

Midterm 1 Solutions

1

- 200 MW fuel plant (steam cycle)
- boiler @ 400°C \rightarrow condenser @ 100°C

$$\text{a) } \eta = 1 - \frac{T_c}{T_H} \quad (\text{convert to } ^{\circ}\text{K})$$

$$\eta = 1 - \frac{373}{67} = 0.446$$

* but this is only 60% efficient

$$\eta_{\text{actual}} = 60\% \eta$$

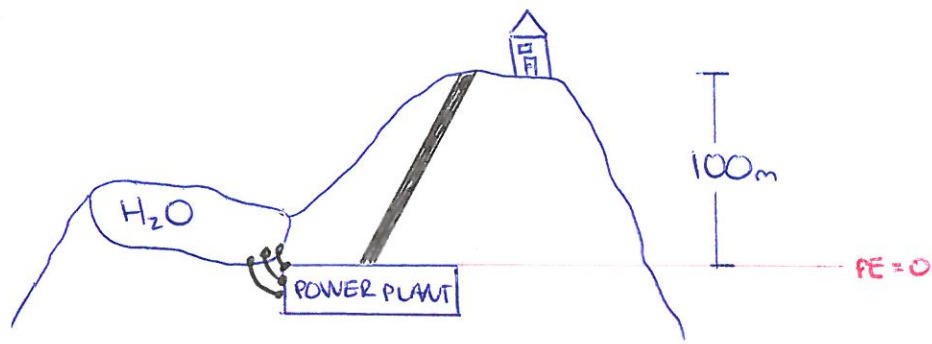
$$\eta_{\text{actual}} = 0.268 = \frac{|W|}{Q_H}$$

$$\text{③ } \therefore \boxed{Q_H = \frac{200 \text{ MW}}{0.268 \times 10^{-1}} = 746 \text{ MW}}$$

$$\text{b) } Q_{\text{out}} = Q_{\text{in}} - W \\ = 746 \text{ MW} - 200$$

$$\text{① } \boxed{Q_{\text{out}} = 546 \text{ MJ/s}}$$

2



① $\frac{\text{energy}}{s} = 500 \times 10^6 \text{ W} \times 70\% = 350 \text{ MJ/s}$

$PE_g = mgh$

① $m = \frac{PE_g}{g \Delta h} = \frac{350 \text{ [MJ/s]}}{9.81 \text{ [m/s}^2\text{]} \cdot 100 \text{ [m]}} = 3.6 \times 10^5 \text{ kg/s}$

3 Entropy Question:

(a) (AD) $m = 10 \text{ kg @ } 60^\circ\text{C}$

lake $m = \text{large!}$ later \rightarrow (AD) lake

After some time, AD has given off heat Q to the lake

$Q_{AD} = m C_{AD} \Delta T = (10 \text{ kg})(895 \text{ J/kg/K})(-50 \text{ K}) = -4.5 \times 10^5 \text{ J}$

$\Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = -\frac{Q_{AD}}{T_{\text{lake}}} = +\frac{4.5 \times 10^5 \text{ J}}{283 \text{ K}} = 1.58 \times 10^3 \frac{\text{J}}{\text{K}}$

(b) Case (b): In this case, the AD receives heat from lake

$Q_{AD} = m C_{AD} \Delta T = (10 \text{ kg})(895 \text{ J/kg/K})(+5) = 44750 \text{ J}$

$\Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = -\frac{Q_{AD}}{T_{\text{lake}}} = -\frac{44750 \text{ J}}{283 \text{ K}} = 158 \frac{\text{J}}{\text{K}}$ ②

4

a) False. The rule is that, in a closed system the change in entropy of the system is zero. If there are two objects in a system, one can decrease in entropy; but the other object will increase such that $\Delta S_{\text{total}} \geq 0$

b) false. A heat pump uses warm air to heat objects (room) that are already hot. It takes work (W) to achieve this, but it is possible.

The correct statement says: No process is possible whose sole result is the removal of heat from one object and the transfer of an equal amount of heat to a hotter object.

[5]

$$(a) \quad \text{COP}_{\max} = \frac{T_H}{T_H - T_c} = \frac{293}{293 - 263} = 9.8$$

$$W_{\min} = \frac{Q_H}{\text{COP}_{\max}} = \frac{20 \text{ kW}}{9.8} = \underline{\underline{2.0 \text{ kW}}}$$

$$(b) \quad \text{Actual COP} = 0.4 \text{ COP}_{\max} = (0.4)(9.8) = \underline{\underline{3.9}}$$

$$\therefore W = \frac{Q_H}{\text{COP}} = \frac{20 \text{ kW}}{3.9} = \underline{\underline{5.1 \text{ kW}}}$$

(c) Advantages: Ground based have smaller difference between T_H & T_c since the ground is warmer than the air in winter. This means higher COP

Disadvantage: Increase cost associated with burial of evaporator lines.

6



molar mass

$$\text{C} = 12 \text{ g/mol}$$

$$\text{H} = 1 \text{ g/mol}$$

$$\text{O} = 16 \text{ g/mol}$$

$$\text{CH}_4 = 12 \text{ g/mol} + 4(1) \text{ g/mol}$$

$$\text{CH}_4 = 16 \text{ g/mol}$$

$$\text{mol}_{\text{CH}_4} = 1000 \text{ g} \cdot \frac{\text{mol}}{16 \text{ g}} = 62.5 \text{ mol}$$

$$\text{CO}_2 = 12 \text{ g/mol} + 2 \cdot 16 \text{ g/mol}$$

$$\text{CO}_2 = 44 \text{ g/mol}$$

$$\text{Amount}_{\text{CO}_2} = \frac{62.5 \text{ mol}}{\text{CH}_4} \cdot \left[\frac{1 \text{ mol CH}_4}{1 \text{ mol CO}_2} \right] \cdot \frac{44 \text{ g}}{1 \text{ mol CO}_2} = 2750 \text{ g}$$

$$= 2.75 \text{ kg}$$

$$\text{b) } Q = mc\Delta T + mL_v$$

$$= (1)(2 \times 10^3)(-161 - 20^\circ \text{C}) + (1)(59 \times 10^3)$$

$$Q = 4.21 \times 10^5 \text{ J} \text{ "removed"}$$

c) ratio is 2:1, so it takes $8.42 \times 10^5 \text{ J}$ of electrical energy for the same job.

But only 40% efficient that means actual

$$\text{energy needed was } Q_{in} = \frac{8.42 \times 10^5}{0.4} = 2.1 \times 10^6 \text{ J}$$

$$\text{amount natural gas} = 2.1 \times 10^6 \text{ J} \cdot \frac{1 \text{ kg}}{55 \times 10^6 \text{ J}} = 0.038 \text{ kg}$$

- 7
- world natural oil resources 1.38×10^{12} bbl
 - consumption rate is 3.2×10^{10} bbl/year

a) "constant rate"

$$\boxed{\text{time}} = \frac{1.38 \times 10^{12} \text{ [bbl]}}{3.2 \times 10^{10} \text{ [bbl/year]}} = \boxed{43 \text{ years}} \quad (1)$$

b) "exponential rate"

- know it takes 23 years to double

$$\boxed{N = N_0 e^{kt}} \quad \text{exponential growth eqn.}$$

$$\frac{N}{N_0} = 2 \longrightarrow 2 = e^{kt} \longrightarrow \ln(2) = \ln(e^{kt})$$

$$\ln(2) = kt \ln(e) \longrightarrow k = \frac{\ln(2)}{t}$$

$$\boxed{k = \frac{\ln(2)}{23} = 0.030 \text{ years}^{-1}} \quad (1)$$

* asks for lifetime

$$t = \frac{1}{k} \ln\left(\frac{kQ_T}{N_0} + 1\right)$$

$$Q_T = 1.38 \times 10^{12} \text{ [bbl]}$$

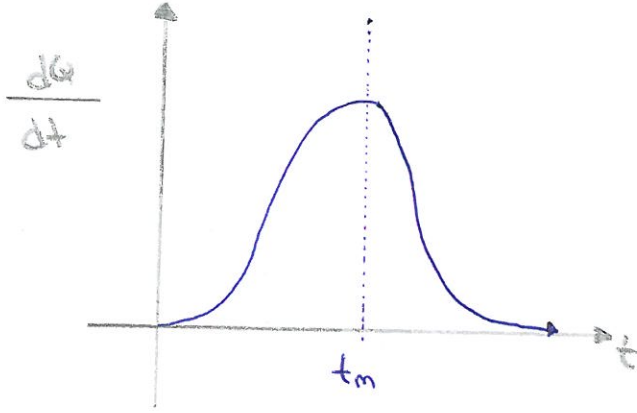
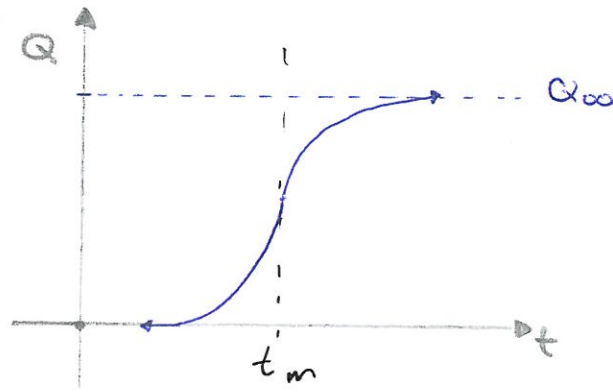
$$N_0 = 3.2 \times 10^{10} \text{ [bbl/year]}$$

$$= \frac{1}{0.030} \ln\left(\frac{0.03 \cdot 1.38 \times 10^{12}}{3.2 \times 10^{10}} + 1\right)$$

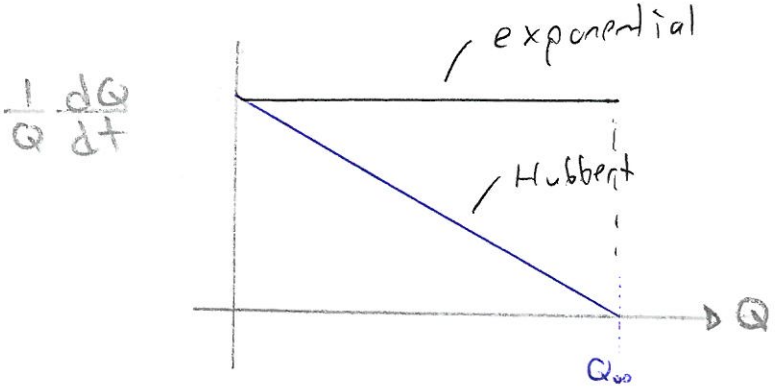
$$\boxed{t = 24 \text{ years}} \quad (1)$$

8

a)



b)



Note: x axis is $Q(t)$, not t

c) takes into account : rate of discovery , rate of production and the size of resources at any time. The resource (assuming fixed amount) is first utilized slowly , then more rapidly as technologies are developed to better use resources potential. The production rate eventually declines because there are less and less of the resource to produce.