Assignment #2 Physics 346

(1) Verify that $Q = \dfrac{Q_\infty}{1+e^{k(t_m-t)}}$ is a solution

of Hubbert's differential equation:

$$\frac{dQ}{dt} = kQ\left(1-\frac{Q}{Q_\infty}\right)$$

First consider the left side: evaluate $\frac{dQ}{dt}$ using the above form of Q

$$LHS = \frac{dQ}{dt} = \frac{Q_\infty (k)\, e^{k(t_m-t)}}{\left(1+e^{k(t_m-t)}\right)^2}$$

Now substitute Q into the right side:

$$RHS = kQ\left(1-\frac{Q}{Q_\infty}\right) = k\left(\frac{Q_\infty}{1+e^{k(t_m-t)}}\right)\left(1-\frac{\cancel{Q_\infty}}{1+e^{k(t_m-t)}}\frac{1}{\cancel{Q_\infty}}\right)$$

$$= \frac{kQ_\infty}{1+e^{k(t_m-t)}}\left(\frac{\cancel{1}+e^{k(t_m-t)}-\cancel{1}}{1+e^{k(t_m-t)}}\right)$$

$$= \frac{kQ_\infty e^{k(t_m-t)}}{\left(1+e^{k(t_m-t)}\right)^2}$$

which is the same as $\frac{dQ}{dt}$

$\therefore$ LHS = RHS

The given form of Q <u>is</u> a solution

(2): (a) From the graph we can estimate

* the initial % growth rate $k \sim .17$ (17%)

we can estimate Q_∞ as ~ 29 Gb (giga barrels or 10^9 barrels)

Also we are given that $t_m \sim 2001$

Now we just plot the expression for $\frac{dQ}{dt}$ vs time

From question 1, $$\frac{dQ}{dt} = \frac{Q_\infty k e^{k(t_m - t_m)}}{\left(1 + e^{k(t_m - t_m)}\right)^2}$$

we set up 2 colums in excel, the first runs from some start time to some finish time

We then enter the above formula in the second column and plot it against the 1st column.

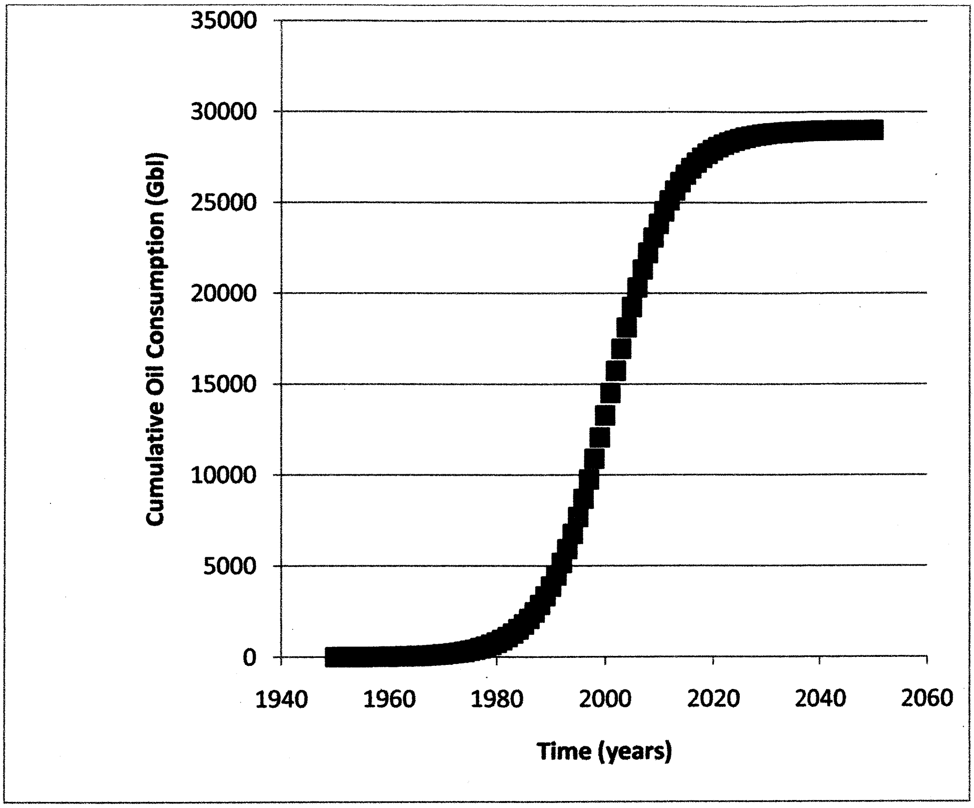
(b) Peak production rate occurs when $\frac{dQ}{dt} = @$ max

From our spreadsheet we can see that this occurs in 2001 at a rate of

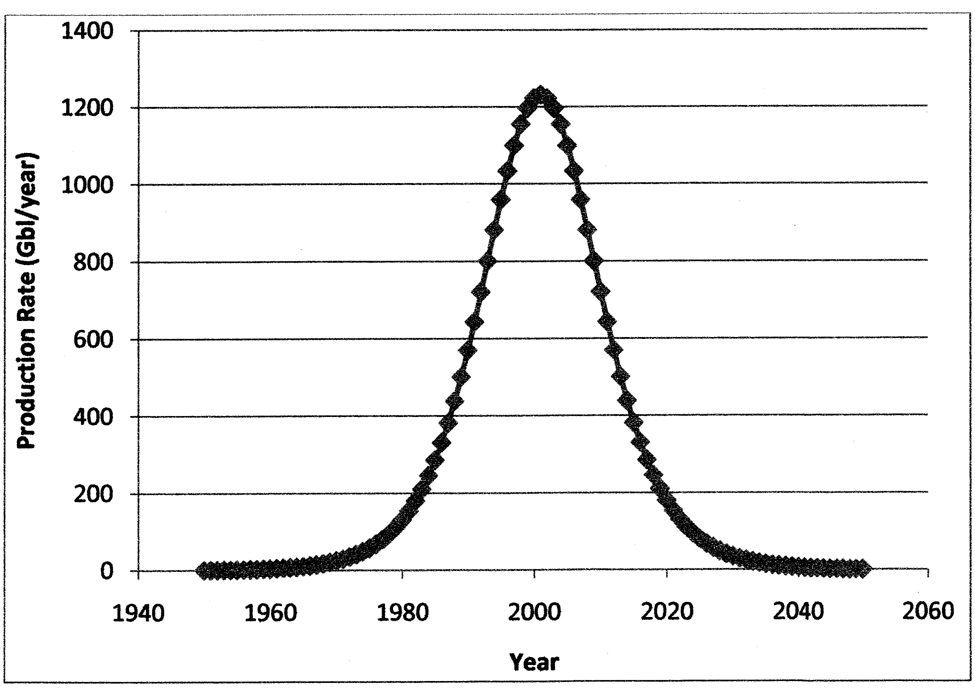
$$\frac{dQ}{dt}_{max} = 1.23 \text{ Gbl/year}$$

(c) Production drops to $\frac{1.23 \text{ Gbl/year}}{3}$ at roughly 2014 - 2015

(d) Production drops to $\frac{1.23 \text{ Gbl/year}}{10}$ at roughly 2022 - 2023.



← total cumulative oil consumption (this was not required)



(3) $T_H = 500°C$

$T_C = 100°C$

$\eta_{max} = \frac{500-100}{773 \cancel{500}} = \frac{400}{773} = 51.7\%$

If T_H is increased to $700°C$

$\eta_{max} = \frac{700-100}{773} = 77.6\%$

(4) useful info: heat capacity of water
latent heat of vaporization (water)
heat capacity of steam:
*

(a) Heat required to raise water from $60°C \to 100°C$

$Q = mC_{pl}\Delta t = (10 kg)(4186\ J\ kg^{-1}K^{-1})(40K)$

$= \underline{1.674 \times 10^6\ J}$

(b) Heat required to vaporize 10kg of water at 100°C

$Q = m\,l_v = (10\ kg)(2.26 \times 10^6 \frac{J}{kg})$

$= \underline{22.6 \times 10^6\ J}$

(c) Heat required to heat steam from 100°C to 120°C:

$Q = m\,C_{pv}\,\Delta t = (10 kg)(2080\ J kg^{-1}K^{-1})(20K)$

$= \underline{4.16 \times 10^5\ J}$

Total heat required = ~~4.16~~ $\underline{\underline{24.7 MJ}}$

(5) Question 9, chapter 4.

60% capacity factor means it produced ~~3~~600 MW of electricity.

$|Q_H|$ = Heat content of 2×10^6 tons of $\frac{\text{coal}}{\text{year}}$ =

$$2 \times 10^6 \frac{\text{tons}}{\text{year}} \times 22 \times 10^6 \frac{\text{Btu}}{\text{ton}} \times \frac{1055\,\text{J}}{\text{Btu}} = 4.642 \times 10^{16} \frac{\text{J}}{\text{year}}$$

Total $\frac{\text{work done}}{\text{year}}$ was $600\,\text{MW} \times 365 \frac{\text{days}}{\text{year}} \times 24 \frac{\text{h}}{\text{day}} \times 3600 \frac{\text{s}}{\text{h}}$

$$= 1.892 \times 10^{16} \frac{\text{J}}{\text{year}}$$

Efficiency $= \frac{|W|}{|Q_H|} = \frac{1.892}{4.642} = 40.8\%$