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Econ 301

THE UTILITY FUNCTION AND INDIFFERENCE CURVES

These notes are intended to make clear the difference and the relationship between an indifference curve (IC) and the utility function.

The **utility function**, $u(x_1, x_2)$ has *two arguments* (inputs) - the quantities of good 1 (x_1) and good 2 (x_2) contained in some consumption bundle (x_1, x_2) and assigns a number (like 1, 10, 20) that corresponds to the level of utility (happiness) obtained from this particular bundle. Take for example the utility function $u(x_1, x_2) = x_1 x_2$. For *any bundle* (x_1, x_2) it assigns its level of utility which can be very different across bundles. For example $u(1, 2) = 2$, $u(100, 3) = 300$, $u(4, 1) = 4$. The utility function represents the preferences of a consumer - knowing the assigned utility value we know if a bundle is preferred to another one. A bundle A will be preferred to B if A yields a higher utility than B. For example the bundle $(100, 3)$ is clearly preferred to the bundle $(4, 1)$ since the first has a utility of 300 while the second has only utility of 4. Bundles that we are indifferent among get assigned *the same level* of utility (i.e. they make us equally happy). For example a person with utility $u(x_1, x_2) = x_1 x_2$ would be indifferent among the bundles (4,3), (12,1), (2,6) since they all yield a utility level of 12.

This is where the indifference curves come into play. An **indifference curve** is a curve that passes through all bundles (x_1, x_2) that yield the same level of utility. Thus an IC is a function between the quantity of good 1 and quantity of good 2, i.e. a function $x_2(x_1) - x_2$ as a function of x_1 which is a function with just *one argument* and represents a set of points that yield the same utility. Thus we can obtain the function corresponding to an indifference curve by setting the utility function to some fixed value, e.g. 12 and then express x_2 in terms of x_1 from there. For example if we want to find the IC corresponding to a utility level of 12 for the utility function $u(x_1, x_2) = x_1 x_2$, we set $x_1 x_2 = 12$ which tells us that the equation of the indifference curve is $x_2 = \frac{12}{x_1}$ which expresses x_2 as a function of the single variable x_1. For any value of x_1 we can find the corresponding value of x_2 such that we remain on the same indifference curve: e.g. if $x_1 = 1, 2, 3, 4$ then we get $x_2 = 12, 6, 4, 3$, saying that the points $(1, 12)$, (2,6), (3,4) and (4,3) are on the same IC - the one that yields a utility level of 12.

For the *same* utility function however we can find the equations of infinitely many ICs - we can set the utility to any arbitrary number, k (like 12) and follow the above procedure to find the IC corresponding to this utility level which in our example would be $x_2 = \frac{k}{x_1}$.

To conclude, we see that the utility function and the indifference curves are **not the same thing**! The indifference curve is just a curve connecting points with the same utility level (same value of $u(x_1, x_2)$) but for any such value we get a different IC while the utility function is kept the same.