A big bounce, slow-roll inflation and dark energy from conformal gravity

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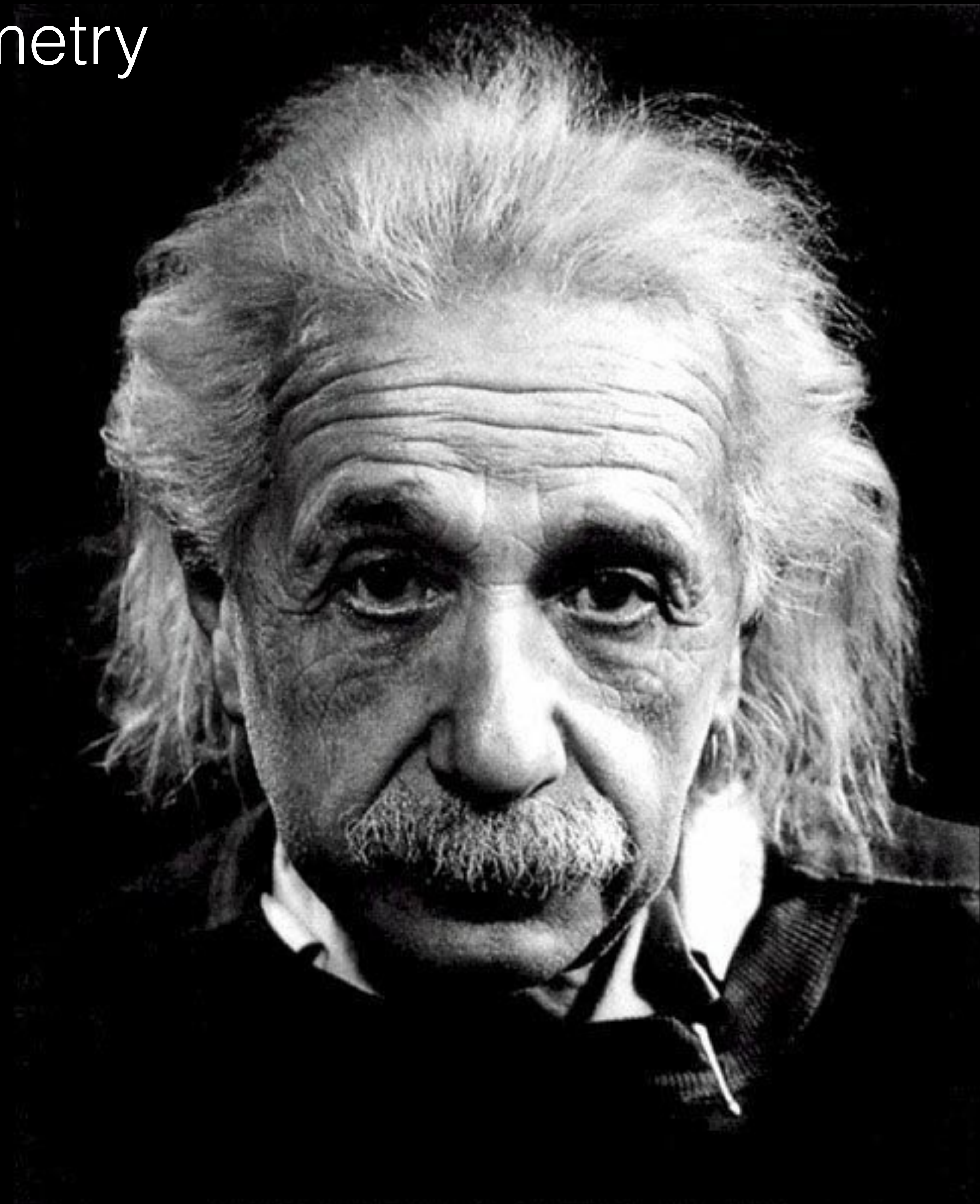
motivation

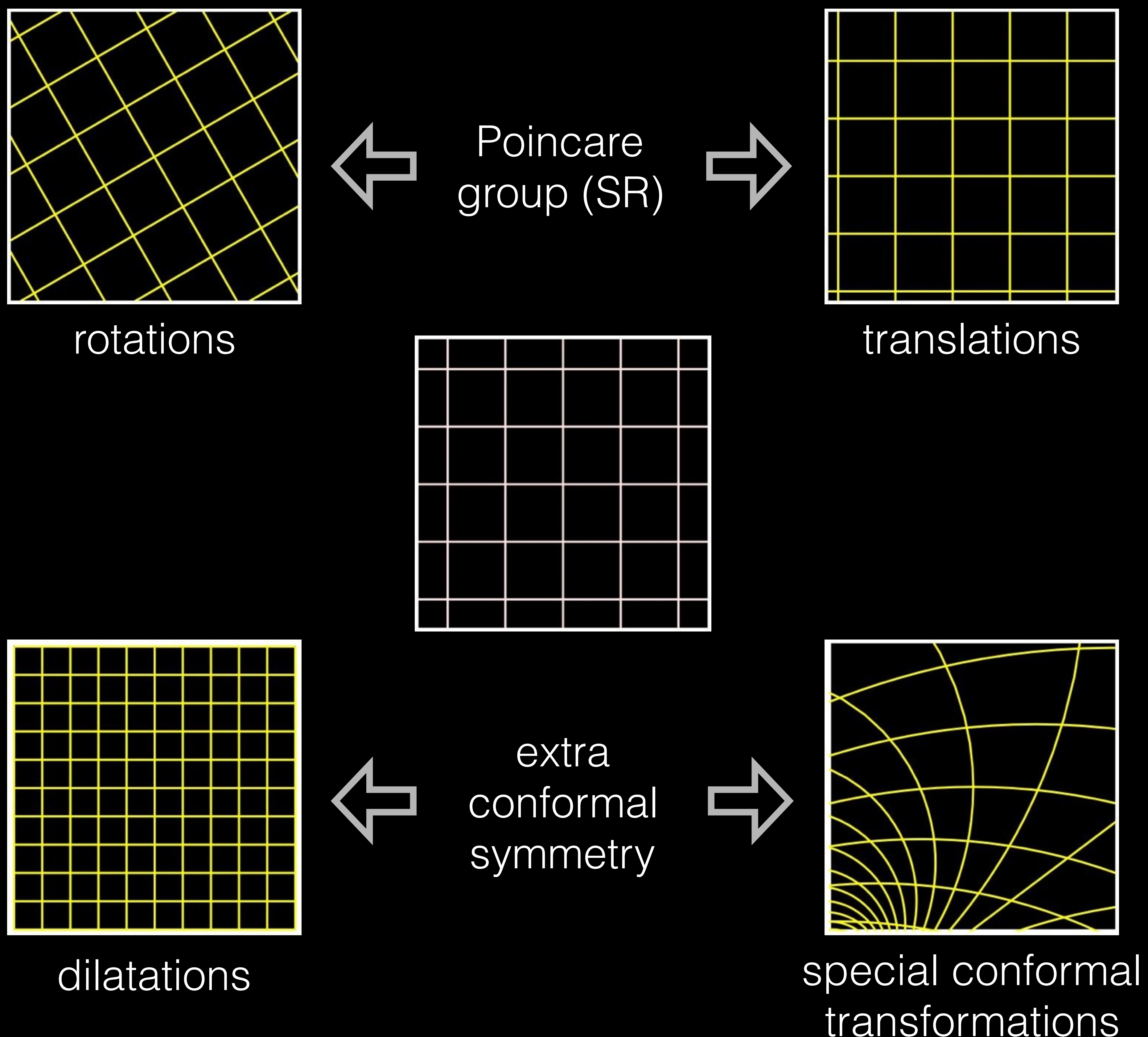
special relativity (SR): a theory of mechanics which respects the Lorentz symmetry of electromagnetism (EM)

general relativity (GR): a theory of gravitation with local Lorentz symmetry

however: EM is invariant under a wider group of symmetries than either SR or GR



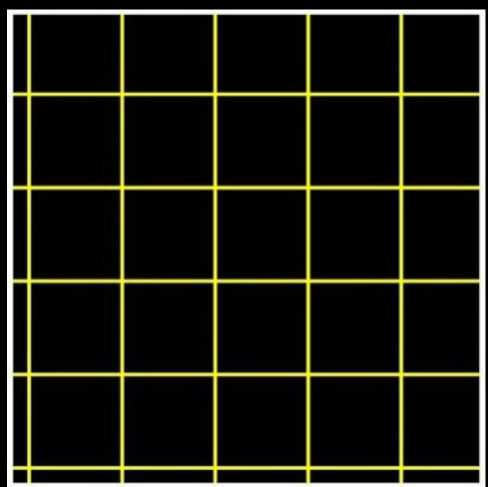
EM is invariant under the conformal group SO(4,2) made of:



can we construct a gravitational theory with conformal symmetries similar to EM?

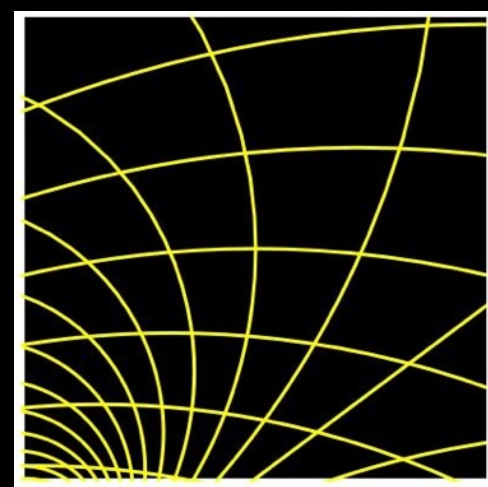
one approach: Yang Mills (YM) gravity

4 generators: $\mathbf{P}_a$



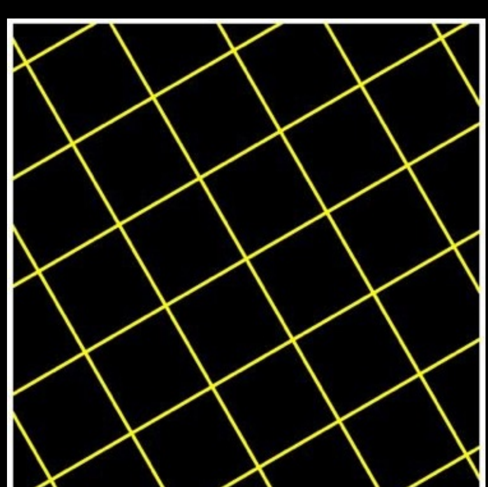
translations

4 generators: $\mathbf{K}_a$



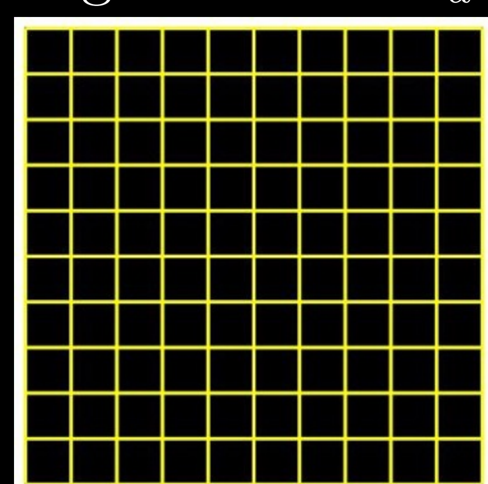
special conformal transformations

6 generators: $\mathbf{J}_{ab} = -\mathbf{J}_{ba}$



rotations

1 generator: $\mathbf{D}_a$



dilatations

SO(4,2)

use generators to form a vector potential:

$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

$$\mathbf{J}_A = \{\mathbf{P}_a, \mathbf{K}_a, \mathbf{J}_{ab}, \mathbf{D}\}$$

define field strength $\mathbf{F}_{\alpha\beta} = F^A_{\alpha\beta}\mathbf{J}_A$ of vector potential:

$$F^A_{\alpha\beta} = \partial_\alpha A^A_\beta - \partial_\beta A^A_\alpha + f^A{}_{BC} A^B_\alpha A^C_\beta$$

$$[\mathbf{J}_A, \mathbf{J}_B] = f^C{}_{AB}\mathbf{J}_C$$

write down YM action

$$S = -\frac{1}{2g^2_{\rm YM}} \int d^4x \sqrt{-g}\, g^{\alpha\mu} g^{\beta\nu} h_{AB} F^A_{\alpha\beta} F^B_{\mu\nu} + S_{\rm m}$$

$$h_{AB} = \mathrm{Tr}(\mathbf{J}_A \mathbf{J}_B) = f^M{}_{AN} f^N{}_{BM}$$

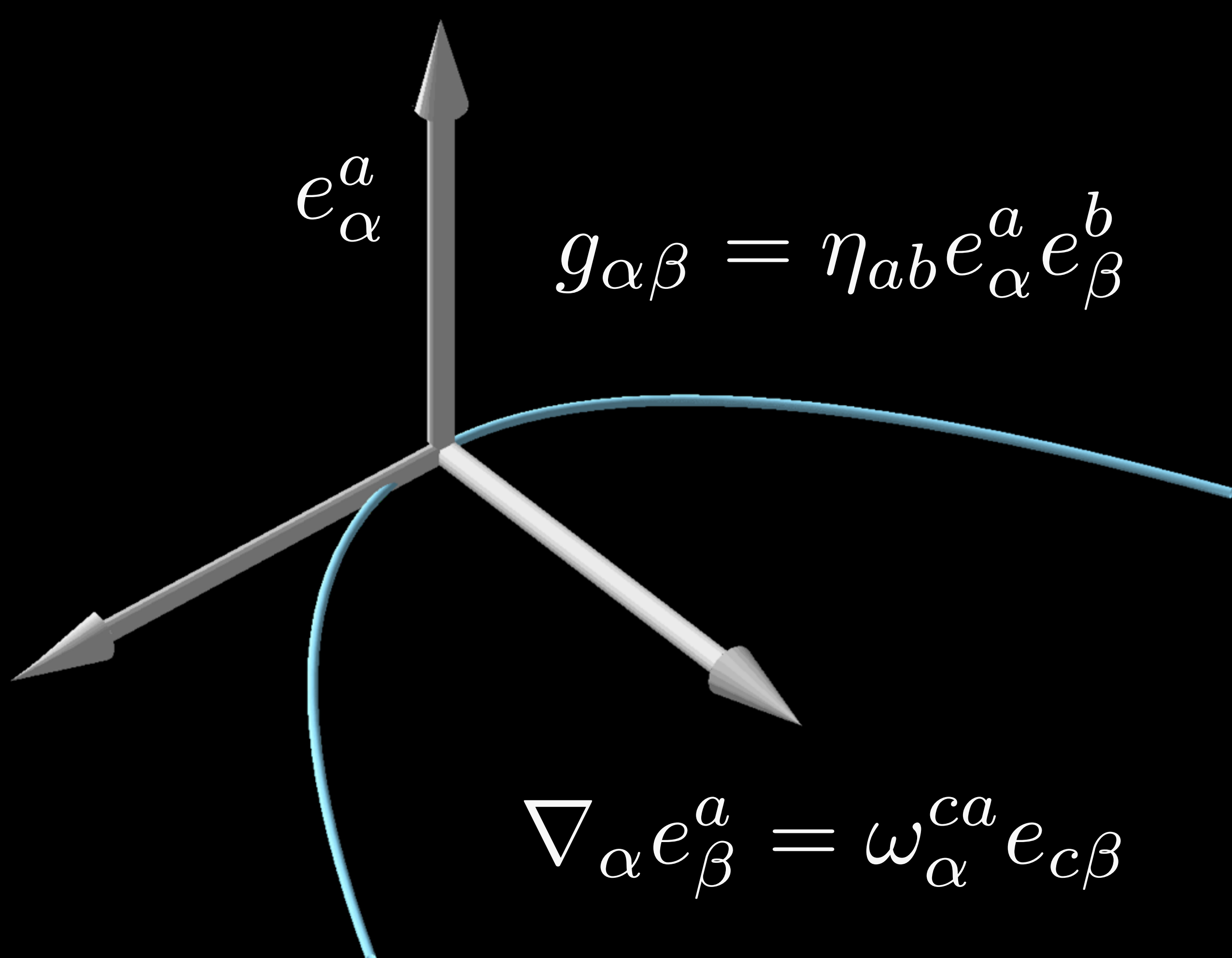
[Utiyama, Kibble, Wheeler + many others]

to obtain a theory of gravity: identify components of vector potential with geometric quantities on a 4-manifold

$$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

$e^a_\alpha \rightarrow$ orthonormal frame

$\omega^{ab}_\alpha \rightarrow$ connection 1-forms



$$g_{\alpha\beta} = \eta_{ab} e^a_\alpha e^b_\beta$$

$$\nabla_\alpha e^a_\beta = \omega^{ca}_\alpha e_{c\beta}$$

affine connection is metric compatible but not necessarily torsion-free

ordinary YM: action invariant under
$\delta A^A_\alpha = \partial_\alpha \epsilon^A + f^A{}_{BC} A^B_\alpha \epsilon^C$ with

$$\epsilon^A \mathbf{J}_A = \epsilon^a \mathbf{P}_a + \lambda^a \mathbf{K}_a + \Lambda^{ab} \mathbf{J}_{ab} + \Omega \mathbf{D}$$

$$S = -\frac{1}{2g^2_{\mathrm{YM}}} \int d^4x \sqrt{-g}\, g^{\alpha\mu} g^{\beta\nu} h_{AB} F^A_{\alpha\beta} F^B_{\mu\nu} + S_{\mathrm{m}}$$

YM gravity: since metric depends on $\{e^a_\alpha\} \subset \{A^A_\alpha\}$,
S invariant under transformations generated by

$$\epsilon^A \mathbf{J}_A = \lambda^a \mathbf{K}_a + \Lambda^{ab} \mathbf{J}_{ab} + \Omega \mathbf{D}$$

simplifying
assumptions

action: $$S = -\frac{1}{2g_{\rm YM}^2}\int d^4x\sqrt{-g}g^{\alpha\mu}g^{\beta\nu}h_{AB}F^A_{\alpha\beta}F^B_{\mu\nu} + S_{\rm m}$$

DOFs: $$\mathbf{A}_\alpha = A^A_\alpha \mathbf{J}_A = e^a_\alpha \mathbf{P}_a + l^a_\alpha \mathbf{K}_a + \omega^{ab}_\alpha \mathbf{J}_{ab} + q_\alpha \mathbf{D}$$

full EOMs pretty complicated, let's simplify things:

gauge choice (without loss of generality):	$q_\alpha = 0$
no torsion assumption (gauge invariant):	$de^a + \omega^{ac} \wedge e_c = 0$
technical assumption (gauge invariant):	$\eta_{ab}(e^a_\alpha l^b_\beta - e^a_\beta l^b_\alpha) = 0$
matter action:	$S_{\rm m} = S_{\rm m}[g_{\alpha\beta}, l^a_\alpha, \psi]$

EOMs now less complicated:

$$0 = B^{\alpha\nu} + \tfrac{1}{16} g_{\rm YM}^2 T^{\alpha\nu} - \nabla_\mu \nabla^{[\nu} \bar{a}^{\mu]\alpha} - Q^{\alpha\nu}$$

$$0 = \nabla^\alpha a_{\alpha\beta}$$

$$0 = g^{\mu\nu} \nabla_\beta a_{\mu\nu}$$

$$\bar{a}_{\alpha\beta} = a_{\alpha\beta} - \tfrac{1}{6} g_{\alpha\beta} a$$

$B^{\mu\nu}$ = Bach tensor involving fourth order metric derivatives

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta g^{\mu\nu}} = \text{familiar stress energy tensor}$$

$$a^{\mu\nu} = \frac{g_{\rm YM}^2}{4\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta l^b_\nu} e^{b\mu} = \text{coupling of matter to } l^a_\alpha$$

$Q^{\mu\nu}$ = tensor quadratic in Ricci, Weyl and $a_{\mu\nu}$ tensors

EOMs now less complicated:

$$0 = B^{\alpha\nu} + \tfrac{1}{16} g_{\mathrm{YM}}^2 T^{\alpha\nu} - \nabla_\mu \nabla^{[\nu} \bar{a}^{\mu]\alpha} - Q^{\alpha\nu}$$
$$0 = \nabla^\alpha a_{\alpha\beta}$$
$$0 = g^{\mu\nu} \nabla_\beta a_{\mu\nu}$$

residual symmetries:

local conformal transformations
$(\delta g_{\alpha\beta} = \epsilon g_{\alpha\beta})$

diffeomorphisms
$(\delta g_{\alpha\beta} = \pounds_\zeta g_{\alpha\beta})$

the cosmological sector

assume cosmological symmetries (isotropy and homogeneity) and FRW metric ansatz:

$$ds^2 = -dt^2 + A^2(t)\left(\frac{dr^2}{1-kr_0^2/r^2} + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2\right)$$

tensor characterizing exotic matter couplings must take form consistent with symmetries:

$$a^{\mu\nu} = \frac{g_{\rm YM}^2}{4\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta l^b_\nu} e^{b\mu} = \left[\xi_1(t) + \xi_2(t)\right] u^\mu u^\nu + \xi_2(t) g^{\mu\nu}$$

$$0 = \nabla^\alpha a_{\alpha\beta}$$

$$0 = g^{\mu\nu}\nabla_\beta a_{\mu\nu}$$

$$\xi_1 = -\frac{\Pi}{A^4} - \Lambda \qquad \xi_2 = -\frac{\Pi}{3A^4} + \Lambda \qquad \Pi, \Lambda \in \text{constants}$$

worthwhile emphasizing all interesting effects we are about to discuss stem from nonzero values of the integration constants Π and Λ

$$a^{\mu\nu} = \frac{g_{\rm YM}^2}{4\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta l^b_\nu} e^{b\mu} = -\frac{4}{3}\frac{\Pi}{A^4(t)}\left(u^\mu u^\nu + \frac{1}{4}g^{\mu\nu}\right) + \Lambda g_{\mu\nu}$$

here: assume both are positive

assume ordinary matter is dust and radiation:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{\rm m})}{\delta g^{\mu\nu}} = T_{\mu\nu}^{(\rm m)} + T_{\mu\nu}^{(\rm r)}$$

remaining EOMs lead to an effective Friedmann equation:

“wrong sign” dark radiation

$$H^2 = \frac{\dot{A}^2}{A^2} = \frac{g_{\rm YM}^2}{8} \left(\frac{\rho_{\rm m} + \rho_{\rm r}}{\Lambda + \frac{\Pi}{A^4}} \right) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

pretty nonstandard

dark energy

late time limit of Friedmann equation $A \gg (\Pi/\Lambda)^{1/4}$:

$$H^2 \approx \frac{\rho_{\rm m} + \rho_{\rm r}}{3M_{\rm Pl}^2} - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3}$$

where we made the identification $M_{\rm Pl}^2 = 8\Lambda/3g_{\rm YM}^2$

recover standard ΛCDM cosmology at late times provided we fix the Yang-Mills coupling can rewrite the Friedmann equation with a time dependent Newton's constant:

$$H^2 = \frac{8\pi G_{\rm eff}(A)}{3}\left(\rho_{\rm m} + \rho_{\rm r}\right) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

$$G_{\rm eff}(A) = \frac{3g_{\rm YM}^2}{64\pi}\left(\frac{A^4}{\Lambda A^4 + \Pi}\right) = \frac{1}{8\pi M_{\rm Pl}^2}\left(\frac{A^4}{A^4 + \Pi/\Lambda}\right)$$

at early times ($A \to 0$) effective Newton's constant goes to zero $\Rightarrow$ gravitational field of ordinary matter is screened in the early time limit…

this must be positive

dominates and diverges to negative infinity

$$H^2 = \frac{8\pi G_{\rm eff}(A)}{3}\left(\rho_{\rm m}+\rho_{\rm r}\right) - \frac{k}{r_0^2 A^2} + \frac{\Lambda}{3} - \frac{\Pi}{3A^4}$$

approximately constant due to screening

range of scale factors near zero are forbidden $\Rightarrow$ there must be a cosmological bounce or Einstein-static past attractor to sort out what happens at early times we'll run some simulations

$$\Omega_{\rm m} = \frac{\rho_{\rm m,0}}{3M_{\rm Pl}^2 H_0^2} \qquad \Omega_{\rm r} = \frac{\rho_{\rm r,0}}{3M_{\rm Pl}^2 H_0^2} \qquad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

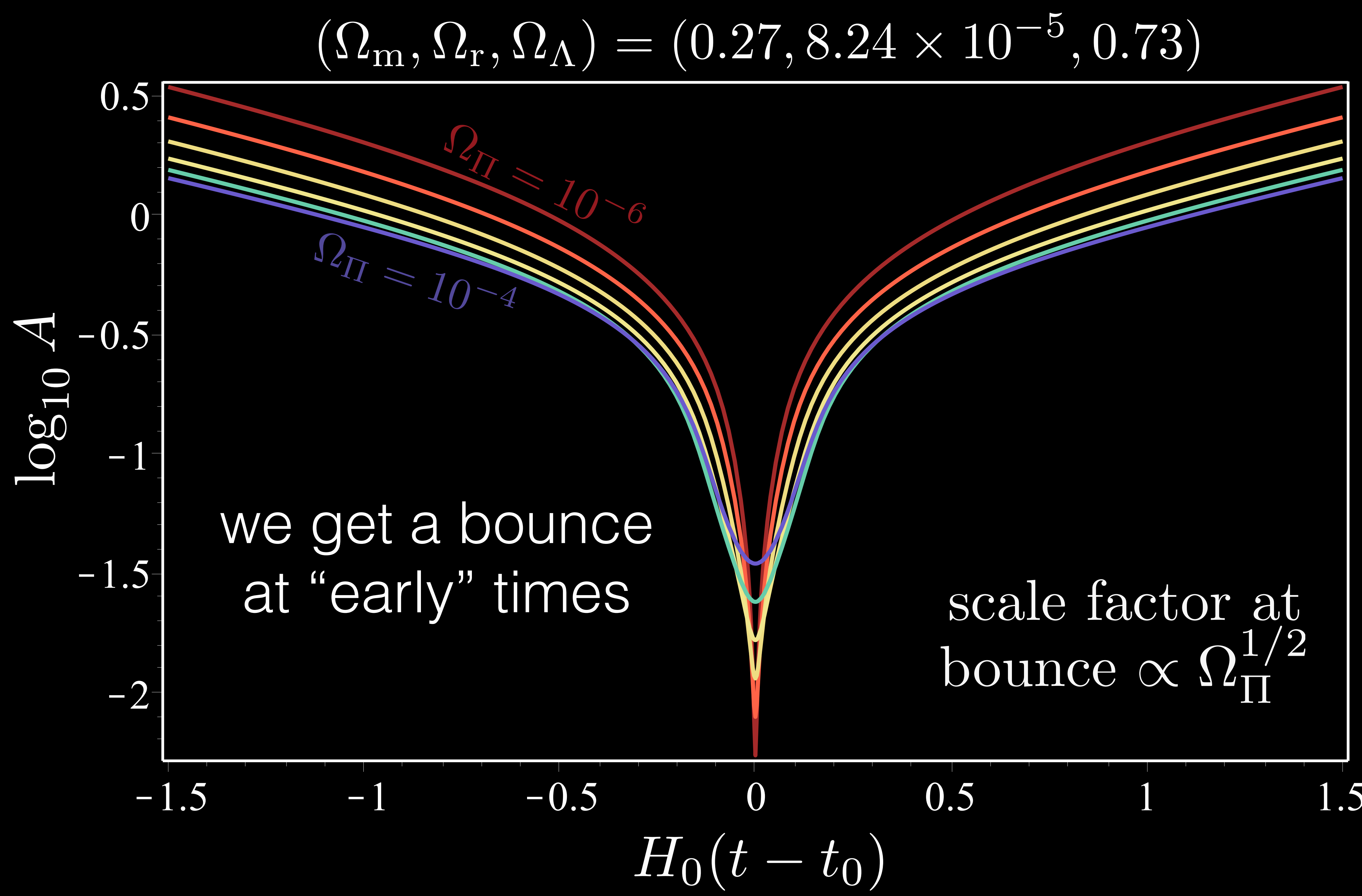
$$\Omega_{\rm k} = -\frac{k}{r_0^2 H_0^2} \qquad \Omega_\Pi = \frac{\Pi}{3H_0^2}$$

$$H_0 = \text{current value of Hubble parameter}$$
$$\rho_{\rm m,0} = \text{current dust density}$$
$$A_0 = \text{current value of scale factor} = 1$$

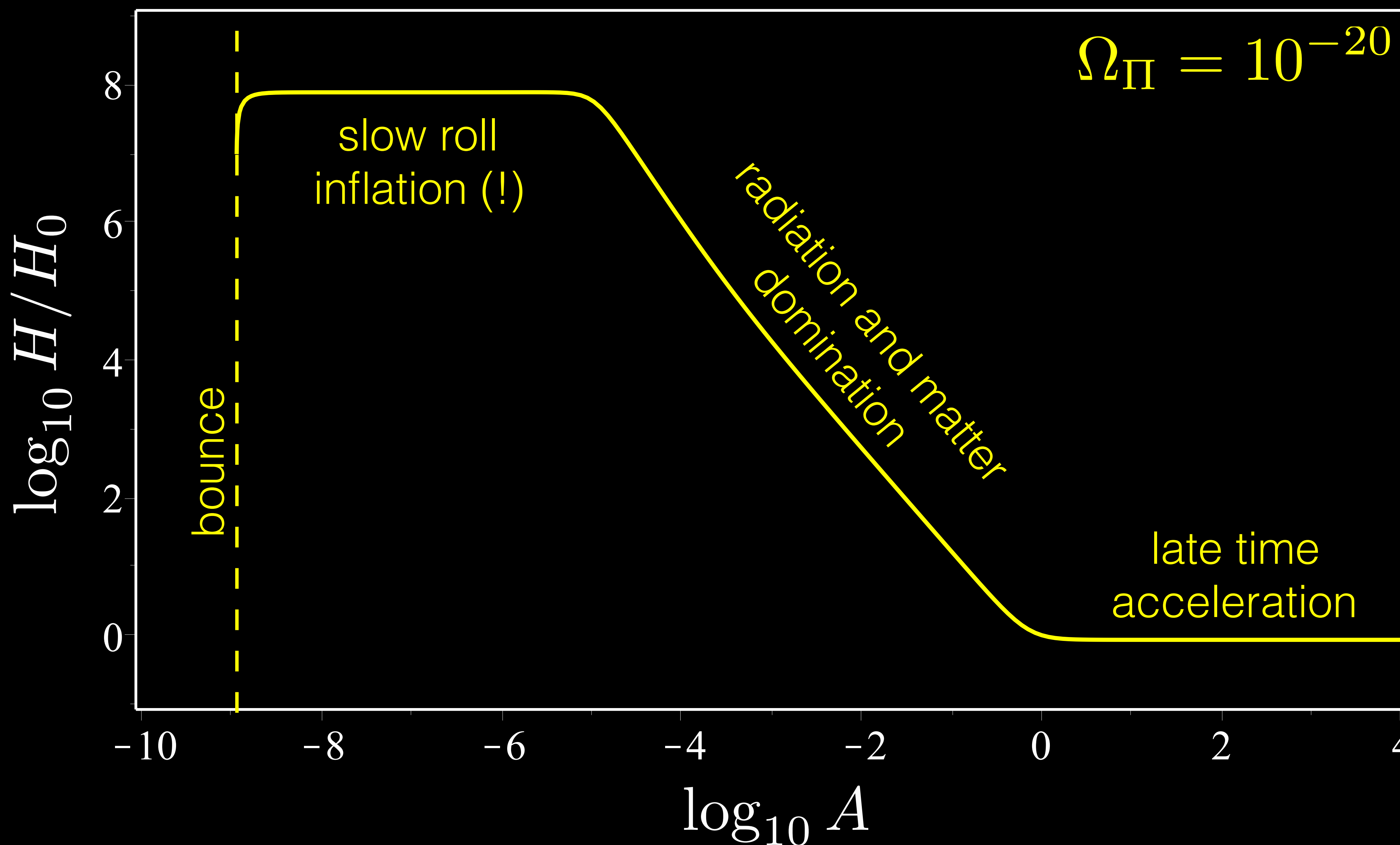
useful to define usual cosmological parameters (plus an extra one)

(we take concordance values for usual parameters)

simulation results



after any cosmological bounce there is an "inflationary period" of accelerated expansion



we can see what kind of inflation is in our model by plotting the Hubble factor numbers

how much inflation?

$$e\text{-folds of inflation} = \ln \frac{A_{\rm end}}{A_{\rm start}} \sim 66 - \frac{1}{4} \ln \frac{\Omega_\Pi}{g^2_{\rm YM}}$$

what's the inflationary energy scale?

$$E_{\rm inf} \sim 5 \times 10^{17}\,{\rm GeV} \left(\frac{\Omega_\Pi}{g^2_{\rm YM}} \right)^{-1/4}$$

what's the Yang-Mills coupling?

$$g^2_{\rm YM} = \frac{8}{3} \frac{\Lambda}{M^2_{\rm Pl}} \sim 10^{-120}$$

(we have similar naturalness problems to other dark energy models)

[using observational values for usual cosmological parameters]

summary

- looked at cosmological solutions of a Yang-Mills gravity based on the conformal SO(4,2) group
- Planck mass given by coupling constant and constant of integration
- solutions exhibit a bounce, long-lived quasi-de Sitter inflation, and late time acceleration
- like Lambda-CDM, no explanation of why the observed cosmological constant is so small in Planck units
- next steps: classical and quantum perturbations, including torsion…