Important Distributions

In testing hypotheses on econometric models, four distributions are mainly used. These are the ***normal***, ***chi-square***, ***Student's t-*** and ***Fisher's F-*** *distributions*.

The Normal Distribution

Also called the Gaussian distribution.

If the random variable X has a normal distribution with mean μ and variance σ^2, then we denote this as $X \sim N(\mu, \sigma^2)$.

Often very useful to **standardize** variables so that they have a mean of 0 and a variance of 1:

$$Z = \frac{X - \mu}{\sigma}$$

The random variable Z has a standard normal distribution, denoted as $Z \sim N(0, 1)$.

The Chi-Square Distribution

The distribution of the sum of squares of *n* independent standard normal random variables is called the chi-square (χ^2) distribution with *n* degrees of freedom. It is written as χ_n^2.

Consider *n* random variables Z_1, Z_2, ..., Z_n, all of which are independently and identically distributed (**iid**) as standard normal N(0,1). If we define a new random variable *U* that is the sum of the squares of the *Z*'s, we have

$$U = Z_1^2 + Z_2^2 + \cdots + Z_n^2.$$

The distribution of *U* is χ_n^2, $U \sim \chi_n^2$.

The Student's t-Distribution

The distribution of the ratio of a normal to the square root of an independent χ_n^2 is called the Student's t-distribution (or just t-distribution) with n degrees of freedom. It is written as t_n.

If $Z \sim N(0,1)$ and $U \sim \chi_n^2$ with Z and U independent, then

$$t = \frac{Z}{\sqrt{U/n}} \sim t_n$$

The F-Distribution

This distribution is the ratio of two independent chi-squares. If $U \sim \chi^2_m$ and $V \sim \chi^2_n$ and U and V are independent, then

$$F = \frac{U/m}{V/n} \sim F_{m,n}$$