SEMICLASSICAL GRAVITY AND ASTROPHYSICS

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WHY PROBE SEMICLASSICAL GRAVITY AT ALL

- quantum gravity is important at Planck scales $l_p = 10^{-33}\mathrm{cm}$
- Semi classical gravity is important, as Hawking estimated for primordial black holes of radius $10^{14} l_p$
- using coherent states I showed that instabilities can happen due to semi classical corrections for black holes with horizon radius of the order of $10^{42} l_p$

WHY COHERENT STATES

- useful semiclassical states in any quantum theory
- expectation values of the operators are closest to their classical values
- fluctuations about the classical value are controlled, usually by `standard deviation' parameter as in a Gaussian, and what can be termed as `semi-classical' parameter.
- in SHM coherent states expectation values of operators are exact, but not so in non-abelian coherent states

LQG: THE QUANTUM GRAVITY THEORY WHERE COHERENT STATES CAN BE DEFINED

- based on work by mathematician Hall.
- coherent states are defined as the Kernel of transformation from real Hilbert space to the Segal-Bergman representation.
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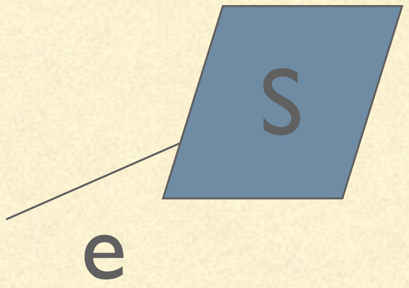
$$h_e \to g_e = e^{iT^I P^I} h_e$$



$$x \to z = x - ip$$

$$\rho(t, g_e) = e^{-t\Delta}\delta(h_e^{-1} g_e)$$

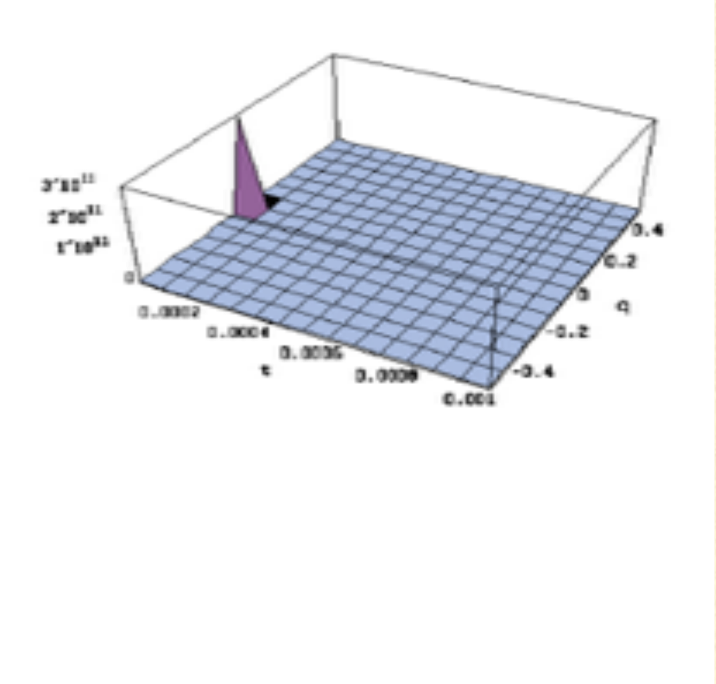
$$\rho(t, z) = e^{-t\Delta}\delta(z)$$

$$h_e(A) = P\exp\left(\int_e A dx\right) \quad P_e^I = -\frac{1}{a}\mathrm{Tr}\left[T^I h_e \int * E\, h_e^{-1}\right]$$

EXPECTATION VALUES OF OPERATORS

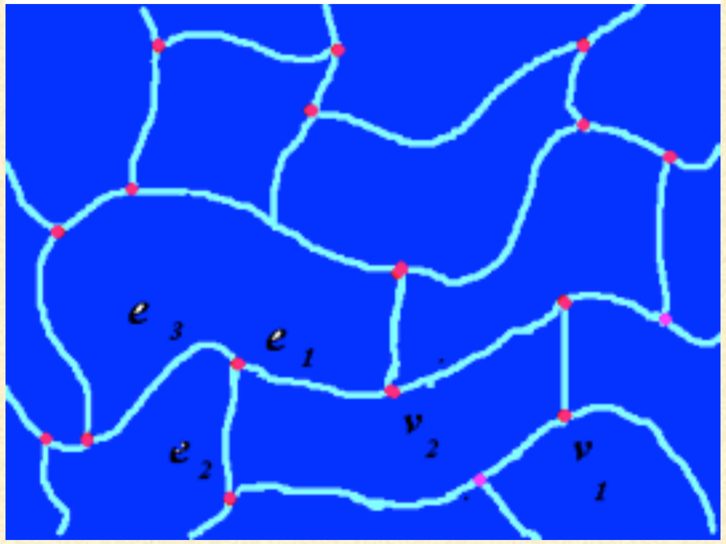
- Coherent State in the holonomy representation

$$\psi^{\tilde{t}} = \sum_j (2j+1) e^{-\tilde{t}j(j+1)/2} \chi_j(g_e h_e^{-1})$$



The expectation values of the momentum operator

$$< \hat{P}^I_{e_a} >= \frac{< \psi^{\tilde{t}} | \hat{P}^I_{e_a} | \psi^{\tilde{t}} >}{||\psi^{\tilde{t}}||} = P^I_{e_a} \left[1 + \tilde{t} \tilde{f}(P) \right]$$



$$\tilde{f}(P) = \frac{1}{P_{e_a}} \left(\frac{1}{P_{e_a}} - \coth(P_{e_a}) \right)$$

NON-POLYNOMIAL CORRECTIONS

CORRECTIONS TO THE HOLONOMY

$$\delta h_{AB} = e^{-\tilde{t}/16} e^{-p^2/\tilde{t}} \frac{z_0}{\sinh(z_0)} \frac{\sinh p}{p} \left[g_{AB} \cosh\left(\frac{z_0}{2}\right) + (\tau_j g)_{AB} \frac{tr(\tau_j g g^\dagger)}{2 \sinh(z_0)} \sinh\left(\frac{z_0}{2}\right) \right]$$

where p is a momentum, and $z_0 = e\,\tilde{t}\,p$, where p corresponds to invariant momentum for a given edge of length e. The corrections tend to zero exponentially as $\tilde{t} \to 0$, nevertheless, a correction exists, and contributes for a possible probe for quantum gravity.

GEOMETRIC INTERPRETATION

$$ds^2 = -d\tau^2 + \frac{dR^2}{\left[\frac{3}{2r_g}(R \pm \tau)\right]^{2/3}} + \left[\frac{3}{2}(R \pm \tau)\right]^{4/3} r_g^{2/3}(d\theta^2 + \sin^2\theta d\phi^2)$$

Lemaitre metric

$$\begin{aligned} g^{tt} &= \frac{dt}{d\tau}\frac{dt}{d\tau}g^{\tau\tau} + \frac{dt}{dR}\frac{dt}{dR}g^{RR} \\ g^{rr} &= \frac{dr}{d\tau}\frac{dr}{d\tau}g^{\tau\tau} + \frac{dr}{dR}\frac{dr}{dR}g^{RR} \\ g^{rt} &= \frac{dr}{d\tau}\frac{dt}{d\tau}g^{\tau\tau} + \frac{dr}{dR}\frac{dt}{dR}g^{RR} \end{aligned}$$

→ DOES NOT CANCEL WHEN CORRECTED

The corrected metric does not transform to the Schwarzschild metric

WHAT DOES THIS `EXTRA TERM' MEAN

$$g_{tr} = \pm \frac{1}{1 - \frac{r_g}{r}} \left(\frac{r_g}{r}\right)^{3/2} \frac{l_p^2}{r_g^2} \tilde{f}$$

It is a `strain' on the space-time fabric. Can LIGO detect such changes from the classical metric? This is a linearized perturbation over a Schwarzschild metric, and would contribute to `even' mode of a spherical gravity wave, though cannot be anticipated in the polynomial linearized mode expansion

THE STRAIN MAGNITUDE

$$\tilde{f}(P_e) = \frac{1}{P_e} = \frac{r_g^2}{r^2 \sin\theta}$$

$$e_{tr} = \pm\frac{1}{2}g^{tr}$$

$r = 1.3$ billion light years$=1.23 \times 10^{25}$m,

$r_g = 2GM = 1.57 \times 10^{22}$ m

A. Dutta & ADG

$$e_{tr} = 7.67 \times 10^{-125}\text{cosec}\theta$$

WHY DOES THIS NUMBER REMIND US OF THE COSMOLOGICAL CONSTANT?

UNSPHERICAL-UNSTATIC COLLAPSE

- difficult numerically, as not due to the addition of a twist vector
- the introduction of a three dimensional grid
- cannot be generalized from the axisymmetric case, work in progress …

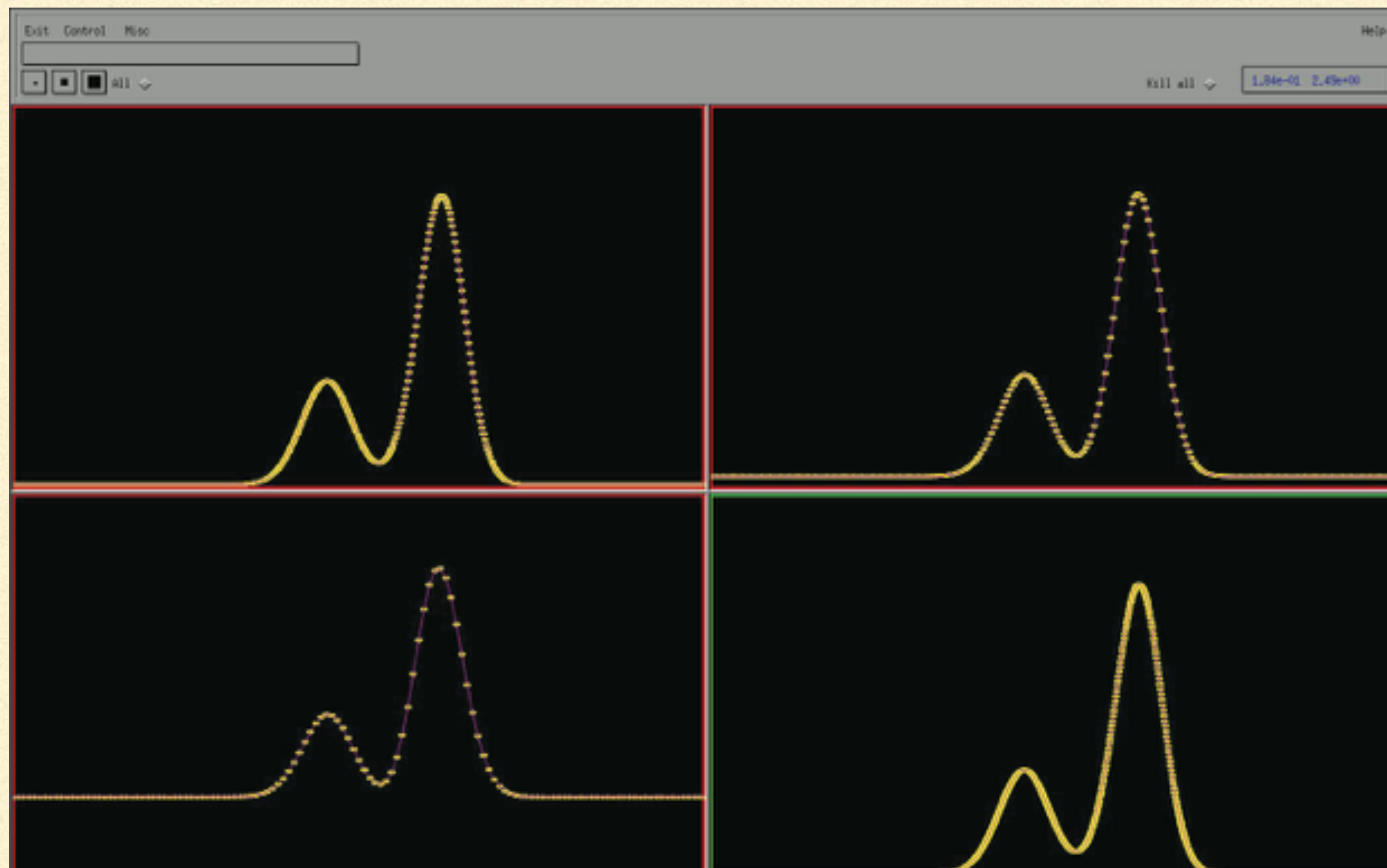
```
program rungekutta
implicit none
real h, t, f
integer n
real a,b,x
integer j
real K1,K2,K3,K4
print *, 'the initial values are'
read(5,*) a,b,x

print *, 'the value of n'
read(5,*) n
h=(b-a)/n
t=a
open(12,file='runge-kutta')
do j=1,n
K1=h*f(t,x)
```

TEST NUMERICAL

$$-\partial_t^2\phi + 2g^{tr}\partial_t\partial_r^*\phi + \frac{1}{r^2}\partial_r^*(g^{tr}r^2)\partial_t\phi + \partial_{r^*}^2\phi + \left((1-\frac{2GM}{r})\right)\nabla_{\theta\phi}\phi \;=\; 0$$

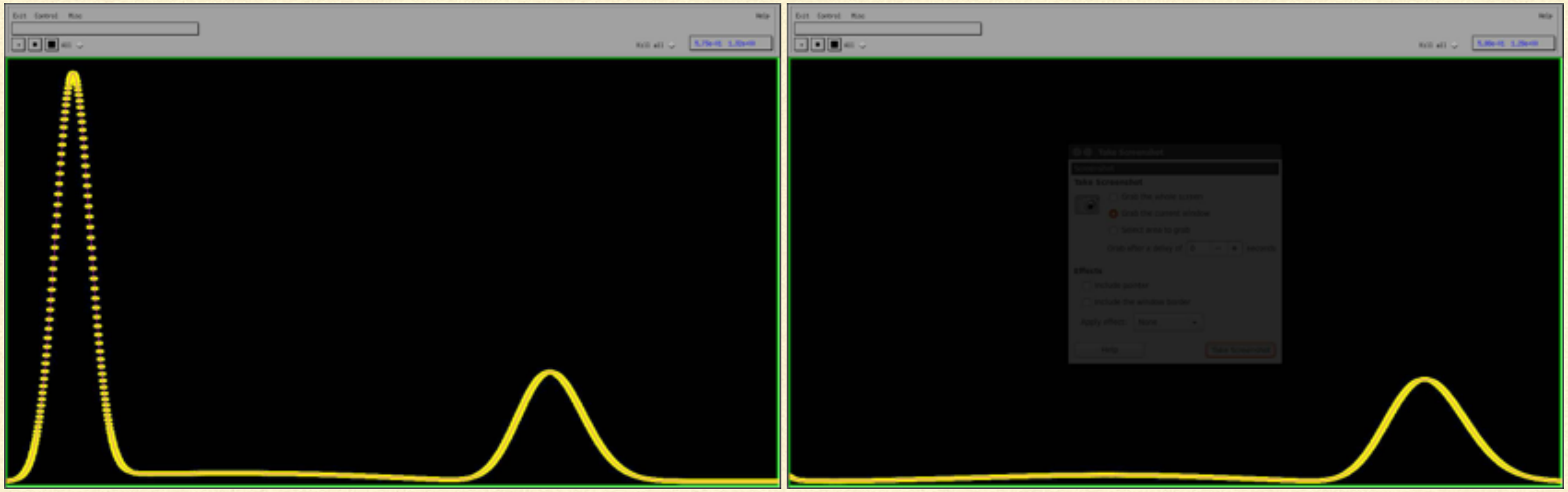
$$r \approx 2GM(1+l_p^2/(2GM)^2).$$

CORRECTION IS FINITE NEAR THE HORIZON

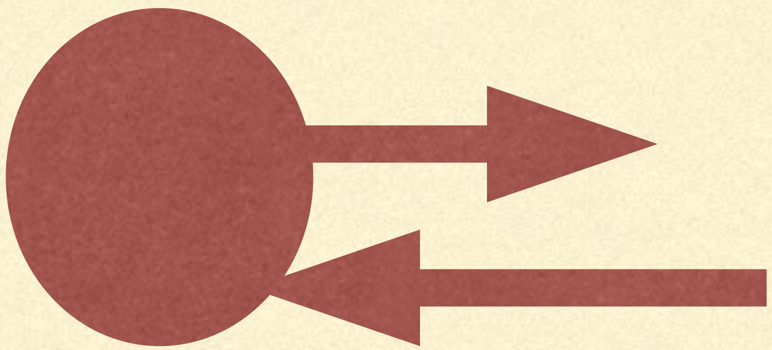


A. Dutta & ADG

LEFT-RIGHT ASYMMETRY



IN-FALLING FIELDS AT THE HORIZON



Imbalance in ingoing and outgoing Flux: Implications to Hawking radiation, and Grey body factors

This is a dynamical process, the imbalance occurs as the scalar field evolves in time.

the computation of back reaction and black hole evaporation will be affected

ASTROPHYSICS: APPLICATIONS

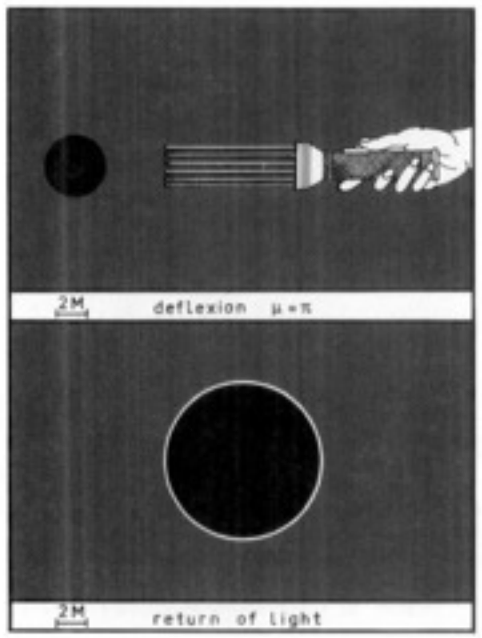


Fig. 2. Return of light deflected by 180° from a bare black hole

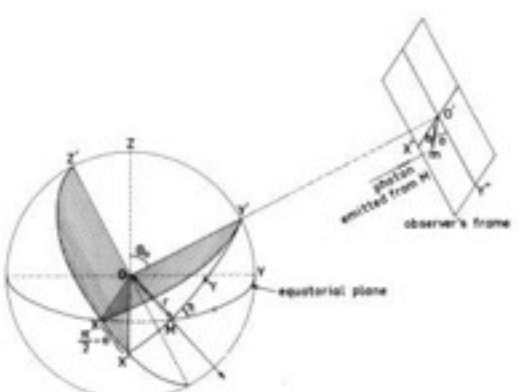


Fig. 3. The coordinate system (see text)

on a Θ-direction. It follows from the above discussion that rays that reach the observer will give a picture consisting of a black disk of radius $b_c = 5.19695\ M$ surrounded by "ghost" rings of different radius and brightness. The exterior ring corresponds to the rays that have not described any circuit; as b approaches its critical value b_c, the rays describe more and more circuits, until in the limit $b_\infty = b_c$ (infinite circuits) the rays are captured.

from the direction of incidence close to π, he will not see rings of brightness since almost all the observed photons at small deviations come from large impact parameter, for which circuiting does not take place. In fact, it is clear that the most favourable condition is $\theta = 0$, i.e. the rays are deflected by $\mu = \pi$. Figure (2) gives the corresponding image; the radius of the external ring is $b_1 = 5.341\ M$; this is already near the rim of the black disk of radius 5.197 M. The following rings fall at distances even closer, equal respectively to 0.00028 M, 0.00000056 M, 0.00000000112 M, etc.

To conclude this section, the only ring practically distinguishable woud be the external one. This is not only a matter of brightness, but also a matter of resolution.

3. Image of a Clothed Black Hole

Let us now assume that the source of radiation is an emitting accretion disk orbiting around the black hole; the astrophysical properties of such an object will be briefly discussed in the next section. Let the thickness of the disk be negligible with respect to M, so that it is considered as lying in the equatorial plane of the Schwarzschild black hole. The coordinate system is chosen as in Fig. 3. The observer lies in the fixed direction θ_0, $\phi_0 = 0$ (plane YOZ) at a distance $r_0 \gg M$. We consider the disk as an assembly of idealized particles emitting isotropically. Starting from an emitting particle with Schwarzschild coordinates (r, ϕ), a typical trajectory whose asymptotic direction is the observer's direction OO' lies in the plane $OX''Y'$ and reaches the photographic plate (which is the plane $O'X''Y''$) at a point m, determined by its polar coordinates (b, α).

Assuming that the observer is practically at infinity and at rest in the gravitational field of the black hole, then the polar distance from m to $0'$ is precisely the impact parameter of the trajectory, and the polar angle α with the "vertical" direction $O'Y''$ is the complement of the dihedral angle between planes OXY' and $OX'Y'$. For a given coordinate r, varying ϕ from 0 to π (the figure being symmetric with respect to $O'Y''$-axis), we get the apparent shape $b(r) = b(r, \alpha)$ on the photographic plate of the circular ring orbiting the black hole at distance r.

As seen in the previous section, rays emitted from a given point M can circle around the black hole before escaping to infinity, giving an infinite series of images on the photographic plate; the same arguments indicate that only the secundary image (which corresponds to rays that have circuited once) has to be taken into account, images of higher order being almost exactly superposed at the critical value b_c. Thus, for a given emitter M, the observer will detect generally two images, a *direct (or primary) image* at polar coordinates $(b^{(d)}, \alpha)$ and a *ghost (or secundary) image* at $(b^{(s)}, \alpha + \pi)$.

Relationships between the different angles involved by the problem follow directly from the resolution of spherical triangles XMY' and XMX'' (Fig. 3). We get:

$$\cos\alpha = \cot\phi\cos\theta_0/\sin\gamma = \cos\phi\cos\theta_0(1 - \sin^2\theta_0\cos^2\phi)^{-1/2} \tag{9}$$

so that α is a monotonic increasing function of ϕ.

We need also the relation

$$\cos\gamma = \cos\alpha(\cos^2\alpha + \cot^2\theta_0)^{-1/2}. \tag{10}$$

Image of a Spherical black hole: the absorption cross section of light

THE IMPACT PARAMETER

$$\left(\frac{1}{r^2}\frac{dr}{d\phi}\right)^2 + \frac{1}{r^2}\left(1 - \frac{2M}{r}\right) = \frac{1}{b^2}$$

The light ray gets `absorbed' for $b < 3\sqrt{3}M$
and `deflected' for $b > 3\sqrt{3}M$

$$b_c = 3\sqrt{3}M$$

Absorption Cross section of the black hole

$$\sigma = 27\pi M^2$$

CORRECTED EQUATION

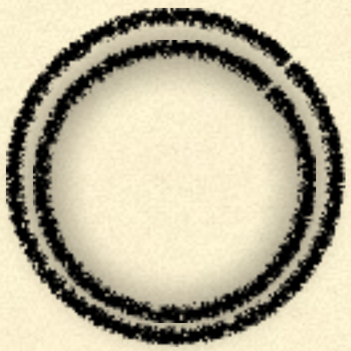
$$\frac{E^2}{f} + g\left(\frac{dr}{ds}\right)^2 + \frac{L^2}{\tilde{q}} = -l$$

$$f(r) = -\left(1 - \frac{r_g}{r}\right) + h_{tt}$$

$$g(r) = \left(\frac{r}{r - r_g}\right) + h_{rr}$$

$$\tilde{q} = r^2 + h_{\phi\phi}$$

$$b_{c\ \mathrm{corr}} = 3\sqrt{3}M\left(1 + \frac{1}{18M^2}\tilde{t} + ..\right)$$



INTERFERENCE FRINGES?
ANGLE DEPENDENCE? INSTABILITIES?

INFINITESIMAL BUT CAN BE CALCULATED AND ONE DAY DETECTED!

SPHERICALLY REDUCED COHERENT STATE

- The break down of spherical symmetry and a correction term which cannot be anticipated by linearized perturbations using spherical harmonics is physically counter-intuitive.

$$h_{\mu\nu} = \sum H_{\mu\nu}(r) e^{-i\omega t} Y_{lm}(\theta, \phi)$$

- A reduction+quantization might retain spherical symmetry in the non-perturbative framework

A REDUCED PHASE SPACE

$$\vec{A} = A_3\tau_3 dx + (A_1\tau_1 + A_2\tau_2)d\theta + (-A_2\tau_1 + A_1\tau_2)\sin\theta d\phi + \tau_3\cos\theta d\phi$$

$$\vec{E} = E_3\tau_3\sin\theta\frac{\partial}{\partial x} + (E_1\tau_1 + E_2\tau_2)\sin\theta\frac{\partial}{\partial\theta} + (-E_2\tau_1 + E_1\tau_2)\frac{\partial}{\partial\phi}$$

Corrections:

$$q^{xx} = \left(\frac{2m}{r} - 1\right)^{-1}\left(1 + 2F\left(\frac{r^2}{a}\right)\right)$$

$$q^{\theta\theta} = \frac{1}{r^2}\left(1 + 2F(\sqrt{r(2m-r)})\right) \quad q^{\phi\phi} = \frac{1}{r^2\sin^2\theta}\left(1 + 2F(\sqrt{r(2m-r)})\right)$$

$$\delta K_{xx} = \left[\delta A_3 - 2\frac{m}{r^2}F\left(r^2/a\right)\right]\frac{\sqrt{r}}{\sqrt{2m-r}}$$

No cross terms! Spherical symmetry is also maintained

TO BREAK SPHERICAL SYMMETRY OR NOT TO BREAK SPHERICAL SYMMETRY

- Nature of course does not `reduce’
- there has to be additional constraints to maintain spherical symmetry for semi-classical corrections
- Experimental predictions
- Generalization to Kerr black holes-in progress