Quantum Gravity and the Unruh Effect

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Outline

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Quantum gravity models suggest modified dispersion relations due to granularity of spacetime at short distance scales

$$\omega_k = k\ f(k/M)$$

for some fixed large mass scale M.

The function f is such that

$$\lim_{x\to 0^+} f(x) = 1,$$

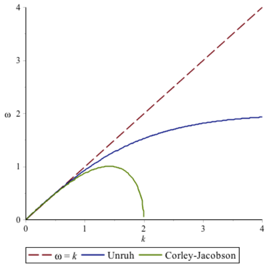
Lorentz invariance is recovered for $\frac{k}{M} \ll 1$.

With Lorentz violation, there is a preferred frame,
... so dispersion relations are frame dependent.

Studied in analog models for Hawking radiation

$$\omega = k_c \tanh^{1/p}(k/k_c)^p \text{ (Unruh);}$$

$$\omega = k\sqrt{1 - k^2/k_c^2} \text{ (Corley-Jacobson)}$$



Spectrum of fluctuations in cosmology (Brandenburger-Martin, Seahra, ...)

$$\omega = k\sqrt{1 + bk^2/a(t)^2 k_C^2}$$

Main result of these studies:

Hawking radiation & inflationary perturbation theory are ROBUST to such changes.

Our question:

How does an Unruh detector respond?

Answer:

Large detection signal even for low velocity inertial detectors !

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Unruh detector

Scalar field $\phi(x)$ coupled to a two state system (E_0, E_1) – the detector $m(\tau)$.

Detector trajectory: $x(\tau)$

Coupling: $\chi(\tau)\phi[x(\tau)]m(\tau)$ ($\chi(\tau)$ is a time dependent coupling field detector)

$|0_M\rangle|E_0\rangle \rightarrow |\Psi\rangle|E_1\rangle$ transition amplitude:

$$\begin{aligned} A_{0\rightarrow 1} &= \langle\psi|\langle E_1| \int_{-\infty}^{\infty} \chi(\tau)\phi[x(\tau)]m(\tau)\, d\tau \quad |E_0\rangle|0\rangle_M \\ &= \langle E_1|m(0)|E_0\rangle \int_{-\infty}^{\infty} d\tau\, \chi(\tau)\, e^{i(E_1-E_0)\tau}\langle\psi|\phi[x(\tau)]|0_M\rangle \end{aligned}$$

Transition probability:

$$|A_{0\to 1}|^2 \sim \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \; \chi(\tau)\chi(\tau') \; e^{-i\Omega(\tau-\tau')} \langle 0_M|\phi[x(\tau)]\phi[x(\tau')]|0_M\rangle$$

– $\Omega = E_1 - E_0$, the detector gap.

– the matrix element is the Wightman 2-point function

$$G(t, x; t', x') = \langle 0_M|\phi(x, t)\phi(x', t')|0_M\rangle$$

evaluated on the detector trajectory.

We will study the transition rate per unit time integral

$$\mathcal{F}(\Omega) = \int_{-\infty}^{\infty} ds \; e^{-i\Omega s} G(x(s), 0).$$

Calculating $G(x(s), 0)$

$$G(t, x; t', x') = \int d^3k \ |\rho_k|^2 e^{ik(x-x')-i\omega_k(t-t'-i\epsilon)}$$

– for the usual scalar field

$$\rho_k = [(2\pi)^3 2k]^{-1/2}$$

– for the deformed theory

$$\rho_k = \sigma(k/M) \ [(2\pi)^3 2k]^{-1/2}$$

Inertial detector: $(t(\tau), \mathbf{x}(\tau)) = (\tau \cosh\beta, 0, 0, \tau \sinh\beta)$

$$
\begin{aligned}
\mathcal{F}(\Omega) &= \int_{-\infty}^{\infty} ds \int d^3k \; |\rho_k|^2 e^{-is(\Omega+\omega_k \cosh\beta - k_3 \sinh\beta)} \\
&= \frac{M}{2\pi \sinh\beta} \int_0^{\infty} dg \; |\sigma(g)|^2 \Theta(g \sinh\beta - |(\Omega/M) + g f(g) \cosh\beta|)
\end{aligned}
$$

$g = k/M$, and recall $\omega_k = k f(g)$.

Note: $\mathcal{F}(\Omega)$ is proportional to M if integral is not zero.

Main point: the behaviour of the argument of Θ function

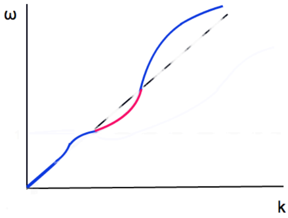
$$\arg(\Theta) > 0 \implies g \cosh\beta \;\; [\tanh\beta - f(g)] > \frac{\Omega}{M} > 0.$$

$$\omega = k, \quad f(g) = 1:$$

$$\mathcal{F}(\Omega) = 0$$

– the usual inertial detector result.

$\omega = kf(g)$, $f(g)$ dips below unity:



$$\mathcal{F}(\Omega) \sim M$$

Large detector excitation occurs for arbitrarily small Ω/M

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Polymer quantization method motivated by loop quantum gravity

Basic idea: $(x, p) \to (\hat{x}, U = \widehat{e^{i\lambda p}})$

- Momentum operator $\hat{p}$ does not exist; quantization comes with mass scale λ.
- Built in fundamental discreteness.
- Kinetic energy defined using U: bounded above!
- 2-point function calculated (G. Hossain, VH, S. Seahra PRD 82 (2010) 124032).

Detector response calculated numerically

$$h = \frac{\Omega}{M}, \quad F = \frac{\mathcal{F}}{M}, \quad v = \tanh \beta$$

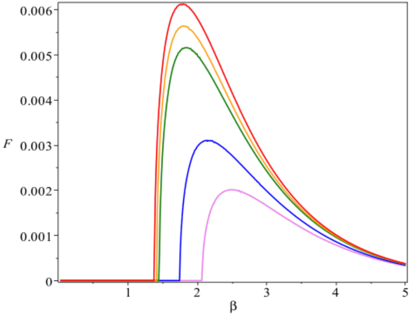

FIG. 1. Transition rate for excitations ($h > 0$) vs β for $h = 0.1$ (lowest curve), 0.05, 0.01, 0.05, and 0.01 (highest curve).

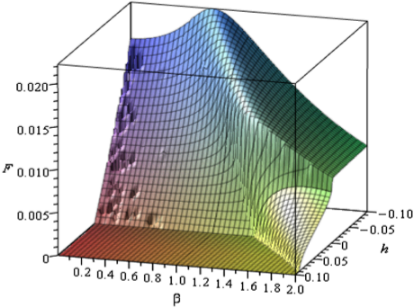

FIG. 3. Transition rate as a function of β and h. The irregularities at small β are numerical noise.

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Summary

- Drastic low energy Lorentz violation in inertial Unruh detectors $\omega = kf(k/M)$ with $\min(f(x) < 1$.
- Polymer quantization provides one example.
- $\beta \approx 3$ (100 GeV) in ion accelerators; dipole-EM field interaction similar to Unruh detector. No detection of Lorentz violation.
- Is polymer quantization experimentally ruled out?