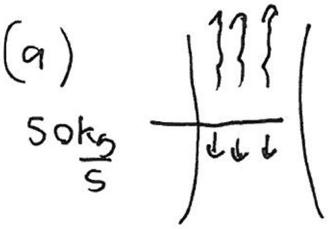
Assignment 4 Solutions   Feb 2012

① (a)



Heat to vaporize 50 kg

$$Q = m\,l^{vap} = (50\ \text{kg})(2.33\times10^{6}\ \tfrac{J}{kg})$$

$$= 1.17\times10^{8}\ J \quad \text{each second}$$

(b) To remove the same heat by cooling water, assuming $\Delta T = 10^\circ C$:

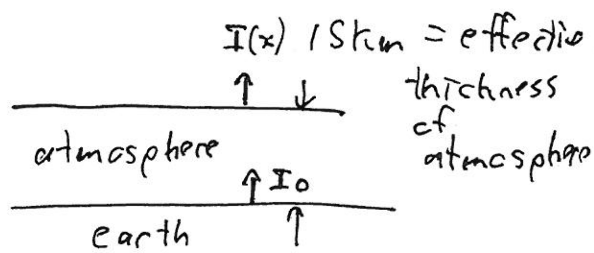
$$Q = m_1\, C_{water}\, \Delta T \qquad C_{water} = 4186\ J/kg/K$$

$$\therefore \quad m_1 = \frac{Q}{C_{water}\,\Delta T} = \frac{1.17\times10^{8}\ \cancel{J}}{(4186\ \tfrac{\cancel{J}}{kg\,\cancel{K}})(10\ \cancel{K})}$$

$$m = 2795\ \text{kg} \gg 50\ \text{kg}!$$

each second.

② Beers Law $I = I_0 e^{-kx}$



(a) Transmitted light $= 0.3\, I_0$

$$\therefore \quad 0.3\,\cancel{I_0} = \cancel{I_0}\, e^{-k(15\ km)}$$

$$\therefore \quad e^{\ln(.3)} = e^{-k(15\ km)}$$

$$\therefore \quad k = \frac{\ln 0.3}{(15\,km)} = 0.0802\ km^{-1}$$

$$k = 0.0802\ km^{-1}$$

(b) Surface temperature is 286 K

$\therefore$ Power intensity emitted is $I_0 = \sigma T^4$
(assuming $\varepsilon = 1$)

$$\therefore I_0 = (5.87 \times 10^{-8}\, W\, m^{-2} K^{-4})(286\, K)^4$$

$$= 392.7\ \frac{W}{m^2}$$ radiated from the surface.

The amount transmitted is $0.3\, I_0$

$\therefore$ The amount absorbed is $(1 - 0.3)\, I_0$

$$= (0.7)(392.7\ \frac{W}{m^2})$$

$\therefore$ Amount absorbed $= 275\ \frac{W}{m^2}$

(c) If we double k the amount transmitted goes from $I = I_0 e^{-kx} \rightarrow I_0 e^{-2kx} = (I e^{-kx})^2$

$\therefore$ But $e^{-kx} = 0.3$ for $x = 15\ km$

$\therefore$ if we double k

we get a transmission of $e^{-2kx} = .3^2 = 0.09$

The amount absorbed is now $1 - 0.09 = 0.91$ ∴

Total amount absorbed is now $I_0(0.91)$

$= \left(392.7 \frac{W}{m^2}\right)(0.91) = \underline{\underline{357.4}}\ W$

Amount absorbed has increased by $\frac{0.91}{0.70}$

$= 1.3$

∴ Even though we have doubled the concentration, the absorbed power has only increased by $\underline{\underline{30\%}}$.

5.10 a)

Fossil Fuel: 1787 TW·h @40% efficiency

Nuclear Plants: 476 TW·h @ 30% efficiency

Total power (P_{in}) for Fossil Fuel:

$$\frac{1787}{P_{in}} = 0.4 \Rightarrow P_{in_{FF}} = 4467.5 \text{ TW·h}$$

$$\text{Waste}_{FF}: \ 4467.5 \text{ TW·h} - 1787 \text{ TW·h} = 2680.5 \text{ TWh}$$

Similarly, Total power for Nuclear plants:

$$P_{in_{NP}} = \frac{476 \text{ TW·h}}{0.3} = 1586.7 \text{ TW·h} \Rightarrow 1586.7 - 476 = 1111 \text{ TW·h waste}_{NP}$$

Total Waste:

$$Q_0 = 2680.5 \text{ TW·h} + 1111 \text{ TW·h} = 3791.5 \text{ TW·h} \times \frac{3600 \text{ sec}}{h} = 13.6 \text{ EJ}$$

How much water needed?

$$Q_0 = mc\Delta T$$

$$m = \rho V$$

$$Q_0 = \rho V \Delta T c \Rightarrow V = \frac{Q_0}{\rho \Delta T c}$$

$\rho = 1000 \text{ kg/m}^3$ $\quad C = 4186 \text{ J/kg°C}$

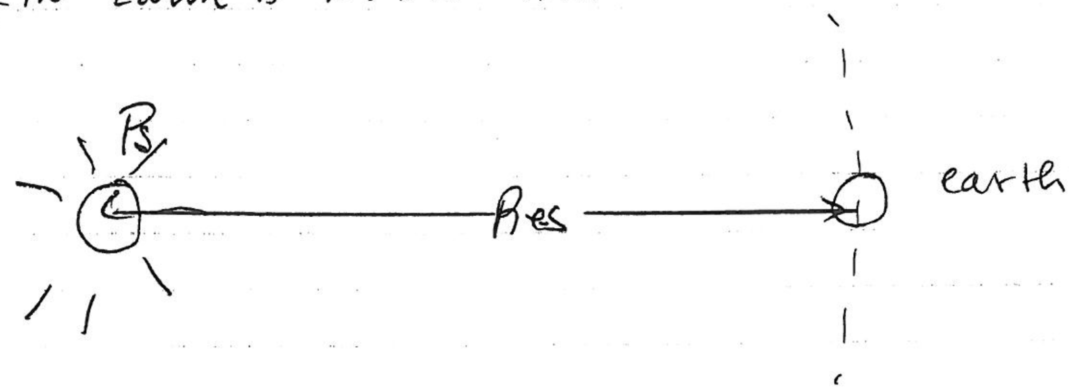
$\Delta T = 10°$

$$V = \frac{13.6 \times 10^{18} \text{ J}}{\left(1000 \frac{kg}{m^3}\right)(10°)\left(4186 \text{ J/kg °C}\right)} = 3.25 \times 10^{11} \text{ m}^3$$

b) $\text{Runoff} = 5.6 \times 10^{12} \text{ m}^3 \times 0.30 = 1.68 \times 10^{12} \text{ m}^3$ $\quad$ 1/5 of this is $\boxed{3.36 \times 10^{11} \text{ m}^3}$

pretty close!

6-6. What is the total power developed by the sun, given the solar constant at the mean position of the Earth is $1.4 \times 10^3 W/m^2$?



By conservation of energy, the power developed by the sun

$$P_S = I_0 \cdot 4\pi R_{es}^2$$

$$= 1.4\times10^3 W/m^2 \cdot 4 \cdot \pi \cdot (149.6\times10^6 \times 10^3 m)^2$$

$$= 3.94 \times 10^{26} \text{ W}.$$

6-13: Assume no atmosphere for this question

From notes/text: $T^4 = \frac{I_0}{4\sigma}(1-\alpha)$, $T = \left(\frac{I_0}{4\sigma}\right)^{\frac{1}{4}}(1-\alpha)^{\frac{1}{4}}$

$$\therefore \frac{dT}{d\alpha} = -\left(\frac{1}{4}\right)\left(\frac{I_0}{4\sigma}\right)^{\frac{1}{4}}(1-\alpha)^{-\frac{3}{4}}$$

using known constants: $\boxed{\frac{dT}{d\alpha} = \frac{-69.7}{(1-\alpha)^{\frac{3}{4}}}}$

For small $d\alpha$:

$$\Delta T = -69.7\,(1-\alpha)^{-\frac{3}{4}}\Delta\alpha$$

Assume $\alpha = 0.31$ and $\Delta\alpha = .0031$

$$\therefore \Delta T = -69.7\,(.69)^{-\frac{3}{4}}(.0031) = \underline{\underline{-.29\text{ K}}}$$

~~-9. If we assume the sun radiates like a black body~~

$$I(T) = \sigma T^4$$

~~we can estimate its surface temperature~~

$$T = \left(\frac{I(T)}{\sigma}\right)^{1/4} = \left[\left(\frac{P_S}{4\pi R_S^2}\right)\frac{1}{\sigma}\right]^{1/4}$$

$$= \left[\frac{3.94\times10^{26}\,W}{4\pi(6.98\times10^{8}\,m)^2}\cdot\frac{1}{5.67\times10^{-8}\,\frac{W}{m^2}\cdot\frac{1}{K^4}}\right]^{1/4}$$

$$= 5800\ K.$$

6-14. We can use Planck's radiation equation to calculate the ratio of the scattered intensities:

$$T = 3000\ K.$$

$$\frac{I(500\,nm, 3000\,K)}{I(1000\,nm, 3000\,K)} = \frac{\frac{2\pi hc^2}{\lambda_1^5}\cdot\frac{1}{e^{hc/\lambda_1\cdot 1/k_BT}-1}}{\frac{2\pi hc^2}{\lambda_2^5}\cdot\frac{1}{e^{hc/\lambda_2\cdot 1/k_BT}-1}}.$$

$$= \left(\frac{\lambda_2}{\lambda_1}\right)^5 \frac{\exp.\left(hc/k_BT\cdot 1/\lambda_2\right)-1}{\exp\left(hc/k_BT\cdot 1/\lambda_1\right)-1}$$

$$= \left(\frac{\lambda_2}{\lambda_1}\right)^5\cdot\frac{\exp\left(hc/k_BT\ 1/2\lambda_1\right)-1}{\exp\left(hc/k_BT\cdot 1/\lambda_1\right)-1}.$$

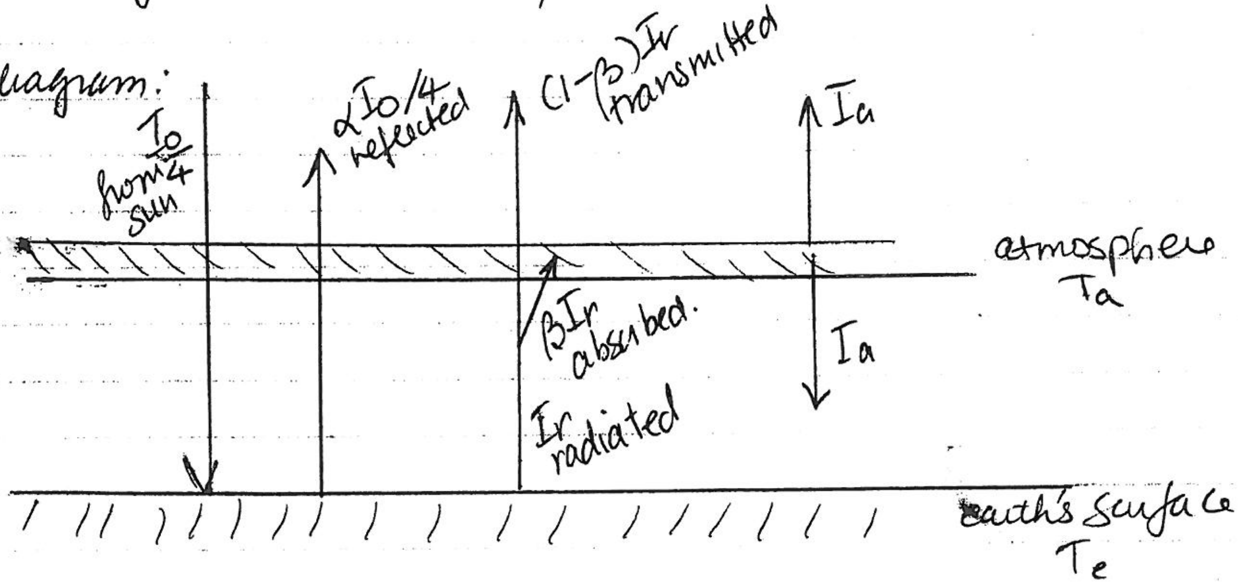
where
$$\frac{hc}{k_BT}\frac{1}{\lambda_1} = \frac{6.63\times10^{-34}\,J\cdot s\cdot 3.0\times10^{8}\,m/s}{1.38\times10^{-23}\,J/K\cdot 3000\,K\cdot 500\times10^{-9}\,m}$$

$$= 9.6$$

$$\therefore\ \frac{I(\lambda_1)}{I(\lambda_2)} = (2)^5\cdot\frac{\exp 4.8-1}{\exp 96-1} = 0.26.$$

6-17. Energy balance for the earth with a semi-transparent atmosphere, where $\beta = 0.75$ or 75% of the radiation from the earth is absorbed.

New diagram:



for earth $\quad \frac{I_o}{4} + I_a = \alpha \frac{I_o}{4} + I_r$

for atmosphere $\quad \beta I_r = 2 I_a.$

$$\therefore I_a = \frac{\beta}{2} I_r.$$

$$\therefore \frac{I_o}{4} - \alpha \frac{I_o}{4} = I_r - \frac{\beta}{2} I_r$$

$$(1-\alpha)\frac{I_o}{4} = \left(1 - \frac{\beta}{2}\right) I_r$$

and $I_r = \dfrac{(1-\alpha)}{1-\frac{\beta}{2}} \dfrac{I_o}{4} = \dfrac{2(1-\alpha) I_o}{2-\beta}$

6-18: from 6-17 we know

$$I_r = \frac{2(1-\alpha)}{2-\beta} \frac{I_0}{4}$$

where $\frac{I_0}{4} = 342 \ W/m^2$ solar insolation

$\alpha = 0.31$ albedo

$\beta = 0.75$ atmospheric absorbtion

$$\therefore \ I_r = \frac{2 \cdot 0.69 \cdot 342 \ W/m^2}{(2-0.75)}$$

$$I_r = 378 \ W/m^2 .$$

assuming the earth radiates like a black body:

$$I_r = \sigma T_e^4$$

$$T_e = \left(\frac{I_r}{\sigma}\right)^{1/4} = \left(\frac{378 \ W/m^2}{5.67 \times 10^{-8} \ W/m^2/K^4}\right)^{1/4}$$

$$= 286 \ K !$$