Linearized Gravity with Matter time

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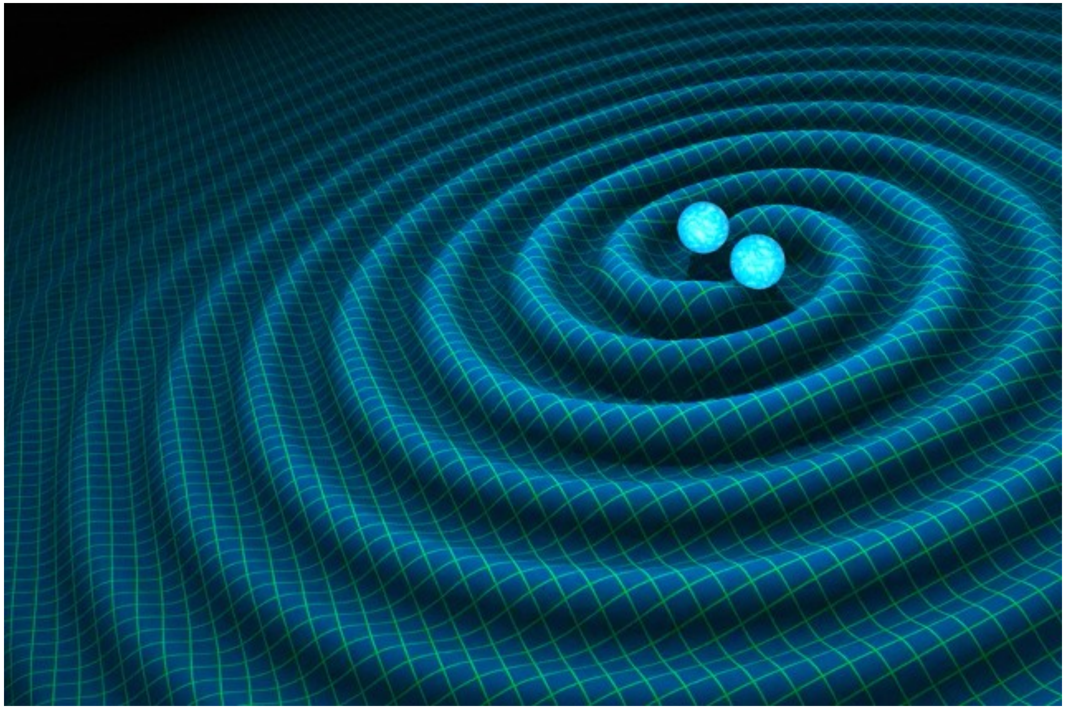
Linearized GR

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$||h_{\mu\nu}|| << 1$$



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Lorenz Gauge:

$$\partial_\mu h^\mu_\nu - \frac{1}{2}\partial_\nu h = 0$$

$$\Box\, \bar{h}_{\mu\nu} = -16\pi G\, T_{\mu\nu}$$

In Vacuum:

$$\Box\, \bar{h}_{\mu\nu} = 0$$

The Lorenz gauge is not a full gauge fixing.

Transverse Traceless Gauge:

$$h^{0i} = 0$$

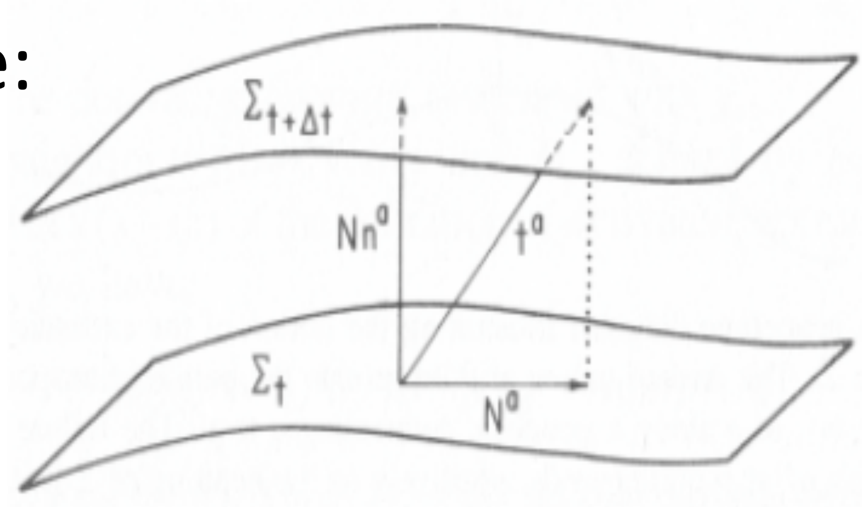
$$\delta^{ij} h_{ij} = 0$$

Can we recover the usual TT graviton modes after using matter degrees of freedom in gauge fixing?

ADM Formalism

Dynamical variables are:

$$(q_{ab}, \tilde{\pi}^{ab})$$



$$S = \int dt d^3x \ \left[\tilde{\pi}^{ab}\dot{q}_{ab} - \underbrace{N\mathcal{H} - N^a\mathcal{C}_a}_{\text{Hamiltonian}}\right]$$

2 physical degrees of freedom

Outline

- Defining the system
- Partial reduction of the theory by fixing a time gauge
- Linearization
- Complete reduction by solving the diffeomorphism constraint
- The physical degrees of freedom

GR with Dust

- Consider GR coupled to dust:

$$S = \int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}M(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1)$$

$$T_{\mu\nu} = M\partial_\mu\phi\partial_\nu\phi$$

This system has 3 physical degrees of freedom

- The ADM action is:

$$S = \int dt d^3x \left[\tilde{\pi}^{ab}\dot{q}_{ab} + p_\phi \dot{\phi} - N\mathcal{H} - N^a \mathcal{C}_a\right]$$

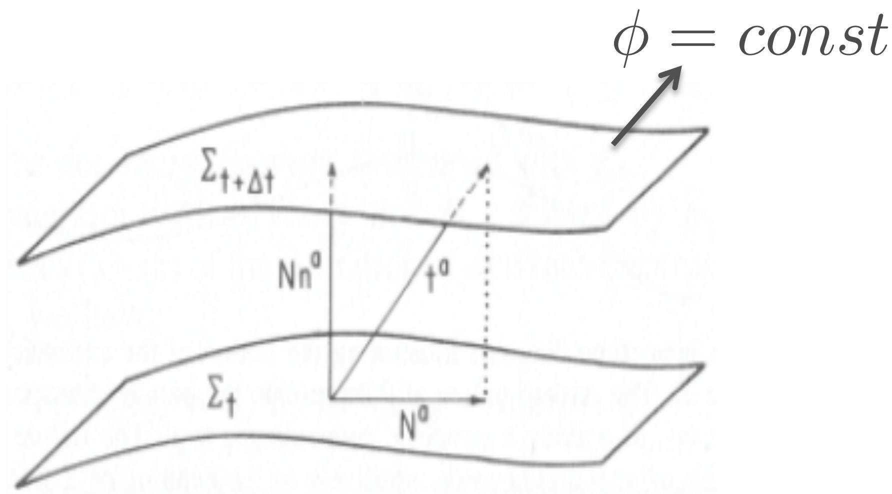
- The Hamiltonian and diffeo constraints are:

$$\mathcal{H} = \mathcal{H}_G + \mathcal{H}_D$$
$$C_a = C_a^G + C_a^D$$

We use the dust to fix the time gauge

Dust time*

Surfaces of constant time = level surfaces of dust



$$\lambda \equiv \phi - t = 0$$

$$N = 1$$

$$H_{physical} = -p_\phi = \int d^3x\, \mathcal{H}_G$$

*V. Husain and T. Pawlowski
Phys.Rev.Lett. 108 (2012) 141301.

Linearized Gravity with Dust time

- Minkowski space is a solution of the equations of motion in the dust time gauge.
- All 3 physical d.o.f are manifested in the metric field
- Linearize about Minkowski

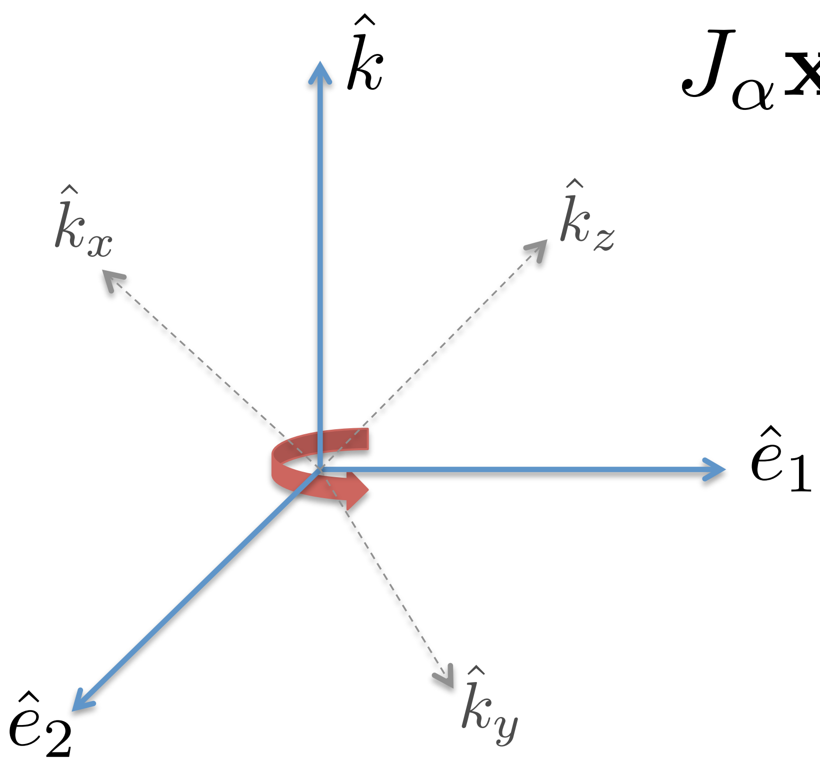
$$q_{ab} = \delta_{ab} + h_{ab}$$

$$\tilde{\pi}^{ab} = 0 + p^{ab}$$

$$N^a = 0 + \xi^a$$

A Suitable Basis

- We want a natural separation of perturbations into scalar, vector and tensor modes



$$J_\alpha \mathbf{x} = e^{im\alpha} \mathbf{x}^*$$

Helicity

Basis

Scalar

$$M_1^{ab} = \delta^{ab},$$

$$M_2^{ab} = f(k, \delta^{ab})$$

Vector

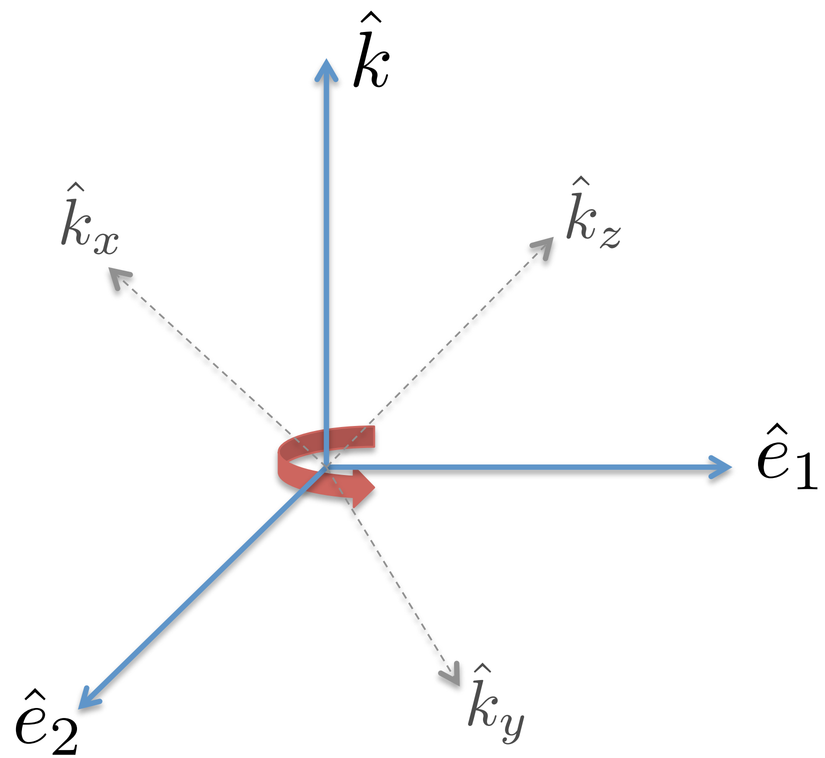
$$M_5^{ab} = f(e_1, e_2, k),$$

$$M_6^{ab} = g(e_1, e_2, k)$$

Tensor

$$M_3^{ab} = f(e_1, e_2),$$

$$M_4^{ab} = g(e_1, e_2)$$



Equations of Motion

- Scalar mode equations:

Only the longitudinal component of the shift appears

$$\dot{h}_1 = -p_1 + |k|\, \xi_{\|},$$

$$\dot{p}_1 = |k|^2\, h_1 - |k|^2\, h_2$$

$$\dot{h}_2 = 2p_2 + |k|\, \xi_{\|}$$

$$\dot{p}_2 = -|k|^2\, h_1 + |k|^2\, h_2$$

- Vector mode equations:

$$\dot{h}_5 = p_5 + |k|\,\xi_2$$

$$\dot{p}_5 = 0$$

$$\dot{h}_6 = p_6 + |k|\,\xi_1$$

$$\dot{p}_6 = 0$$

Only the transverse components of the shift vector appear

- Tensor mode equations:

$$\dot{h}_3 = p_3, \quad \dot{p}_3 = -|k|^2\, h_3,$$
$$\dot{h}_4 = p_4, \quad \dot{p}_4 = -|k|^2\, h_4,$$

No components of the shift vector!

The Diffeomorphism Constraint

$$k_a \bar{p}^{ab} = k_a p^I(k,t) M_I^{ab} = 0$$

$$\Longrightarrow \; (\, p_1 + p_2) \, k^b + |k| \; \left(p_5 \;\, e_2^b + p_6 \;\, e_1^b\right) = 0$$

Only the scalar and vector modes are constrained

The Final Reduction

Transverse components:

Vector modes

$$\left(\, p_1 + p_2\right) k^b + |k| \left(p_5 \; e_2^b + p_6 \; e_1^b\right) = 0$$

Gauge choice:

$$h_5 = 0 = h_6$$

Longitudinal component:

$$(p_1 + p_2)\, k^b + |k| \left(p_5 \; e_2^b + p_6 \; e_1^b\right) = 0$$

Scalar modes

Gauge choice:

$$h_2 = 0$$

Physical degrees of freedom

One ultra-local scalar mode

$$\ddot{h}_1 = 0$$

Two Lorentz covariant tensor modes

$$\ddot{h}_I = -|k|^2 \, h_I \qquad I = 3, 4$$

Results

- The graviton modes propagate at the speed of light
- The scalar mode is ultra-local. None of the consequences associated with dynamical scalars.
- GR + dust in the dust time gauge appears to be consistent with standard field theory on Minkowski spacetime

- If we were to deform the Hamiltonian as:

$$H := -\sqrt{q}R^{(3)} + \frac{1}{\sqrt{q}}\left(\tilde{\pi}^{ab}\tilde{\pi}_{ab} - \alpha\tilde{\pi}^2\right)$$

the scalar mode no longer remains ultra-local

$$\ddot{h}_1 = (1-2\alpha)|k|^2 h_1$$

- If we include a potential for the dust field this would act as a dynamical cosmological constant

Thank you for listening.

Counting the degrees of freedom

Covariant Picture:

$$(g_{\mu\nu})$$

10 metric components

–

4 coordinate choices

–

4 constraint equations

=

2 physical d.o.f

Canonical Picture:

$$(q_{ij}, \pi^{ij})$$

12 components

–

4 coordinate choices

–

4 first class constraints

=

4 phase space d.o.f

Linearized Theory

$$q_{ij} = \delta_{ij} + h_{ij}$$

ADM showed that for any symmetric tensor:

$$h_{ij} = h_{ij}^{TT} + h_{ij}^{T} + h_{ij}^{L}$$

$$N = 1, \quad N^a = 0$$

$$\Rightarrow \quad h_{\mu\nu} = h_{\mu\nu}^{TT}$$