Assignment 1 Solutions

Friday January 18, 2008

1-9) In this process, the gravitational potential energy of the water in the catchment is harnessed. We assume that the catchment is emptied directly at the base. When one considers the gravitational potential energy of an object, the height of the object must be used. In this case, the height varies for each water molecule, so one could integrate over all heights. However, because the water is distributed throughout the height of the catchment and we will be considering all the water, we can simplify our consideration of the height by using the height of the center of mass (for a uniform density this is the center of the catchment).

The potential energy is found by:

$$PE = mgh_{cm},$$

where

$$m = \rho V = \rho l^2 h$$

is the total mass of the water and

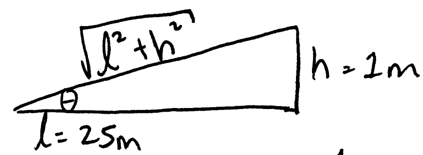
$$h_{cm} = \frac{h}{2}$$

is the height of the centre of mass. The power is the rate of energy consumption, given by:

$$P = \frac{PE}{t} = \frac{(\rho l^2 h) g (\frac{h}{2})}{t} = \frac{(\rho l^2 h^2) g}{2t} = \frac{1000\frac{kg}{m^3} \times (1200m)^2 \times (3.7m)^2 \times 9.8m/s^2}{2 \times 3600s}$$
$$= 26.8 \times 10^6 J/s = 26.8MW$$

1-10) a) The energy loss is due to heat, which can be accounted for by friction within the engine.

1-10) b)



$$\sin\theta = \frac{h}{\sqrt{l^2+h^2}} = \frac{1m}{\sqrt{(1m)^2+(25m)^2}} = \frac{1}{\sqrt{626}}$$

Convert

$$V = 90\,\frac{km}{h} \times \frac{1000\,m}{1km} \times \frac{1\,h}{3600\,sec} = 25\,m/s$$

We know that the power needed is due to the change in potential energy:

$$P = \frac{\Delta PE}{t}$$

We want to know how much the potential energy changes in 1 sec:

$$\Delta PE = mg\Delta h$$

Where

$$\Delta h = V_y t = V\sin\theta\, t = \frac{V \cdot t}{\sqrt{626}}$$

Then,

$$P = \frac{\Delta PE}{t} = \frac{mg\Delta h}{t} = V\sin\theta\, mg = \frac{(25m/s)(9.8m/s^2)(1300\,kg)}{\sqrt{626}}$$

$$P = 12.7 \times 10^3\ kg\,m^2/s^3$$

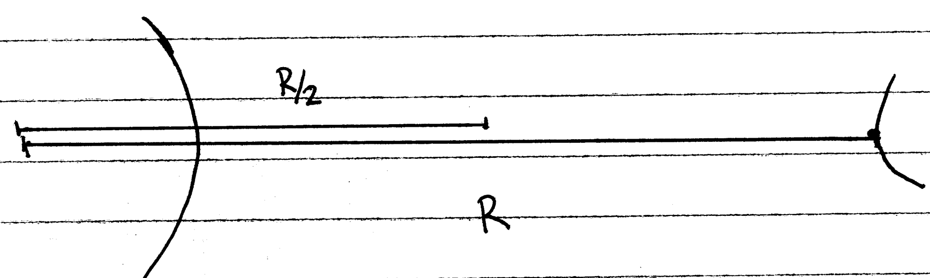
$$\boxed{P = 13 \times 10^3\ W}$$

1-11) We know that

$$I(R) \propto \frac{1}{R^2}$$

And because we can assume that all other variables remain constant in this problem, it may be easier to absorb all other constants into some factor C:

$$I(R) = \frac{C}{R^2}$$



The 2nd position of the satalite is a distance $R/2$ from the sun, where R is the first position. Then we know:

$$I(R/2) = \frac{C}{(R/2)^2} = \frac{4C}{R^2} = 4I(R)$$

So we can calculate the solar energy recieved at distance $R/2$, it is a factor of 4 more intense.

$$E_T = I(R/2) \cdot V \cdot t$$

$$= 4 \cdot (1.35\ kW/m^2)(125\ m^2) \cancel{\times 24 =}\ 675\ kW \times 24h = 1.62 \times 10^4\ kW \cdot h$$

Convert to Joules:

$$1.62 \times 10^4\ kW \cdot h \times \frac{10^3\ W}{kW} \times \frac{3600\ sec}{hr} = 5.83 \times 10^{10}\ J$$

2-6) a) 70×10^6 tonnes of oil equivalent

b) $70 \times 10^6 \text{ tonnes} \times 4 \times 10^7 \frac{\text{Btu}}{\text{tonne}} \times 1055 \frac{\text{J}}{\text{Btu}} = 3.0 \times 10^{18} \text{ J} = 3.0 \text{ EJ}$

c) Just as in example 2-4, the growth constant is the slope of the line. We must take the ln of the y-axis, as it is a semi-logarithmic graph:

$$k = \frac{\ln(80/45)}{2005-2000} = 0.12 \text{ yr}^{-1}$$

d) To calculate the change in production due to exponential growth we use

$$N = N_0 e^{kT}$$

$$\frac{N}{N_0} = e^{kT}$$

$$\ln\left(\frac{N}{N_0}\right) = kT$$

$$T = \frac{1}{k} \ln\left(\frac{N}{N_0}\right)$$

Choose a data point: year 2000 at 45 million tonnes of oil equivalent

$$T = \frac{1}{0.12 \text{ yr}^{-1}} \ln\left(\frac{400 \times 10^{16} \text{ tonne}}{45 \times 10^{16} \text{ tonne}}\right)$$

$$T = 18.2 \text{ yr}$$

So, it would reach this rate at the year 2018.

2-7) The tables in the book state that world oil production is currently at 164 EJ/yr and resources are 12000EJ.

The rate 1.7% gives a growth constant of

$$k = \ln\left(1 + R/100\right) = \ln(1.017) = 0.0169\ yr^{-1}$$

Because we want to know how long we have until the total amount of oil is used, we must ~~...~~ use the integrated equation:

$$T = \frac{1}{k}\ln\left(\frac{kQ_T}{N_0} + 1\right)$$

$$= \frac{1}{0.0169\ yr^{-1}} \ln\left(\frac{(0.0169\ yr^{-1})(12000EJ)}{164\ EJ/yr} + 1\right)$$

$$T = 47.6\ yr$$

2-8) a)

$$N = N_0 e^{kT}$$

$$\ln\left(\frac{N}{N_0}\right) = kT$$

$$k = \frac{1}{T}\ln\left(\frac{N}{N_0}\right)$$

$$= \frac{1}{6\,yr}\ln\left(\frac{681\times10^6\,tonnes}{542\times10^6\,tonnes}\right)$$

$$= 0.038\,yr^{-1}$$

$$k = \ln(1 + R/100)$$

$$\frac{R}{100} = e^k - 1$$

$$\frac{R}{100} = 0.039$$

which is a 3.9% growth rate.

b) $T = \frac{1}{k}\ln(2)$

$$T = \frac{1}{0.038\,yr^{-1}}\ln(2)$$

$$T = 18\,yr$$

c) i) A constant rate is $681\times10^6\,\frac{tonnes}{yr}\times 4\times10^7\,\frac{Btu}{tonne} = 27.2\,quads/yr$

$$T = \frac{(18000\,quads)}{(27.2\,quads/yr)} = 661.2\,yr$$

ii) $T = \frac{1}{k}\ln\left(\frac{kQ_\infty}{N_0}+1\right) = \frac{1}{0.038\,yr^{-1}}\ln\left(\frac{(0.038\,yr^{-1})(18000\,quads)}{(27.2\,quads/yr)}+1\right) = 86\,yr$

2-9. Resource production increasing exponentially at 12%/yr.

growth constant $k = \ln(1+0.12) = 0.113$.

At this rate, expect depletion in 47 yr = T_0.

$$T_0 = \frac{1}{k} \ln\left(\frac{k\,Q_T^0}{N_0} + 1\right) \quad (1)$$

If $Q_T = 3Q_T^0$, resource will last longer

$$T_1 = \frac{1}{k} \ln\left(\frac{3kQ_T^0}{N_0} + 1\right) \quad (2)$$

T_0, k known

T_1, Q_T^0 N_0 unknown but only the ratio Q_T^0/N_0 is needed.

Use (1) :
$$e^{T_0 k} - 1 = \frac{kQ_T^0}{N_0}$$

$$\therefore \frac{kQ_T^0}{N_0} = \exp\left[47 \cdot 0.113\right] = 202.6$$

$$\text{so } T_1 = \frac{1}{k} \ln\left\{3\frac{kQ_T^0}{N_0} + 1\right\} = \frac{1}{0.113} \cdot \ln\left\{607.7+1\right\}$$

$= 56.7$ yr - only 10 yr longer!