Y \leq 1.9)$.
   b. If $Y \sim t_{60}$ find $\Pr(Y \geq 1.5)$.

9. A sample of randomly selected economics majors shows that when they took their first job they received the following starting salaries:

$30,200	$37,500	$36,700	$40,300	$30,300	$43,200
$40,500	$37,100	$30,400	$29,900	$40,900	$23,100
$38,800	$37,800	$31,200	$36,200	$34,100	$36,700

Use a 5% level of significance to test whether the true mean starting salary differs from $39,000. Conduct this test three ways using: a test statistic and a critical value, a p-value, and a confidence interval.

10. For a random sample $X_1, X_2, \ldots, X_n$ with $Var(X_i) = \sigma^2$ for all observations, show that $Var(\bar{X}) = \sigma^2/n$.