A wide-angle aerial photograph of a coastal town nestled in a valley. In the background, a range of mountains is heavily covered in snow. The town below is surrounded by green forests and includes several buildings, roads, and a bridge crossing a river. The sky is clear and blue.

What can Cosmology tell us about Gravity?

Levon Pogosian
Simon Fraser University

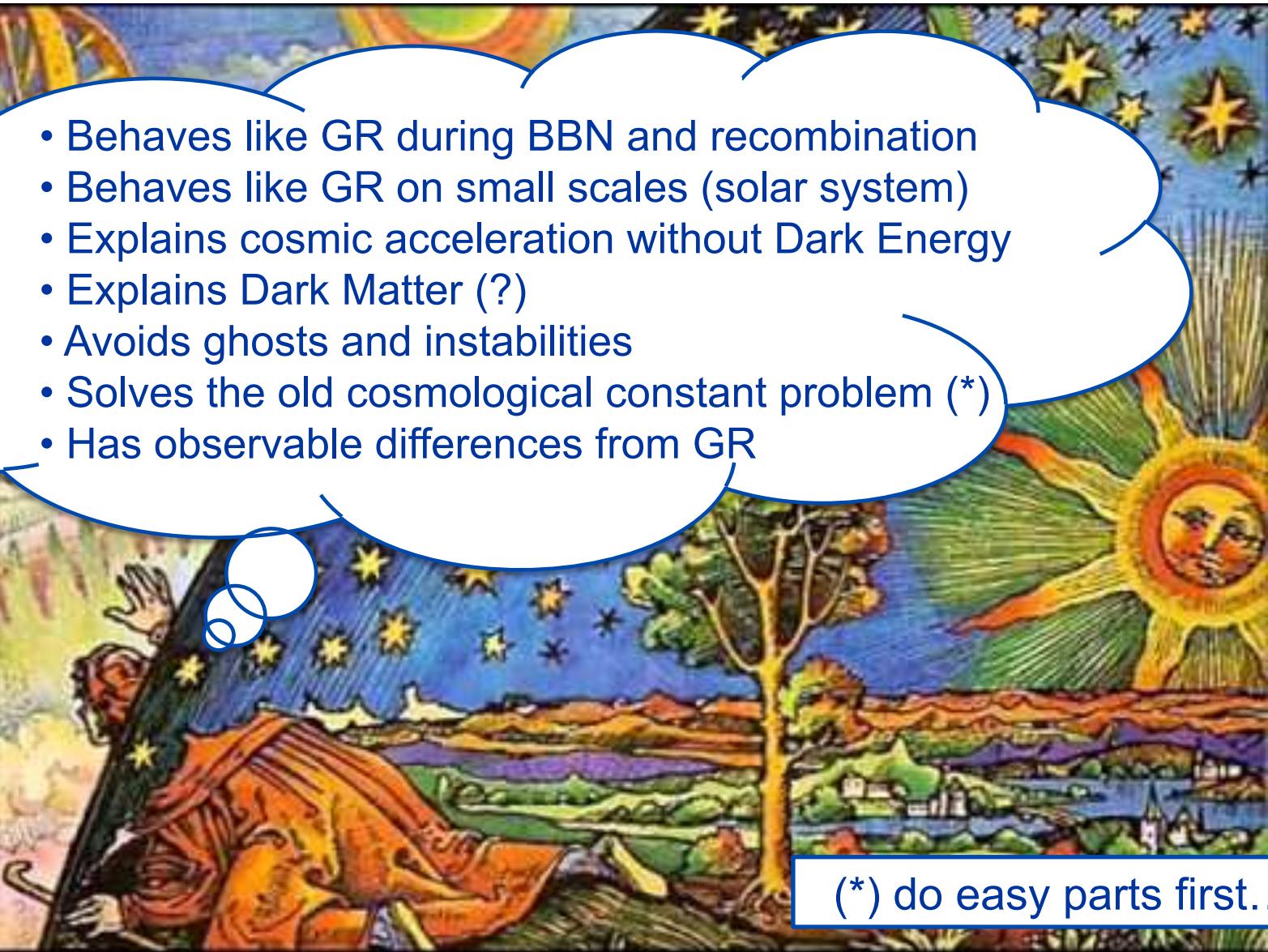
SFU

Based on work with Alireza Hojjati, Alessandra Silvestri, Gong-Bo Zhao, Alex Zucca

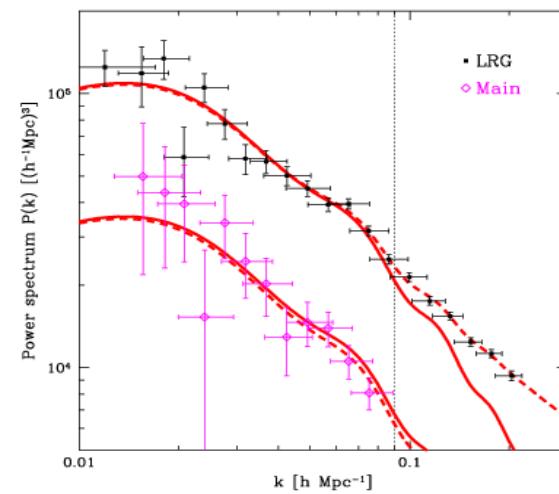
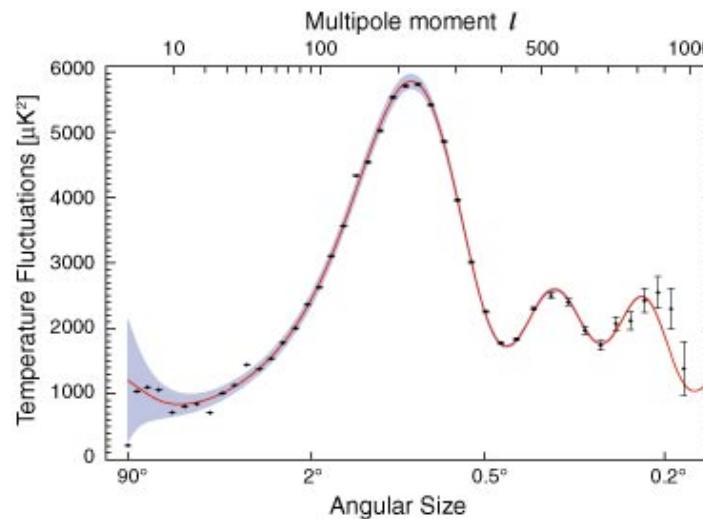
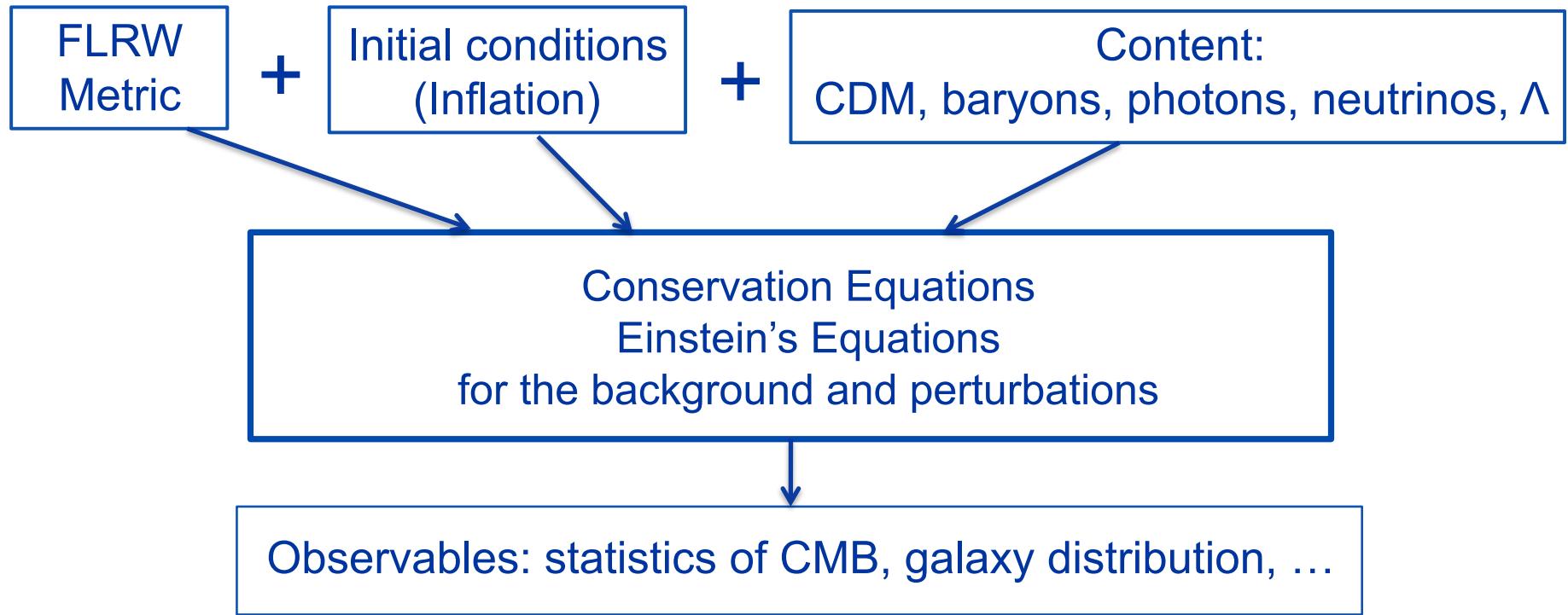
Are there reasons to expect GR to be modified on cosmological scales?

| A popular viewpoint | A different viewpoint |
|---|---|
| GR has, so far, passed all experimental tests | GR is yet to be seriously tested on cosmological |
| The LCDM model is in good agreement with observations | We do not know how the vacuum gravitates and why the universe is accelerating at the current rate |
| Alternative models tend to create more problems than they solve | We do not know what Dark Matter is |
| GR is appealing for its beauty alone | |

Cosmologists' Dream Modified Gravity Theory



What does Cosmology test?



Linear perturbations in FLRW universe

$$ds^2 = a^2(\eta) [-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)d\mathbf{x}^2]$$

$$\boxed{D_\mu T^{\mu\nu} = 0} \quad \longrightarrow \quad \boxed{\begin{aligned} \delta' + \frac{k}{aH}V - 3\Phi' &= 0 \\ V' + V - \frac{k}{aH}\Psi &= 0 \end{aligned}}$$

Einstein's equations

$$\boxed{\begin{aligned} k^2\Phi &= -4\pi Ga^2\rho \left(\delta + \frac{3aH}{k}V \right) \equiv -4\pi Ga^2\rho\Delta \\ k^2(\Phi - \Psi) &= 12\pi Ga^2(\rho + P)\sigma \end{aligned}}$$

Linear perturbations in FLRW universe

$$ds^2 = a^2(\eta) [-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)d\mathbf{x}^2]$$

$$\boxed{D_\mu T^{\mu\nu} = 0} \quad \longrightarrow \quad \boxed{\begin{aligned} \delta' + \frac{k}{aH}V - 3\Phi' &= 0 \\ V' + V - \frac{k}{aH}\Psi &= 0 \end{aligned}}$$

Einstein's equations with dust

$$\boxed{\begin{aligned} k^2\Phi &= -4\pi G a^2 \rho \Delta \\ \Phi &= \Psi \end{aligned}}$$

Things we can agree to keep

FLRW background with small perturbations:

$$ds^2 = a^2(\eta) [-(1 + 2\Psi)d\eta^2 + (1 - 2\Phi)d\mathbf{x}^2]$$

Conservation of matter energy-momentum:

$$\boxed{D_\mu T^{\mu\nu} = 0} \quad \longrightarrow \quad \boxed{\begin{aligned} \delta' + \frac{k}{aH}V - 3\Phi' &= 0 \\ V' + V - \frac{k}{aH}\Psi &= 0 \end{aligned}}$$

(!) Need two additional equations to close the system of four variables

Two ways of modeling linear perturbations

(1) Parameterize new terms in the effective action

(Gubitosi et al, 1210.0201; Bloomfield et al, 1211.7054, Gleyzes et al, 1304.4840, Bellini & Sawicki, 1404.3713)

$$S_g^{(2)} = \int d^3x dt a^3 \left[\frac{M^2}{2} \left(\delta K_j^i \delta K_i^j - \delta K^2 \right) + (1 + \alpha_T) \delta_2 \left(\sqrt{h} R/a^3 \right) + \alpha_K H^2 \delta N^2 + 4\alpha_B H \delta K \delta N \right],$$

- New terms determined by symmetries
- Perturbations in broad classes of theories described by a few functions of time
- Difficult to constrain individual functions simultaneously

(2) Use algebraic relations in Fourier space

(MGCAMB, Hojjati, Zhao, Zucca, LP, Silvestri)

- Closely related to observables
- Maps onto theories in the quasi-static limit

$$\begin{aligned} k^2 \Psi &= -\mu(a, k) 4\pi G a^2 \rho \Delta \\ \Phi &= \gamma(a, k) \Psi \end{aligned}$$

GR+ Λ CDM: $\mu = \gamma = 1$

$$\begin{aligned} k^2 \Psi &= -\mu(a, k) 4\pi G a^2 \rho \Delta \\ \Phi &= \gamma(a, k) \Psi \end{aligned}$$

GR+ Λ CDM

$$\mu = \gamma = 1$$

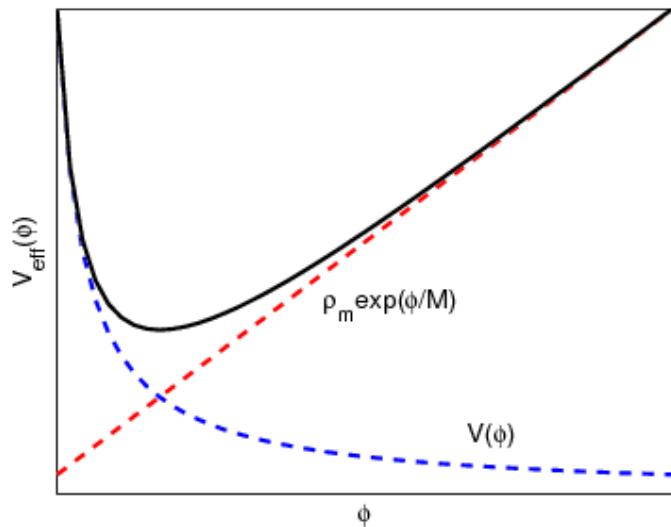
Example: scalar-tensor models of chameleon type

Khoury & Weltman, astro-ph/0309300, PRL '04

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_P^2}{2} \tilde{R} - \frac{1}{2} g^{\tilde{\mu}\tilde{\nu}} (\tilde{\nabla}_\mu \phi) \tilde{\nabla}_\nu \phi - V(\phi) \right] + S_i \left(\chi_i, e^{-\kappa \alpha_i(\phi)} \tilde{g}_{\mu\nu} \right)$$

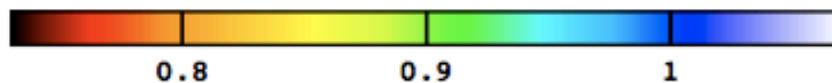
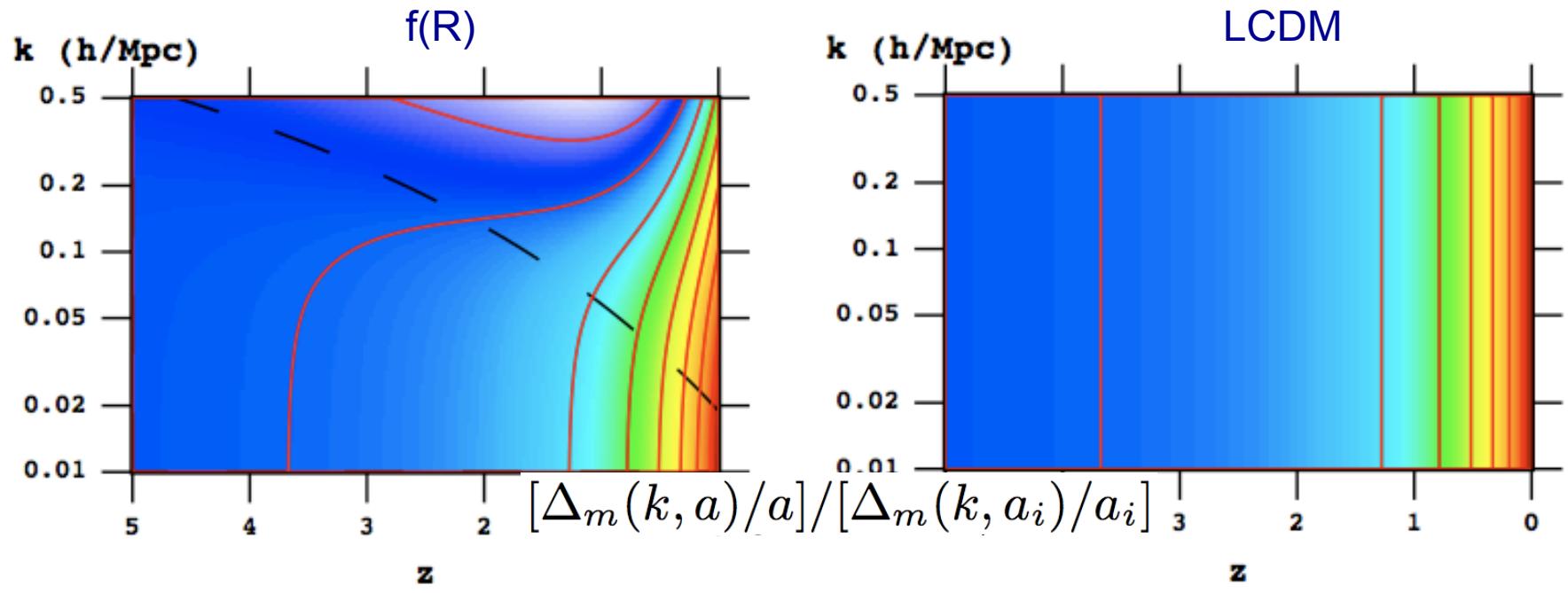
$$\mu(a, k) \approx e^{-\kappa \alpha(\phi)} \frac{1 + \left(1 + \frac{1}{2} \alpha'^2\right) \frac{k^2}{a^2 m^2}}{1 + \frac{k^2}{a^2 m^2}}$$

$$\gamma(a, k) \approx \frac{1 + \left(1 - \frac{1}{2} \alpha'^2\right) \frac{k^2}{a^2 m^2}}{1 + \left(1 + \frac{1}{2} \alpha'^2\right) \frac{k^2}{a^2 m^2}}$$



The growth of cosmological perturbations in $f(R)$

L.P. and A. Silvestri, PRD (2008)



$$\alpha' = \frac{d\alpha}{d\phi} = \sqrt{\frac{2}{3}}$$

$$\mu(a, k) \approx \frac{1 + 4/3k^2\lambda_C^2}{1 + k^2\lambda_C^2} \xrightarrow{k^{-1} \ll \lambda_C} \frac{4}{3}$$

$$\gamma(a, k) \approx \frac{1 + 2/3k^2\lambda_C^2}{1 + 4/3k^2\lambda_C^2} \xrightarrow{k^{-1} \ll \lambda_C} \frac{1}{2}$$

An alternative choice of modified relations

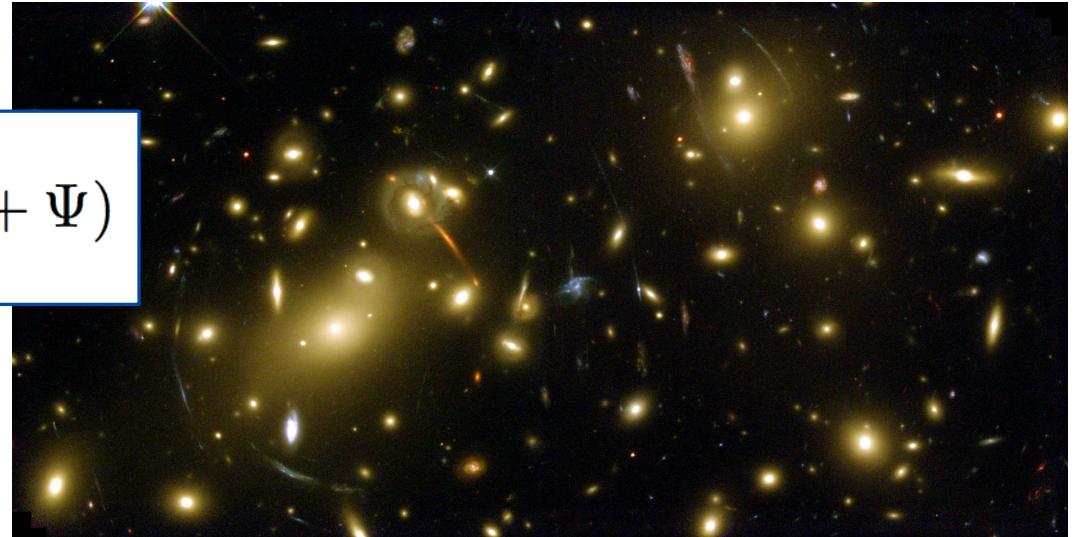
$$k^2 \Psi = -4\pi \mu(a, k) G a^2 \rho \Delta$$
$$k^2 (\Psi + \Phi) = -8\pi \Sigma(a, k) G a^2 \rho \Delta$$

A smoking gun of new gravitational physics

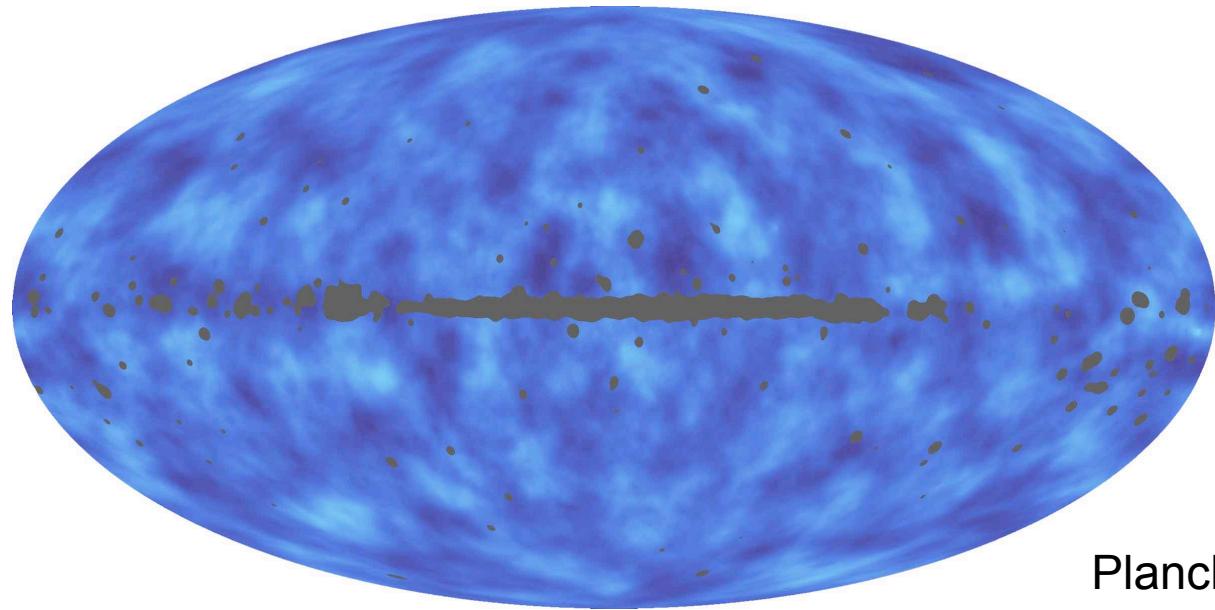
$$G_{matter} \neq G_{light} \quad \text{or} \quad \Phi \neq \Psi$$

Gravitational Lensing

$$\text{Distortion} \propto \int dz \ \partial_{\perp}(\Phi + \Psi)$$



Hubble

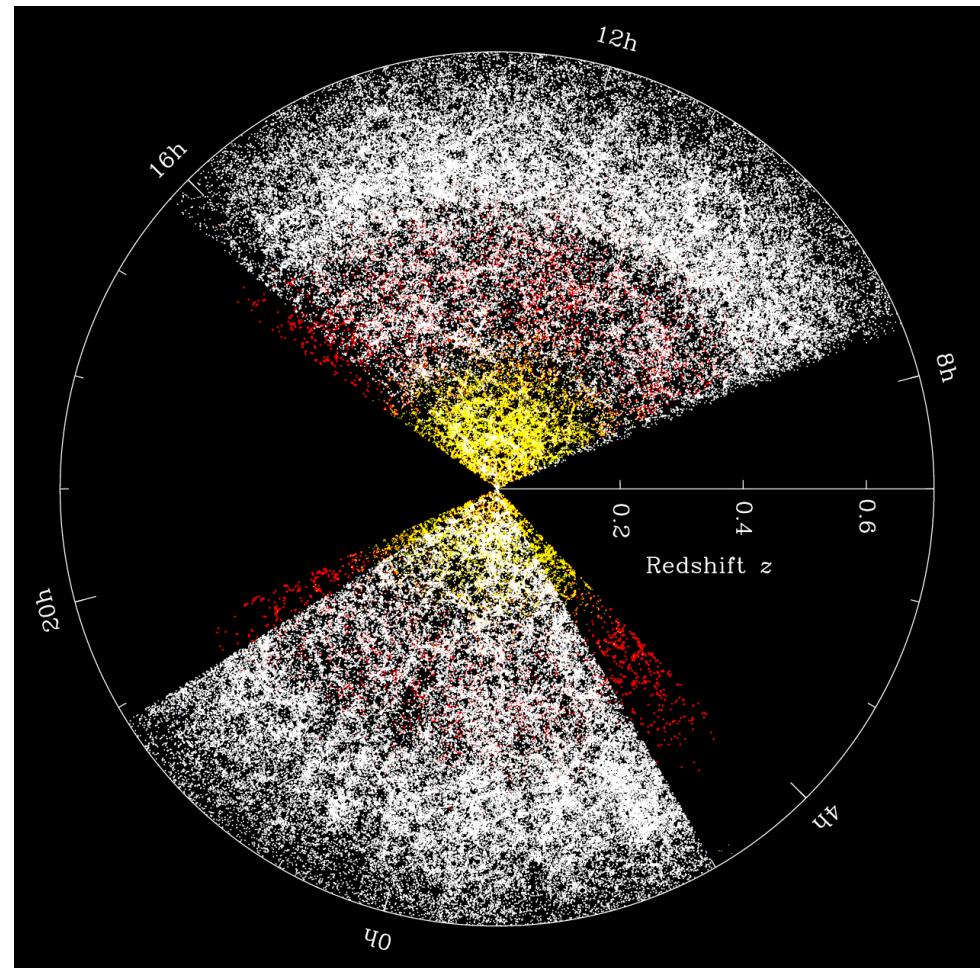


Planck

Galaxy Clustering

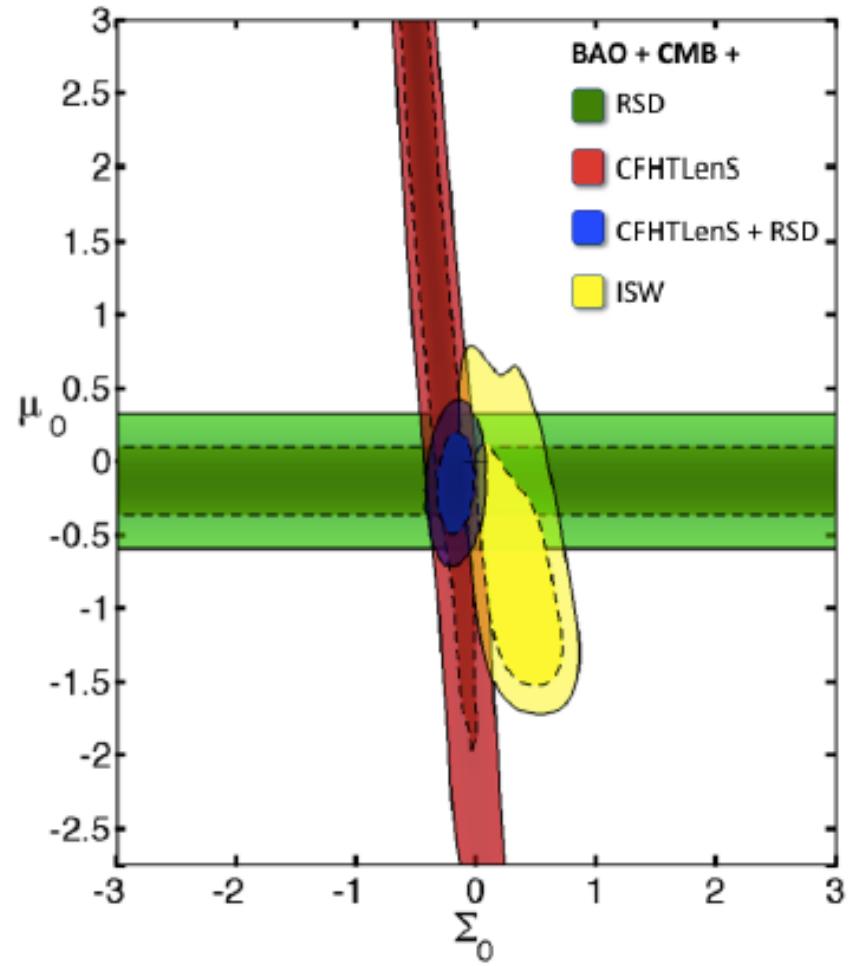
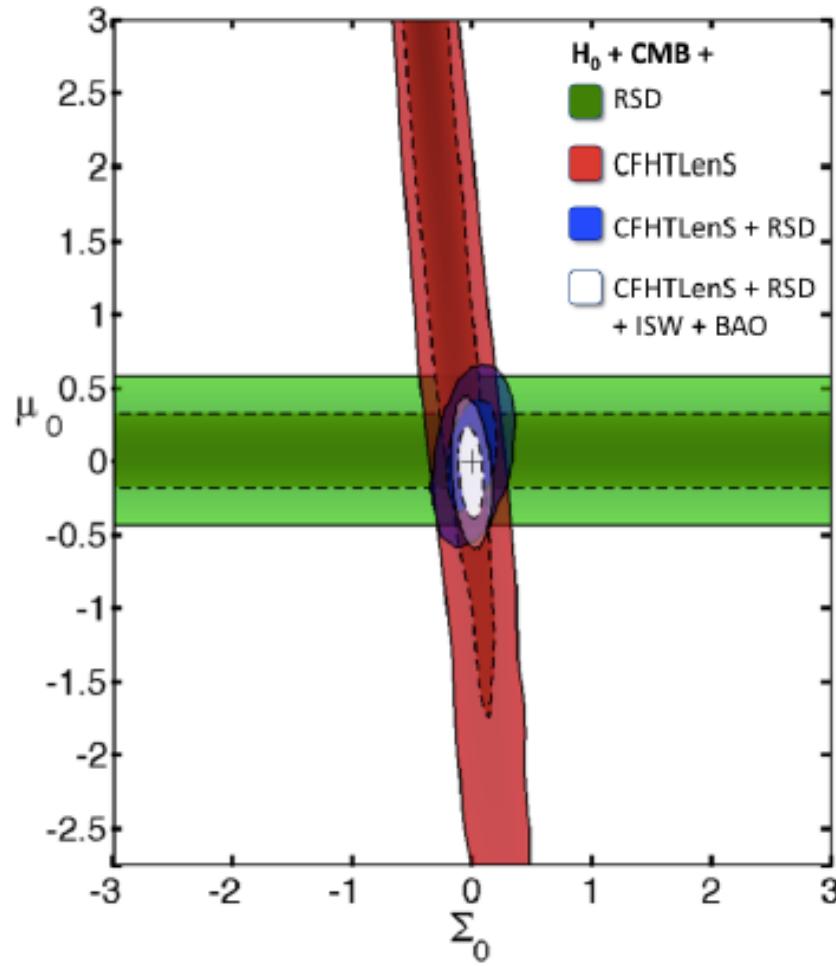
Redshift space distortions
due to peculiar motion

$$V' + V = \frac{k}{aH} \Psi$$



Fergus Simpson^{1*}, Catherine Heymans¹, David Parkinson², Chris Blake³,
Martin Kilbinger^{4,5,6}, Jonathan Benjamin⁷, Thomas Erben⁸, Hendrik Hildebrandt^{7,8},
Henk Hoekstra^{9,10}, Thomas D. Kitching¹, Yannick Mellier¹¹, Lance Miller¹²,
Ludovic Van Waerbeke⁷, Jean Coupon¹³, Liping Fu¹⁴, Joachim Harnois-Déraps^{15,16},
Michael J. Hudson^{17,18}, Konrad Kuijken⁹, Barnaby Rowe^{19,20}, Tim Schrabback^{8,9,21},
Elisabetta Sembolini⁹, Sanaz Vafaei⁷, Malin Velander^{12,9}.

$$\mu = 1 + \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}, \quad \Sigma = 1 + \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$



Complementarity of Weak Lensing and Peculiar Velocity Measurements in Testing General Relativity

Yong-Seon Song^{1,2}, Gong-Bo Zhao², David Bacon², Kazuya Koyama², Robert C Nichol², Levon Pogosian³

¹*Korea Institute for Advanced Study, Dongdaemun-gu, Seoul 130-722, Korea*

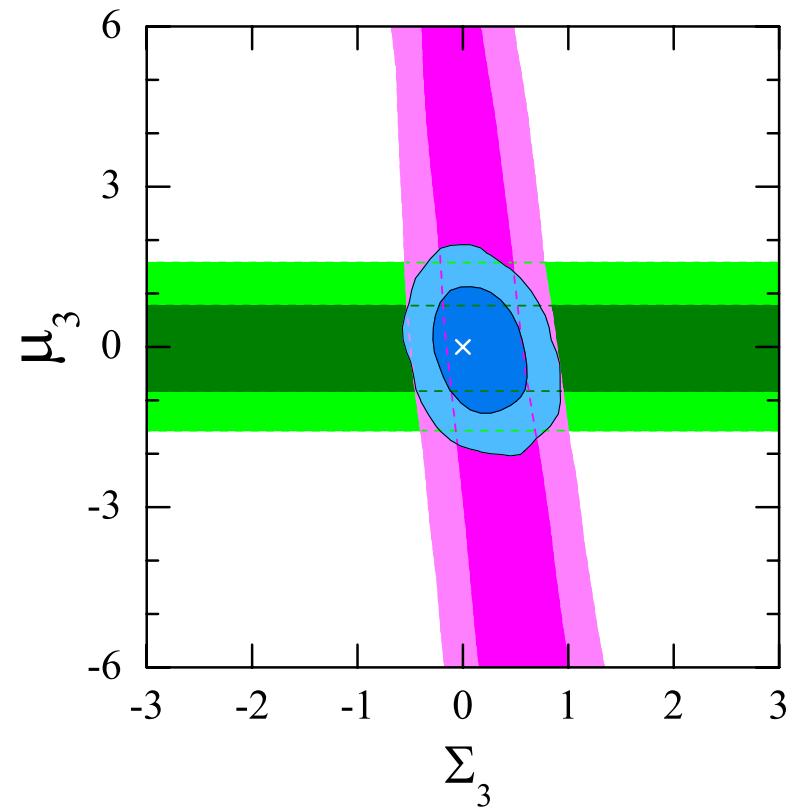
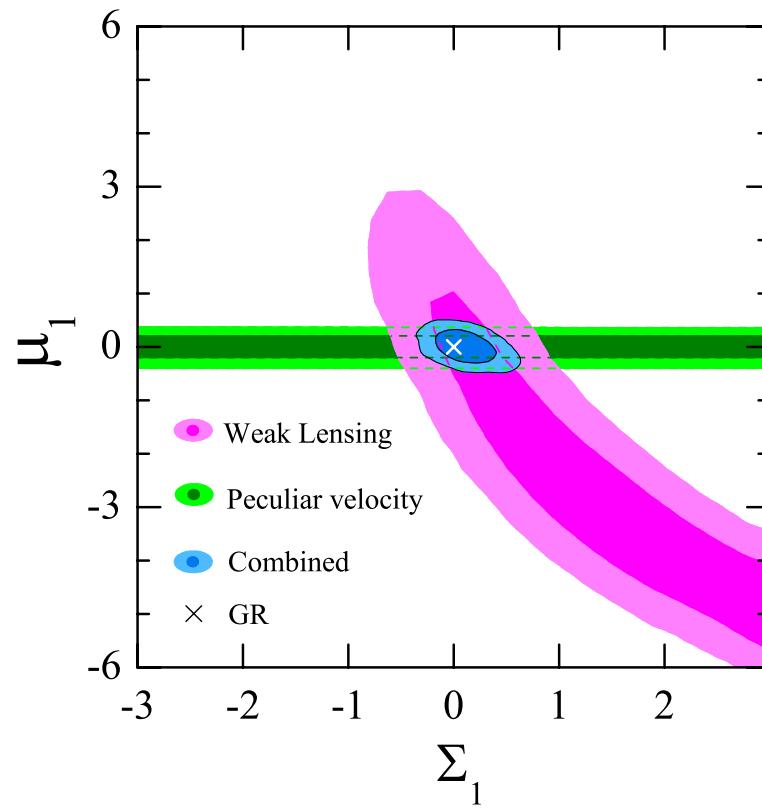
²*Institute of Cosmology & Gravitation, University of Portsmouth,*

Dennis Sciama Building, Portsmouth, PO1 3FX, United Kingdom

³*Department of Physics, Simon Fraser University, Burnaby, BC, V5A 1S6, Canada*

CFHTLS-Wide T003 (Fu et al, 2008), SDSS DR7

$$\Sigma = 1 + \Sigma_s a^s, \quad \mu = 1 + \mu_s a^s$$



Planck 2015 results. XIV. Dark energy and modified gravity

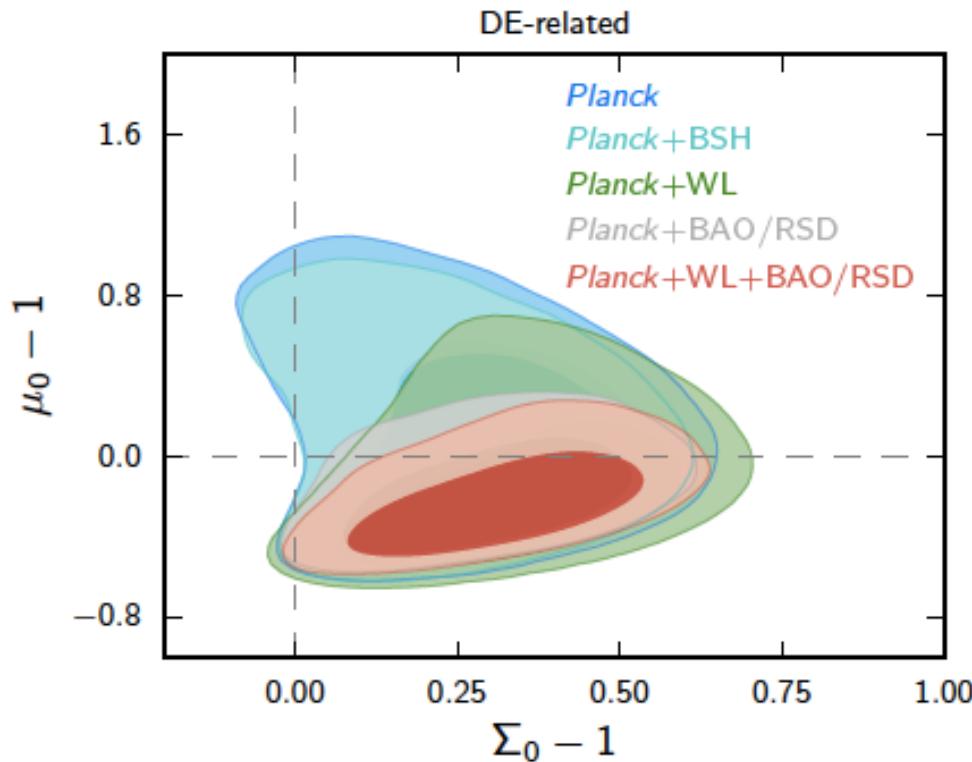


Fig. 15. Marginalized posterior distributions for 68% and 95% C.L. for the two parameters $\{\mu_0 - 1, \Sigma_0 - 1\}$ obtained by evaluating Eqs. (46) and (47) at the present time in the DE-related parametrization when no scale dependence is considered (see Sect. 5.2.2). Σ is obtained as $\Sigma = (\mu/2)(1 + \eta)$. The time-related evolution would give similar contours. In the labels, *Planck* stands for *Planck* TT+lowP.

Most general second-order scalar-tensor theory

$$S = \int d^4x \sqrt{-g} \left[\sum_{i=2}^5 \mathcal{L}_i + \mathcal{L}_m[g_{\mu\nu}] \right]$$

$$X = -\phi^{;\mu}\phi_{;\mu}/2$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X) \left[(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} \right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X) \left[(\square\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\square\phi \right]$$



Generalized Brans-Dicke models

Includes models with “chameleon”, “symmetron” and “dilaton” type screening

$$K(\phi, X) = h(\phi)X - V(\phi)$$

$$G_4(\phi, X) = \frac{A^{-2}(\phi)}{16\pi G}$$

$$G_3 = G_5 = 0$$

$$S = \int d^4x \sqrt{-g} \left[\frac{A^{-2}(\phi)}{16\pi G} R - \frac{h(\phi)}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathcal{L}_M(g_{\mu\nu}, \psi) \right]$$

In the Einstein frame: $\tilde{g}_{\mu\nu} = A^{-2}(\phi)g_{\mu\nu}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{16\pi G} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \tilde{V}(\phi) + \mathcal{L}_M(A^2(\phi)\tilde{g}_{\mu\nu}, \psi) \right]$$

Phenomenology of generalized Brans-Dicke models

$$\begin{aligned}
 -k^2\Psi &= \mu(k, a) \quad 4\pi G a^2 \rho \delta \\
 \Phi &= \gamma(k, a) \quad \Psi \\
 -k^2(\Psi + \Phi) &= \Sigma(a, k) \quad 8\pi G a^2 \rho \Delta
 \end{aligned}$$

Additional force mediated by the scalar: $\vec{f} = -\vec{\nabla}\Psi - \frac{d \ln A(\phi)}{d\phi} \vec{\nabla}\phi$

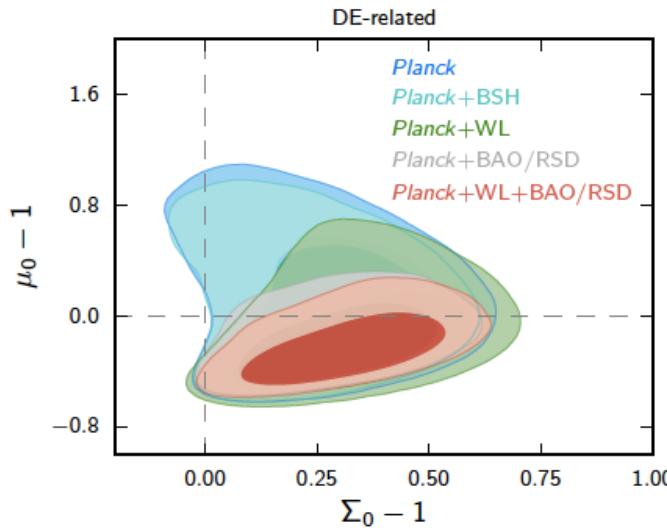
The range set by the density dependent mass: $m^2 = \frac{d^2 V_{\text{eff}}}{d\phi^2}$

The coupling strength: $\beta = m_{\text{Pl}} \frac{d \ln A}{d\phi}$

$$\epsilon(k, a) = \frac{2\beta^2(a)}{1 + m^2(a)a^2/k^2}$$

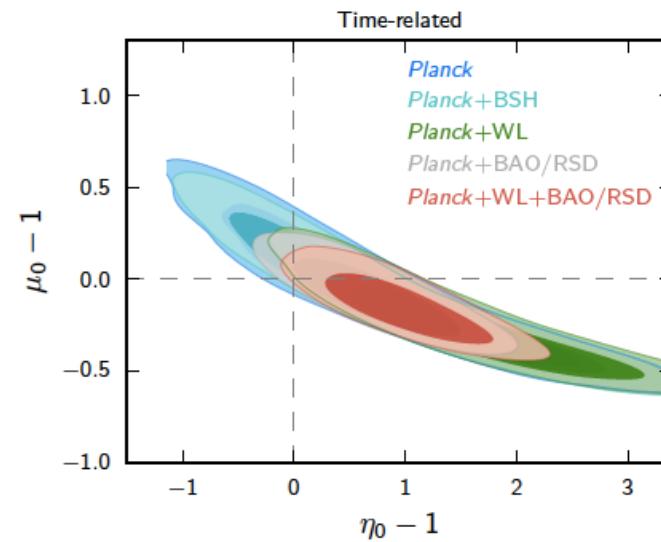
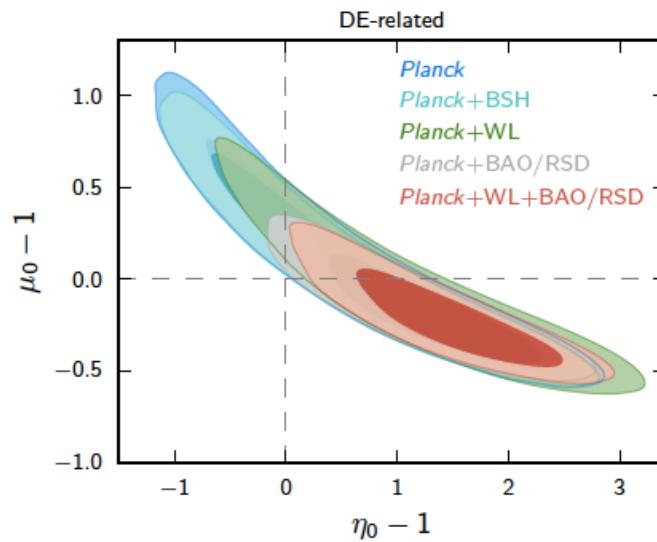
| | | |
|----------|-----|--|
| μ | $=$ | $A^2(\phi)[1 + \epsilon(k, a)] \geq 1$ |
| γ | $=$ | $\frac{1 - \epsilon(k, a)}{1 + \epsilon(k, a)} \leq 1$ |
| Σ | $=$ | $A^2(\phi) \approx 1$ |

Planck 2015 results. XIV. Dark energy and modified gravity



$$\begin{aligned}\mu &< 1 \\ \gamma &> 1 \\ \Sigma &> 1\end{aligned}$$

would rule out all Brans-Dicke type models
(and pretty much all other scalar-tensor models)



Beyond Brans-Dicke: a general Horndeski model

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X) \left[(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} \right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X) \left[(\square\phi)^3 + 2\phi_{;\mu}^{\ \nu}\phi_{;\nu}^{\ \alpha}\phi_{;\alpha}^{\ \mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\square\phi \right]$$

$$S = \int dt d^3x a^3 \left[\text{background terms} + \frac{M_*^2}{4} \left(\dot{h}_T^2 - \frac{1 + \alpha_T}{a^2} (\vec{\nabla} h_T)^2 \right) \right]$$

Modified speed of gravity waves if $G_{4,X}$ is not zero, or G_5 is not constant

$$M_*^2 = 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}H X G_{5,X})$$

$$\alpha_T = 2X(2G_{4,X} - 2G_{5,\phi} - (\ddot{\phi} - \dot{\phi}H)G_{5,X})M_*^{-2}$$

Gao & Steer, 1107.2642; De Felice & Tsujikawa, 1110.3878; Bellini & Sawicki, 1404.3713

Consistency tests of Horndeski theories

The large scale limit: $k/a \ll m_C$

$$\begin{aligned}\Sigma_0 &= \frac{m_{\text{Pl}}^2}{M_*^2} \left(1 + \frac{\alpha_T}{2}\right) \\ \gamma_0 &= \frac{1}{1 + \alpha_T} \\ \mu_0 &= \frac{m_{\text{Pl}}^2}{M_*^2} (1 + \alpha_T)\end{aligned}$$

A deviation from unity on large scales indicates a modified speed of GW. Can be compared to GW speed bounds from compact sources and CMB

Pulsars constrain α_T today < 0.02 , but it can, in principle, vary in the past

Jimenez, Piazza, Velten, 1507.05047

Consistency tests of Horndeski theories

The small scale limit: $k/a \gg m_C$

$$\mu_\infty = \frac{m_0^2}{M_*^2} (1 + \alpha_T + \beta_\xi^2),$$

$$\gamma_\infty = \frac{1 + \beta_B \beta_\xi}{1 + \alpha_T + \beta_\xi^2},$$

$$\Sigma_\infty = \frac{m_0^2}{M_*^2} \left(1 + \frac{\alpha_T + \beta_\xi^2 + \beta_B \beta_\xi}{2} \right)$$

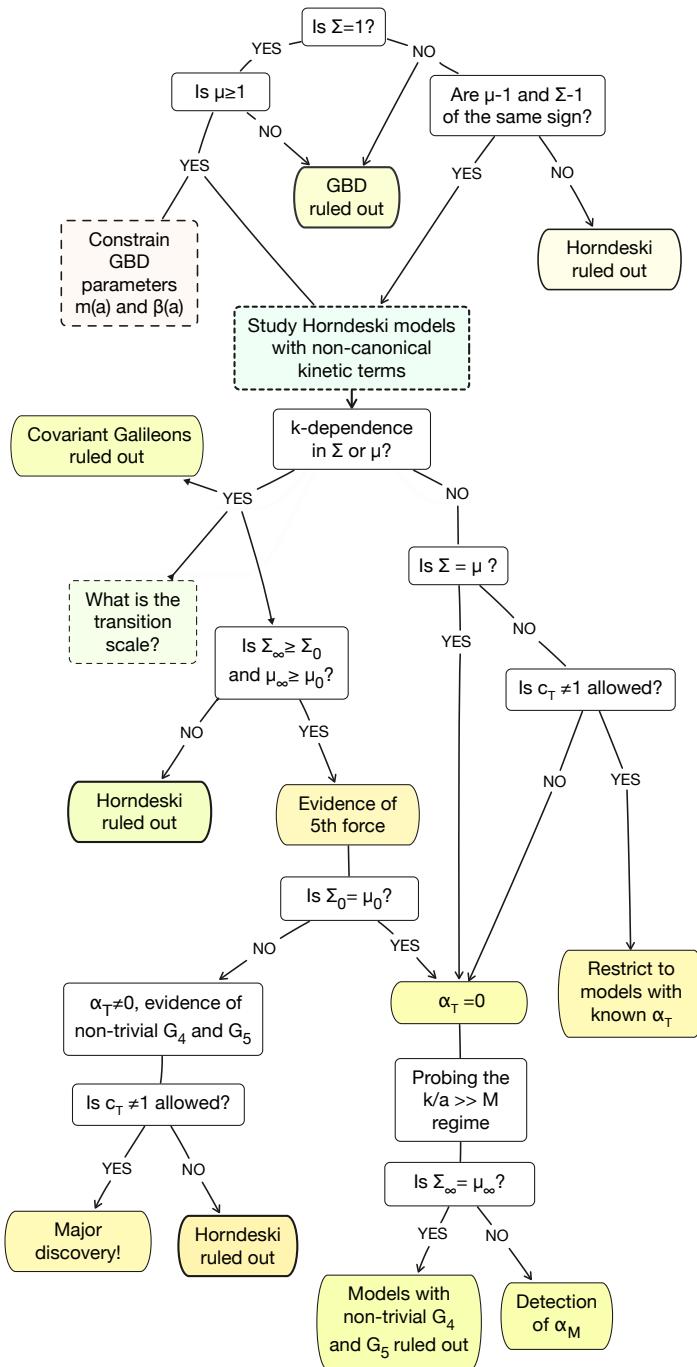
Pulsars constrain α_T today < 0.02 , but it can, in principle, vary in the past

Jimenez, Piazza, Velten, 1507.05047

Evolution of the Planck constant is constrained by BBN and CMB (at 10% level)

Different signs of $\mu-1$ and $\Sigma-1$ would effectively rule out all Horndeski models

LP & Silvestri, 1606.05339



Summary

Key observational tests:

Is $w_{DE} = -1$?

Are the Newtonian and the Weyl potentials the same?

Is the speed of gravitational waves the same as the speed of light?

It is possible to constrain large classes of modified gravity models using a few phenomenological functions

Future surveys, such as Euclid and LSST, will measure a lot of numbers

The challenge for theorists is to find meaningful questions they can answer