

### Sample Term Test

Duration: 90 minutes

Answer all 4 questions. Non-programmable calculators are permitted. **The test should be written in ink. If it is written in pencil it will not be accepted for re-grading.**

Show all work and formulas used.

1. Suppose you would like to learn about the demand for new cars. The population regression model is given by:

$$cars_i = \beta_0 + \beta_1 price_i + \beta_2 income_i + \beta_3 intrate_i + \varepsilon_i$$

where, *cars* is the number of new car sales, *price* is the new car price index, *income* is per-capita real disposable income, *intrate* is the interest rate. The following table presents estimates and standard errors of this model:

Dependent Variable: CARS

Method: Least Squares

Sample: 1 35

Included observations: 35 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PRICE	-0.07	0.04		
INCOME	0.003	0.001		
INTRATE	-0.15	0.07		
C	-7.45	13.5		

The adjusted R-squared from this regression is 0.63. The R-squared from this regression is 0.66.

- How useful is this regression for predicting the variation in the demand for new cars? [5]
- You expect the variable *price* to have a negative coefficient. Test the appropriate hypothesis to evaluate this expectation at the 5% level. Use a p-value. [5]
- You expect the variable *income* to have a positive coefficient. Test the appropriate hypothesis to evaluate this expectation at the 10% level. Use a test statistic and critical value. [5]
- Test at the 1% level whether or not the *interest rate* is significantly different from -0.20. Use a confidence interval. [5]

2. The following table gives the joint probability distribution between employment status and university graduation status among those either employed or looking for work (unemployed) in the working age population in Canada.

	<i>Unemployed Y=0</i>	<i>Employed Y=1</i>
<i>Non-University grads X=0</i>	0.045	0.709
<i>University grads X=1</i>	0.005	0.241

- Calculate the mean of  $Y$ . [3]
  - Demonstrate the Law of Iterated Expectation explicitly for  $E(Y)$ . [6]
  - Are university graduation status and employment status independent? [3]
  - What is the unemployment rate for university graduates? The unemployment rate is defined as the fraction of the labour force that is unemployed. [3]
3. Show the following property holds for any random variables  $X, Y$  and  $V$  and any constants  $a$  and  $b$  [5]:

$$\text{Cov}(aX + bV, Y) = a\sigma_{XY} + b\sigma_{VY} .$$

4. Consider the simple regression model  $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ . Suppose instead of using the least squares estimator  $\hat{\beta}_1$  you use another estimator, say  $b_1$ , which is given by

$$b_1 = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}$$

- Determine whether  $b_1$  is an unbiased estimator of  $\beta_1$ . Justify all steps. [5]
- Determine the value of  $R^2$  for a regression that yields a  $\hat{\beta}_1 = 0$ . Justify your answer. [5]