Validity of Quasi-Static Approximation in Dark Energy / Modified Gravity Theories

16th CCGRRA

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Work in Progress

Joint work with L.Pogosian, D.Steer, D.Langlois.

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Motivation

Why looking beyond the standard cosmological model (ΛCDM+GR)?

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Conference Advertisement!

Testing Gravity. 25-29 January, 2017, Vancouver.

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Explore parameter space of the model. For example Hu-Sawicki $f(R)$ gravity: f_{R_0}, n

$$f(R) = R - 2\Lambda + \frac{f_{R_0}}{n} \frac{R_0^{n+1}}{R^n}$$

Testing Gravity

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1. "From scratch": modify the action S. Solve all the equations.
2. Parametrize the deviations from GR (phenomenological).

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Phenomenological parametrization.

- $\mu(a,k)$: deviation from GR in the Poisson equation
  $k^2\Psi = -\frac{3}{2}\Omega_M \mu(a,k)\rho(a)\delta(a,k)$
- $\gamma(a,k)$: deviation from GR in the gravitational slip equation
  $\Phi/\Psi = \gamma(a,k)$

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)\delta_{ij}dx^i dx^j$$

For a specific model, the functions μ and γ are obtained using the Quasi-Static Approximation.

Quasi-Static Approximation

Scalar-Tensor theory:

$$S_{\text{tot}} = S_g(g_{\mu\nu}, \pi) + S_m(g_{\mu\nu}, \chi)$$

The variational principle gives us

- $\delta_{g_{\mu\nu}} S_g = 0 \rightarrow$ (Modified) Einstein equations.
- $\delta_\pi S_{\text{tot}} = 0 \rightarrow$ Dynamical equation for π.
- $T^{\mu\nu}_{;\nu} = 0$. (Minimally coupled matter)

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Combine above equations to get:

$$\ddot{\Psi} + A(k_H^2, t)\dot{\Psi} + B(k_H^2, t)\Psi = C(k_H^2, t)\delta + D(k_H^2, t)v$$

$$E(\Psi, \Phi) = 0$$

Quasi-Static Approximation

Consider sub-horizon scales $k_H^2 \gg 1$.
Quasi-Static Limit: assume $k_H^2 \Psi \gg \ddot{\Psi}, \dot{\Psi}$. The equation for Ψ becomes

$$k^2 \Psi = -\frac{3}{2} \Omega_M f(a,k) \delta$$

This is the reason why MG is often parametrized through $\mu(a,k), \gamma(a,k)$ functions.

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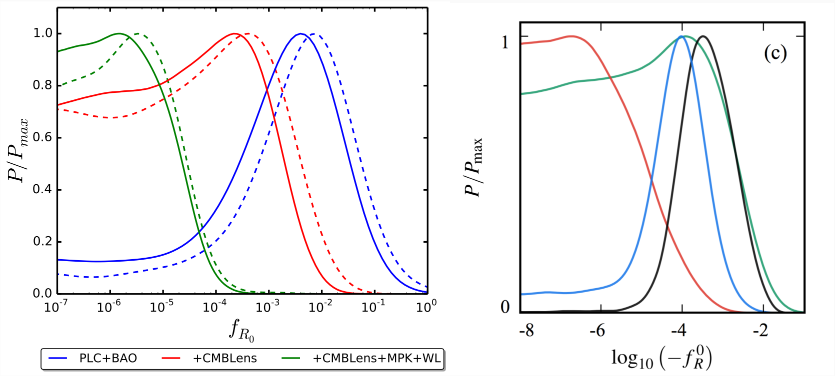
Problem!
The scalar field π could induce oscillations in the potential Ψ endangering the quasi-static approximation validity.

People don't want to rely on the Quasi-Static Approximation.

Quasi-Static Approximation



using QS approx.: $f_{R_0} < 3 \times 10^{-3}$

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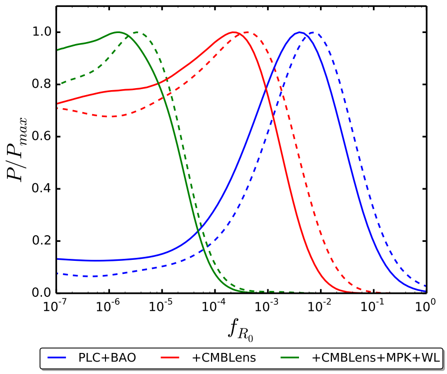

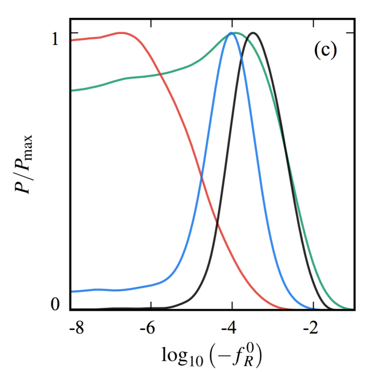

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1. What are the MG/DE theories for which the QS approximation works?
2. What is the range of validity of the QS approximation?
3. How sensitive is cosmological data to deviations from the QS approximation?

Validity of Quasi-Static Approximation

Approach

EFT of Dark Energy

Most general Lagrangian in the context of linear perturbations about FLRW.

Each MG/DE theory is specified by 5 time dependent functions:

$$\alpha_B, \alpha_T, \alpha_M, \alpha_K, \alpha_H.$$

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Starting Point:

Equation for Ψ:

$$\begin{aligned}
&\ddot{\Psi} + \frac{\beta_1\beta_2 + \beta_1\beta_3 k_H^2}{\beta_1 + \alpha_B^2 k_H^2} H\dot{\Psi} + \frac{\beta_1\beta_4 + \beta_1\beta_5 k_H^2 + c_s^2\alpha_B^2 k_H^4}{\beta_1 + \alpha_B^2 k_H^2} H^2\Psi \\
&= -\frac{3}{2}\Omega_M H^2 \frac{\beta_1\beta_6 + \beta_7\alpha_B^2 k_H^2}{\beta_1 + \alpha_B^2 k_H^2}\delta - \frac{3}{2}\Omega_M H^2 \frac{\beta_1\beta_8 + \beta_9\alpha_B^2 k_H^2}{\beta_1 + \alpha_B^2 k_H^2} Hv.
\end{aligned} \tag{1}$$

Gravitational "slip" equation:

$$\begin{aligned}
&\alpha_B^2 k_H^2 (\Phi - \Psi\frac{\xi}{\alpha_B}) + \beta_1\left[\Phi - \Psi(1+\alpha_T)\left(1 + \hat{\alpha}\frac{\alpha_T - \alpha_M}{2\beta_1}\right)\right] \\
&= \frac{\alpha_T - \alpha_M}{2}\left\{\hat{\alpha}\frac{\dot{\Psi}}{H} + 3\Omega_M\alpha_B\delta + 3\Omega_M\frac{\alpha_K - 6\alpha_B}{2}Hv\right\}
\end{aligned} \tag{2}$$

Approach

Sub-horizon scales $k_H^2 \gg 1$. Natural expansion in powers of $1/k_H^2$.

$$\ddot{\Psi} + \beta_3 H \dot{\Psi} + \left[\frac{\beta_1}{\alpha_B^2}(\beta_5 - c_s^2) + c_s^2 k_H^2\right] H^2 \Psi$$
$$= -\frac{3}{2}\Omega_M H^2 \left[\beta_7 + \frac{1}{k_H^2}\frac{\beta_1}{\alpha_B^2}(\beta_6 - \beta_7)\right]\delta - \frac{3}{2}\Omega_M H^2 \left[\beta_9 + \frac{1}{k_H^2}\frac{\beta_1}{\alpha_B^2}(\beta_8 - \beta_)\right] H v. \tag{3}$$

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Continuity and Euler equations:

$$\dot{\delta} = k_H^2 H^2 v + 3\dot{\Psi}, \quad (4)$$
$$\dot{v} = -\Phi. \quad (5)$$

Hierarchy of variables:

$$H^2 v \sim \mathcal{O}\left(\frac{1}{k_H^2}\delta\right), \quad \Psi, \Phi \sim \mathcal{O}\left(\frac{1}{k_H^2}\delta\right).$$

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Define a parameter ϵ that *counts* the intrinsic dependence on $1/k_H^2$.

Splitting the potential

IDEA (*Bellini, Sawicki (2015)*): split the potential Ψ in

$$\Psi = \underbrace{\Psi_{QS}}_{\text{quasi-static part}} + \underbrace{\psi}_{\text{oscillation}}$$

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We need an "ansatz" for Ψ_{QS}. Following the usual QS approximation we define (up to first order in ϵ)

$$c_s^2 k_H^2 \Psi_{QS} = -\frac{3}{2}\Omega_M \left[A_1 + \frac{\epsilon}{k_H^2} A_2\right]\delta - \frac{3}{2}\Omega_M \epsilon B_1 H v.$$

Oscillation equation

Substitute in the QS ansatz in the equation for Ψ (up to first order in ϵ)

$$\epsilon(\ddot{\Psi}_{QS} + \ddot{\psi}) + \beta_3 H \epsilon(\dot{\Psi}_{QS} + \dot{\psi}) + \left[\frac{\beta_1}{\alpha_B^2}(\beta_5 - c_s^2) + c_s^2 \frac{k_H^2}{\epsilon}\right] H^2 \epsilon(\Psi_{QS} + \psi)$$

$$= -\frac{3}{2}\Omega_M H^2 \left[\beta_7 + \frac{\epsilon}{k_H^2}\frac{\beta_1}{\alpha_B^2}(\beta_6 - \beta_7)\right]\delta - \frac{3}{2}\Omega_M H^2 \beta_9 H \epsilon v + \mathcal{O}(\epsilon^2)$$

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Take derivatives w.r.t. t of the QS *ansatz*.

Use continuity and Euler equations to eliminate the dependence on $\dot{\delta}$ and $\dot{v}$.

$$\epsilon\ddot{\psi} + \beta_3 H \epsilon \dot{\psi} + \left[\frac{\beta_1}{\alpha_B^2}(\beta_5 - c_s^2) + \frac{3}{2}\Omega_M \frac{A_1}{c_s^2}\frac{\xi}{\alpha_B} + c_s^2 \frac{k_H^2}{\epsilon}\right] H^2 \epsilon \psi$$
$$= -\frac{3}{2} H^2 \Omega_M \left[\Big(\beta_7 - A_1\Big) + \frac{\epsilon}{k_H^2}\mathcal{G}_1\right]\delta - \frac{3}{2} H^2 \Omega_M \Big(\beta_9 - \mathcal{G}_2\Big)\epsilon H v + \mathcal{O}(\epsilon^2),$$

Gravitational Instability

How does ψ affect the growth of δ?

$$\dot{\delta} = k_H^2 H^2 v + 3\Psi, \dot{v} = -\Phi.$$

$$\ddot{\delta} + \left(2H + \frac{\epsilon}{\mathcal{C}_1}\right)\dot{\delta} - \left(\frac{3}{2}\Omega_M \frac{\beta_7}{c_s^2} H^2 + \frac{3}{2}\Omega_M \frac{\alpha_T - \alpha_M}{\alpha_B} - \frac{\epsilon}{k_H^2}\mathcal{C}_2\right)\delta = S[\psi]$$

$$S[\psi] = -k_H^2 H^2 \psi + \epsilon \mathcal{C}_3(k_H^2, \psi, \dot{\psi}, \ddot{\psi})$$

Work in Progress..

- Gravitational Collapse δ
- Lensing: Weyl Potential $\Phi_{+} = (\Phi + \Psi)/2$
- Check the QS limit for specific models

Thank you!