Calculus Notes, Physics 346

If $y = f(x)$, the derivative is written as either $\frac{dy}{dx}$ or $f'(x)$. Physicists usually prefer the form $\frac{dy}{dx}$ since it resembles the ratio $\frac{\Delta y}{\Delta x}$ which equals the derivative for the limit where Δx approaches zero. Below are some examples of simple differentiation relevant to this course:

1. Derivative of a polynomial: If $y = Ax^n$ where n is an integer $\frac{dy}{dx} = (n-1)Ax^{n-1}$

   Example: If $y(x) = Ax^3$ then $\frac{dy}{dx} = 3Ax^2$

2. Note that the derivative of $y(x) = \frac{1}{x^n}$ can also be calculated using the above

   Example: If $y(x) = \frac{A}{x^3}$ then $\frac{dy}{dx} = (-3)Ax^{-4} = \frac{-3A}{x^4}$

3. Derivative of the exponential function (special property)

   If $y(x) = e^x$, then $\frac{dy}{dx} = e^x$

4. Chain rule: Sometimes we can calculate derivatives of a more complicated function by using the chain rule:

   If $y(x)$ can be written in the form $y(x) = y(u(x))$ then the derivative is given by $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

   Example: $y = e^{-kx}$ can be written in the form $y(u) = e^u$ where $u(x) = -kx$

   $\frac{dy}{du} = e^u$ and $\frac{du}{dx} = -k$

   Therefore $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = e^u \times (-k) = -ke^{-kx}$

   Example $y = \frac{1}{(A+Bx^3)^2}$. In this case define $y = \frac{1}{u^2}$ where $u = (A + Bx^3)$.

   The chain rule says $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

   $\frac{dy}{du} = \frac{-2}{u^3}$

   The derivative of $u = (A + Bx^3)$ is $\frac{du}{dx} = 3Bx^2$

   The complete answer is $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{(-2)}{(A+Bx^3)^3}(3Bx^2)$