Electricity & Magnetism
Lecture 9: Electric Current

Today's Concept:

Electric Current

How do you feel about circuits

A) I completely understand them from high school

B) Need Review

C) Still hopeless

D) Circuit? What's that?

Your stuff

- so which way does DC current flow? -_-
- "How many different things will sigma symbolize???"
- "Since R = ρL/A, the greater the cross sectional area, the smaller the resistance, but the greater the length the higher the resistance. Is that why long cables have to be very thick?"
- "What if I put ammeter right between + and -?"
- "the part relating to the ohm's law and current density stuff makes no sense to me."

A Big Idea Review

Coulomb's Law
Force law between point charges

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

Electric Field
Force per unit charge

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Field
Property of Space
Created by Charges
Superposition

Gauss' Law
Flux through closed surface is always proportional to charge enclosed

$$\oint_{\text{surface}} \vec{E}\cdot\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Gauss' Law
Can be used to determine E field → Spheres, Cylinders, Infinite Planes

Electric Potential
Potential energy per unit charge

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Capacitance
Relates charge and potential for two conductor system

$$C \equiv \frac{Q}{V}$$

Electric Potential
Scalar Function that can be used to determine E

$$\vec{E} = -\vec{\nabla}V$$

A Note on Units

- Force is newtons: N = kg•m/s^2
- Electric Field: newton/coulomb (N/C = V/m)
- Electric potential: newton-meter/coulomb = volt
  - kg•m^2/s^{2}C = V
- Capacitance: farad = coulomb/volt
  - farad is big, we usually use
    - μF = 10^{-6} F
    - pF = 10^{-12} F (μμF in olden days, "puffs" now)
    - (nF = 10^{-9} not customary in N. America)

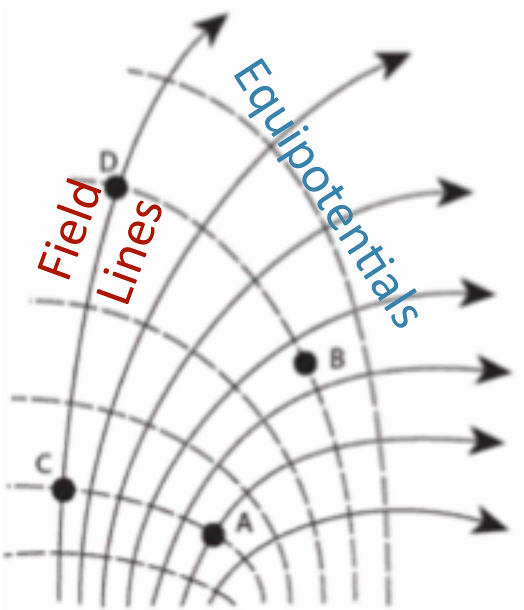
Applications of Big Ideas

Conductors
Charges free to move → What Determines How They Move? → They move until $E = 0$!

$E = 0$ in conductor determines charge densities on surfaces

Gauss' Law → Spheres, Cylinders, Infinite Planes

Field Lines & Equipotentials



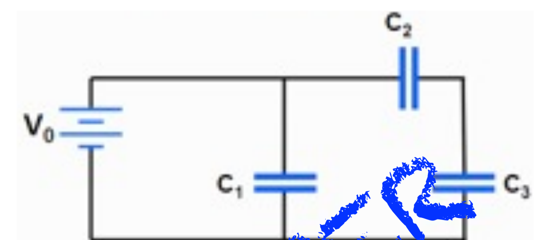
Work Done By E Field

$$W_{a\rightarrow b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q\vec{E}\cdot d\vec{l}$$

Change in Potential Energy

$$\Delta V_{a\rightarrow b} \equiv \frac{\Delta U_{a\rightarrow b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Capacitor Networks



LATER

Series:

$(1/C_{23}) = (1/C_2) + (1/C_3)$

Parallel

$C_{123} = C_1 + C_{23}$

Key Concepts:

1) How resistance depends on A, L, σ, ρ

σ is **conductivity** here (not surface charge density)
ρ is **resistivity** here (not volume charge density).

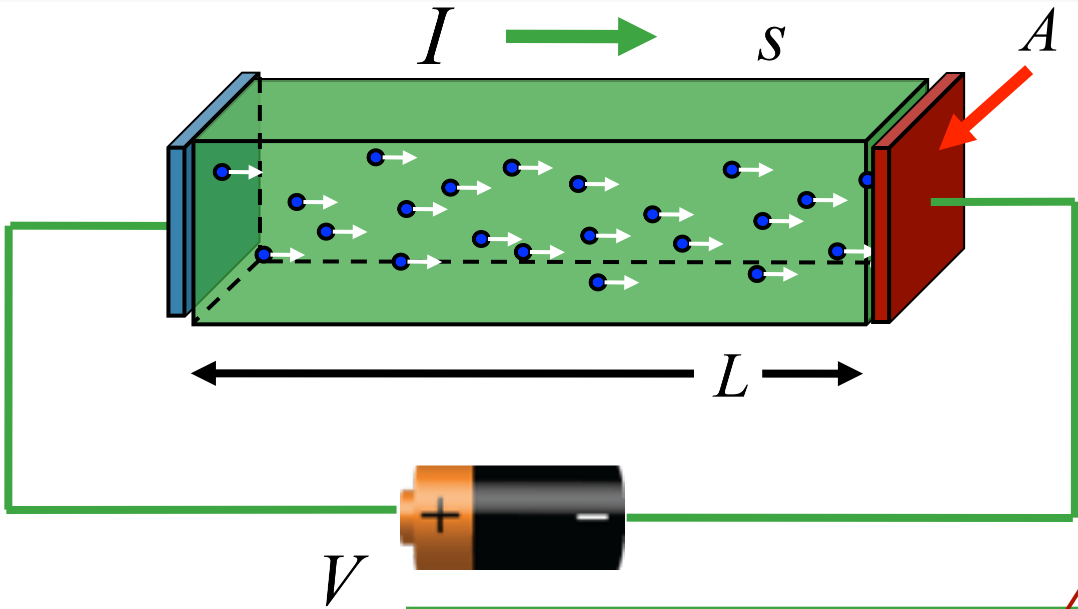
2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

Today's Plan:

1) Review of resistance & preflights

2) Work out a circuit problem in detail



Conductivity – high for good conductors.

Ohm's Law: $J = \sigma E$

Observables:

$$V = EL$$
$$I = JA$$

$$\rightarrow \quad I/A = \sigma V/L \quad \rightarrow \quad I = V/(L/\sigma A)$$

R = Resistance

$$\rho = 1/\sigma$$

$$I = V/R \quad \leftarrow \quad R = \frac{L}{\sigma A}$$

This is just like Plumbing!

I is like flow rate of water

V is like pressure

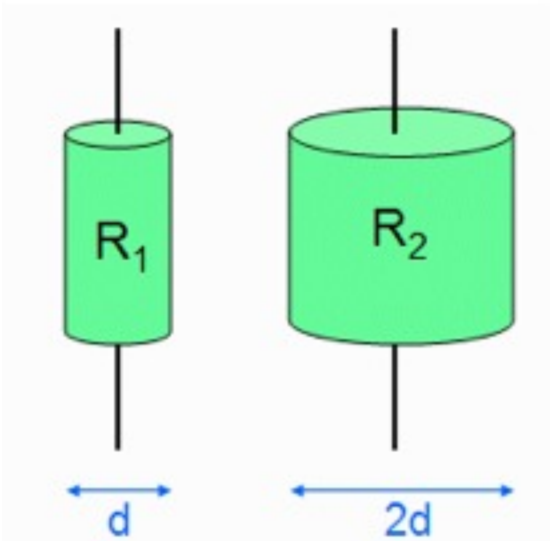
R is how hard it is for water to flow in a pipe

$$R = \frac{L}{\sigma A}$$

To make *R* big, make *L* long or *A* small

To make *R* small, make *L* short or *A* big

1 CheckPoint: Two Resistors 2



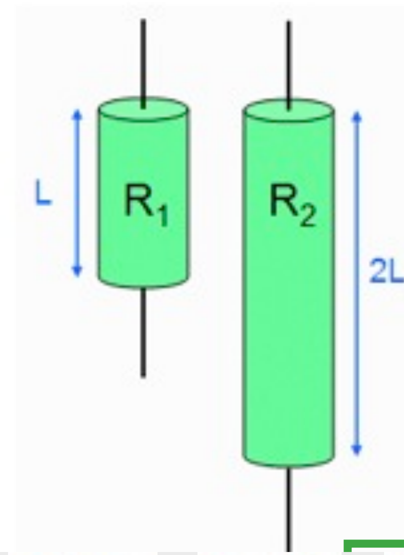
A) $V_1 > V_2$ B) $V_1 = V_2$ C) $V_1 < V_2$

Same current through both resistors

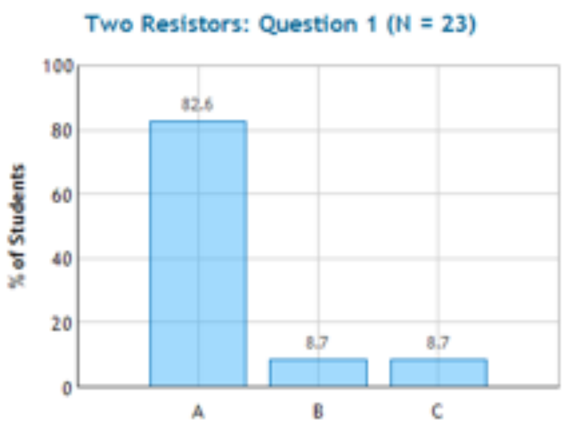
Compare voltages across resistors

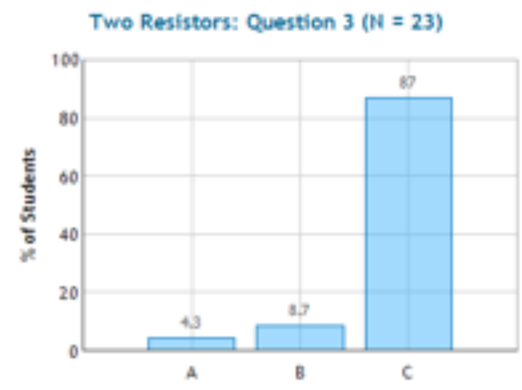
$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$



A) $V_1 > V_2$ B) $V_1 = V_2$ C) $V_1 < V_2$



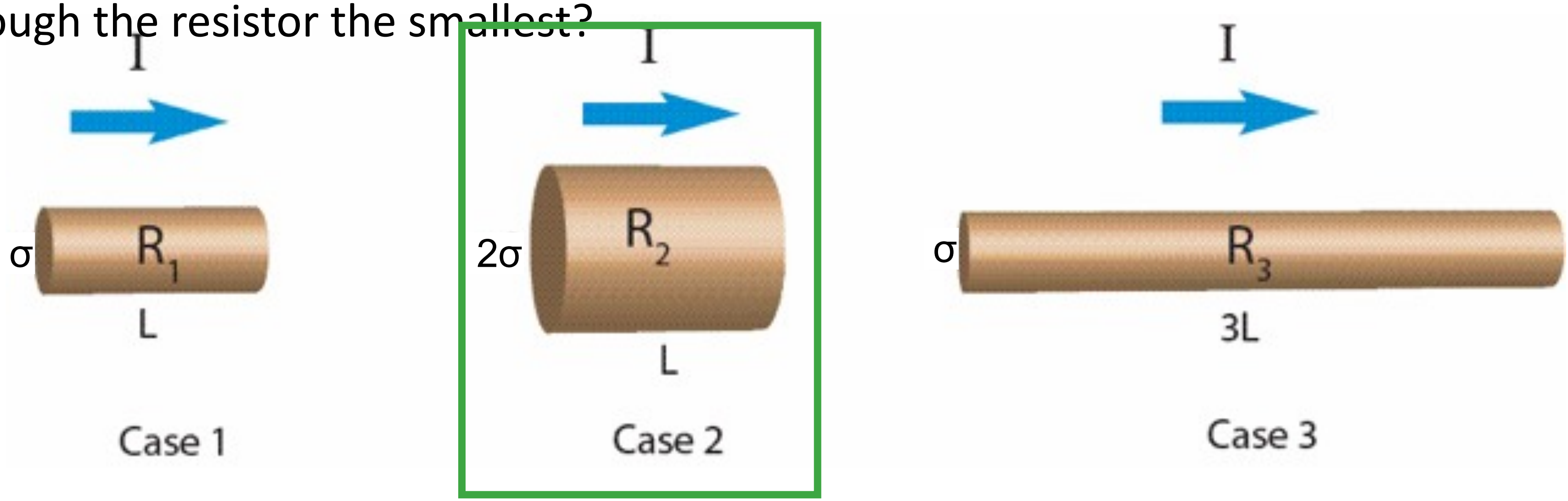


$$A_2 = 4A_1 \Rightarrow V_2 = \tfrac{1}{4}V_1$$

$$L_2 = 2L_1 \Rightarrow V_2 = 2V_1$$

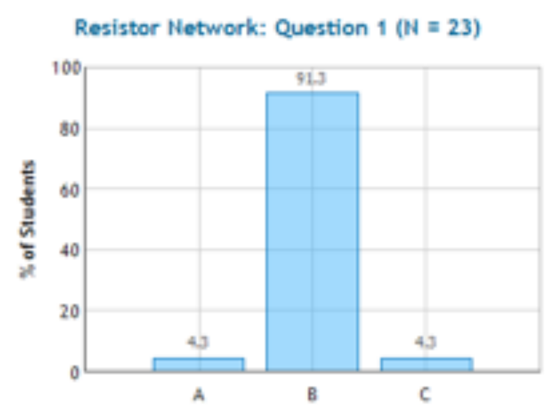
CheckPoint: Current Density

The SAME amount of current I passes through three different resistors. R_2 has twice the cross-sectional area and the same length as R_1, and R_3 is three times as long as R_1 but has the same cross-sectional area as R_1. In which case is the CURRENT DENSITY through the resistor the smallest?

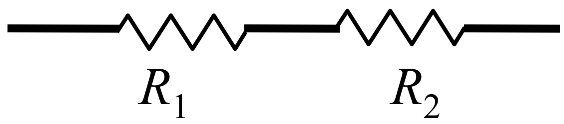
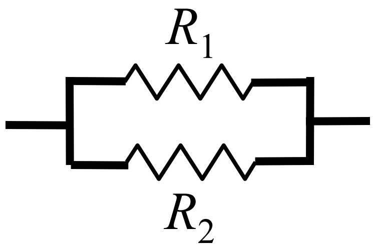


$$J \equiv \frac{I}{A} \rightarrow J_1 = J_3 = 2J_2$$

Same Current $\rightarrow$ $J \propto \frac{1}{A}$



Resistor Summary

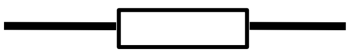
	Series	Parallel
		
Wiring	Each resistor on the <u>same</u> wire.	Each resistor on a <u>different</u> wire.
Voltage	<u>Different</u> for each resistor. $V_{total} = V_1 + V_2$ Voltage Divider	<u>Same</u> for each resistor. $V_{total} = V_1 = V_2$
Current	<u>Same</u> for each resistor $I_{total} = I_1 = I_2$	<u>Different</u> for each resistor $I_{total} = I_1 + I_2$ Current Divider
Resistance	<u>Increases</u> $R_{eq} = R_1 + R_2$	<u>Decreases</u> $1/R_{eq} = 1/R_1 + 1/R_2$

Symbols

- Resistor symbol (ANSI)
  - N. America, Japan, China(?)

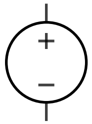
4.7 k = 4700 ohm
1.8 Ω = 1.8 ohm

- Alternate resistor symbol (DIN)
  - Europe, Middle East, Aus/NZ, Africa(?)



4k7 =4700 ohm
1R8= 1.8 ohm

- Voltage Source

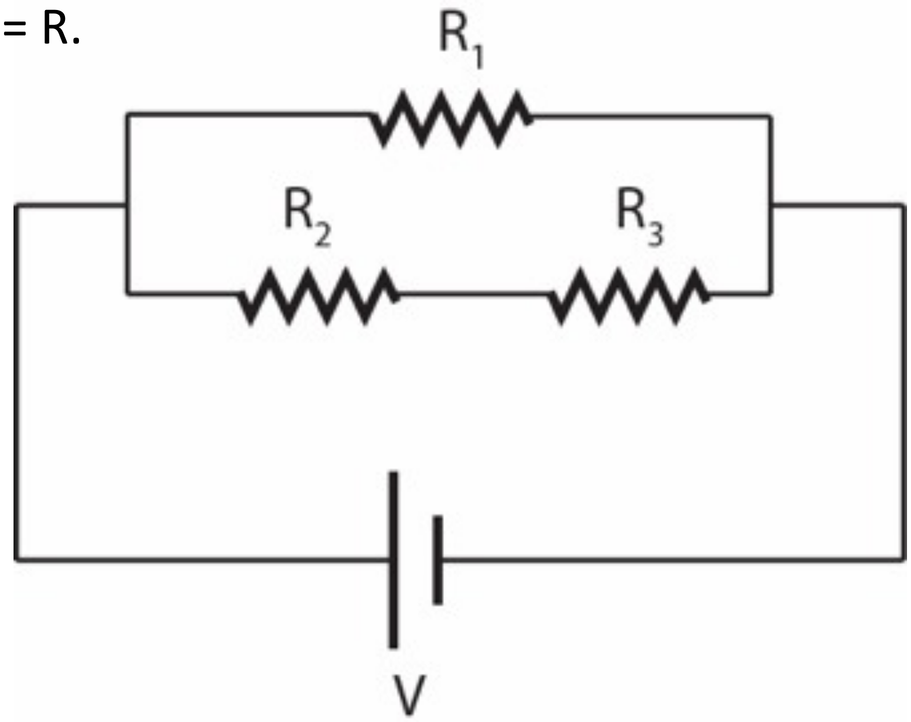


- Electrochemical Cell (“battery”)

sometimes used for voltage source

CheckPoint: Resistor Network 1

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.

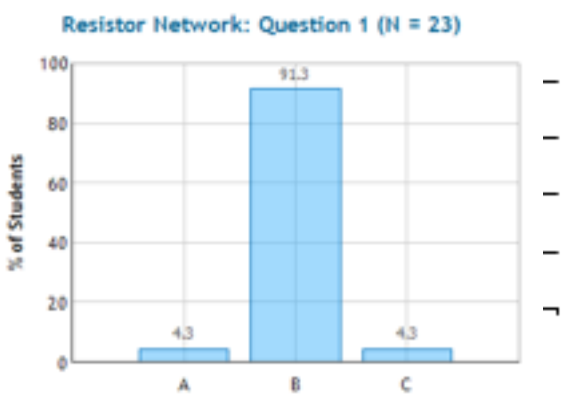


Compare the current through R_2 with the current through R_3:

A. $I_2 > I_3$

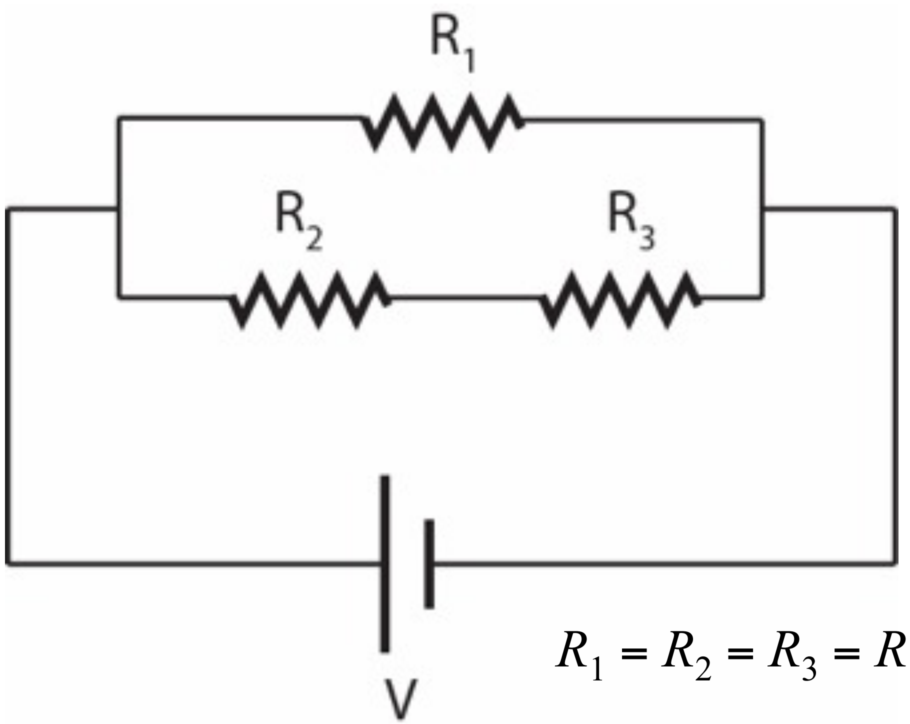
B. $I_2 = I_3$

C. $I_2 < I_3$



R_2 in series with R_3 → Current through R_2 and R_3 is the same

$$I_{23} = \frac{V}{R_2 + R_3}$$



$R_1 = R_2 = R_3 = R$

CheckPoint 2

Compare the current through R_1 with the current through R_2

$I_1 \longleftrightarrow I_2$

CheckPoint 3

Compare the voltage across R_2 with the voltage across R_3

$V_2 \longleftrightarrow V_3$

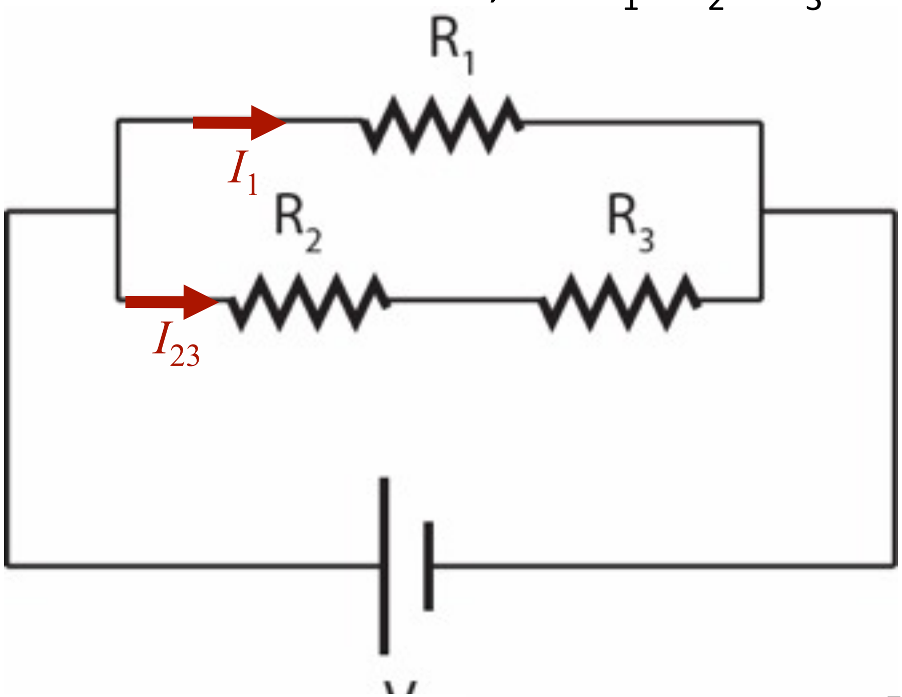
CheckPoint 4

Compare the voltage across R_1 with the voltage across R_2

$V_1 \longleftrightarrow V_2$

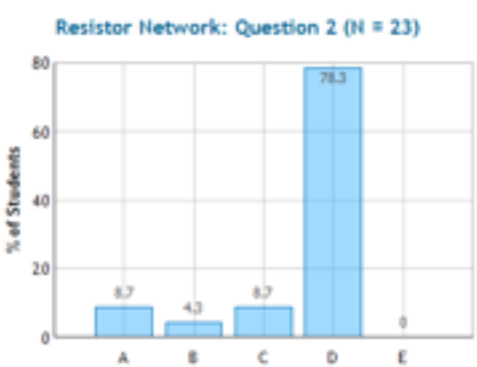
CheckPoint: Resistor Network 2

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the current through R_1 with the current through R_2:

A. $I_1/I_{23}=1/2$
B. $I_1/I_{23}=1/3$
C. $I_1 = I_{23}$
D. $I_1/I_{23}=2$
E. $I_1/I_{23}=3$



We know: $$I_{23} = \frac{V}{R_2 + R_3}$$

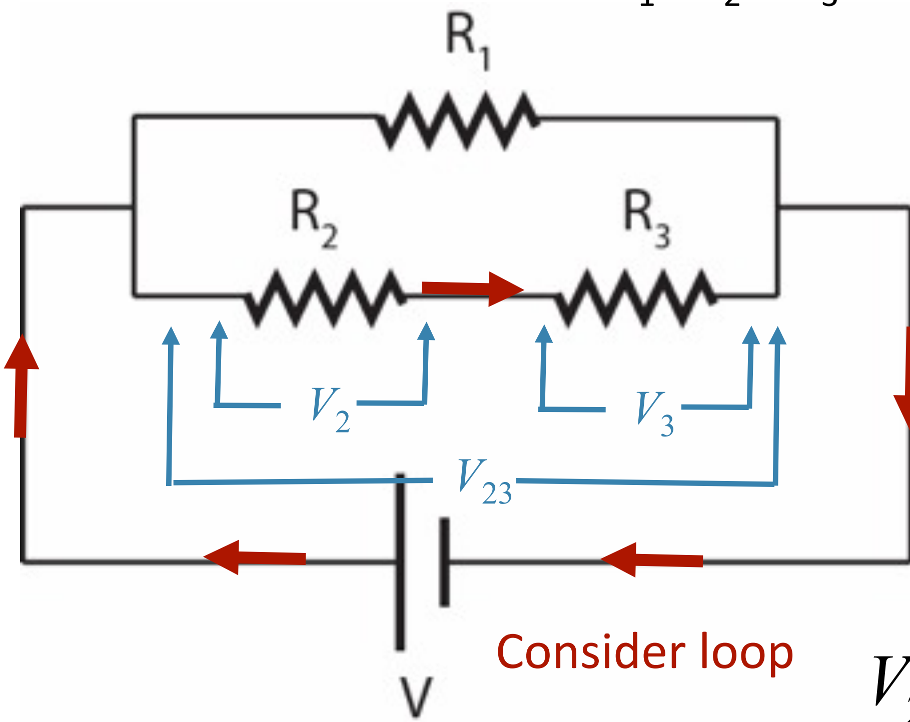
Similarly: $$I_1 = \frac{V}{R_1}$$

$$I_1 = I_{23}\frac{R_2 + R_3}{R_1}$$

$$\frac{I_1}{I_{23}} = \frac{R_2 + R_3}{R_1} = 2$$

CheckPoint: Resistor Network 3

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the voltage across R_2 with the voltage across R_3:

A. $V_2 > V_3$

B. $V_2 = V_3 = V$

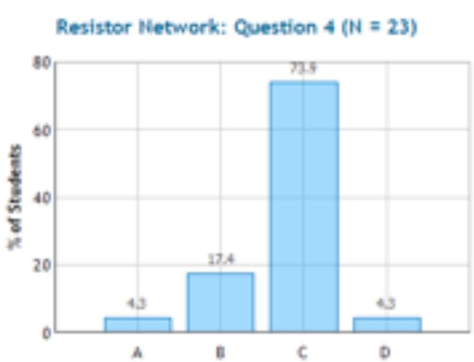
C. $V_2 = V_3 < V$

D. $V_2 < V_3$

$$V_{23} = V$$

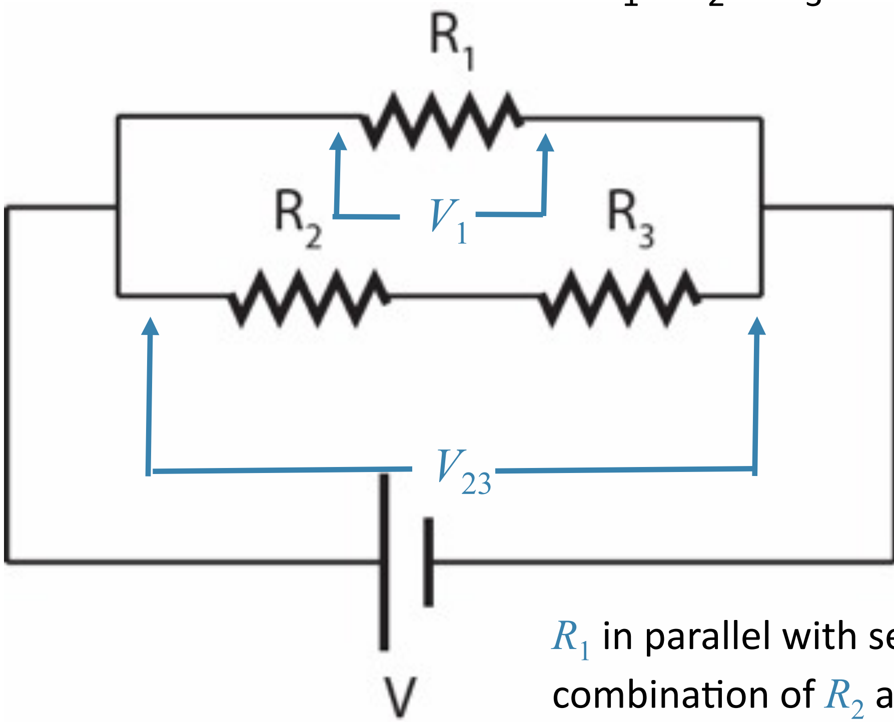
$$V_{23} = V_2 + V_3 \quad \longrightarrow \quad V_2 = V_3 = \frac{V}{2}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3$$



CheckPoint: Resistor Network 4

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the voltage across R_1 with the voltage across R_2.

A. $V_1 = V_2 = V$

B. $V_1 = 1/2\ V_2 = V$

C. $V_1 = 2V_2 = V$

D. $V_1 = 1/2\ V_2 = 1/5\ V$

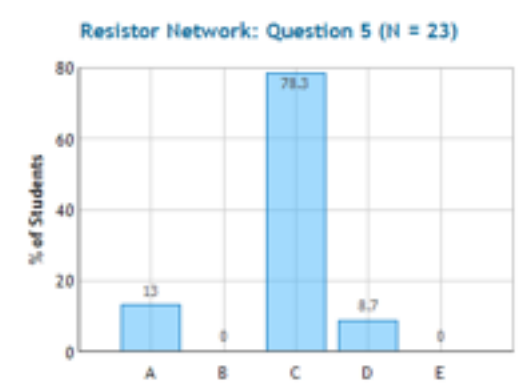
E. $V_1 = 1/2\ V_2 = 1/2\ V$

R_1 in parallel with series combination of R_2 and R_3

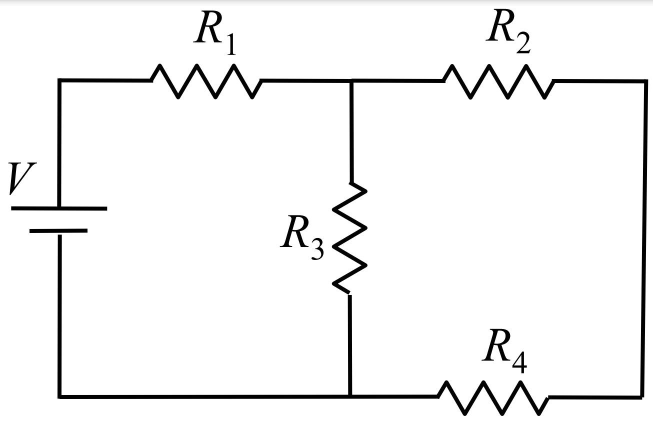
$$V_1 = V_{23}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3 \longrightarrow V_1 = 2V_2 = V$$

$$V_{23} = V_2 + V_3 = 2V_2$$



Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Conceptual Analysis:

Ohm's Law: when current *I* flows through resistance R, the potential drop V is given by:
$V = IR$.

Resistances are combined in series and parallel combinations

$$R_{series} = R_a + R_b$$
$$(1/R_{parallel}) = (1/R_a) + (1/R_b)$$

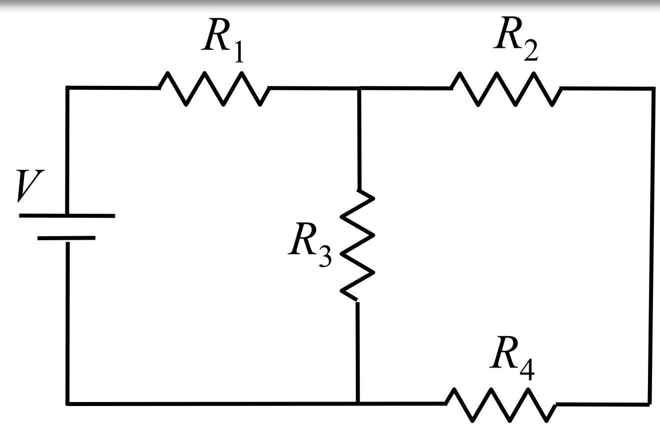
Strategic Analysis:

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm's Law

Expand circuit back using knowledge of voltages and currents

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

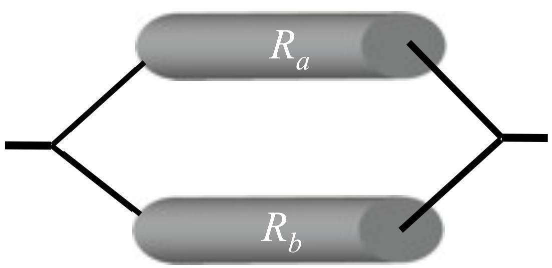
R_1 and R_2 are connected:

A) in series

B) in parallel

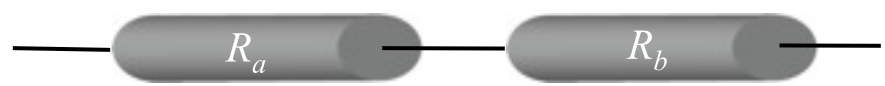
C) neither in series nor in parallel

Parallel Combination



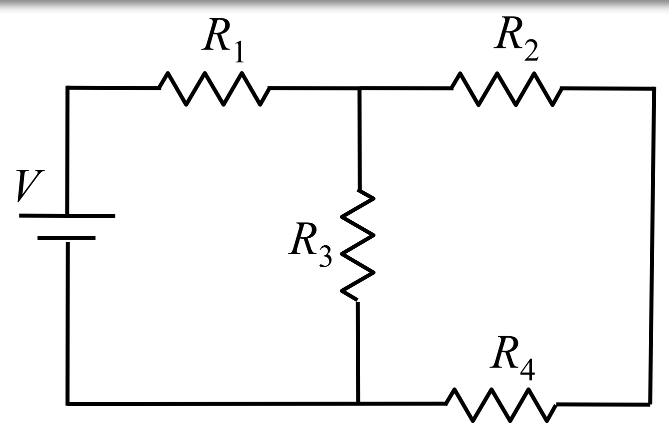
Parallel: Can make a loop that contains only those two resistors

Series Combination



Series : Every loop with resistor 1 also has resistor 2.

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

We first will combine resistances $R_2 + R_3 + R_4$:

Which of the following is true?

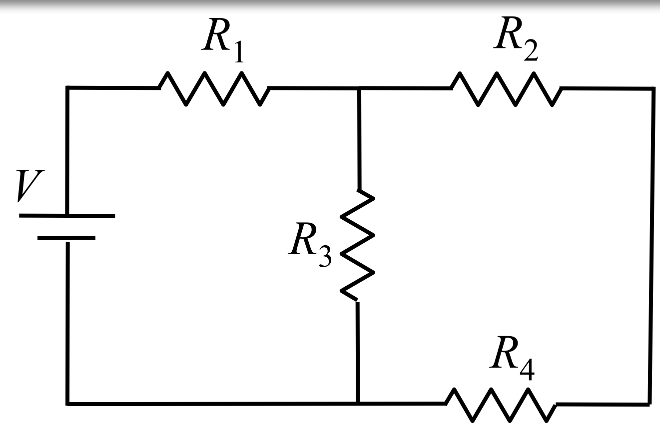
A) R_2, R_3 and R_4 are connected in series

B) R_2, R_3, and R_4 are connected in parallel

C) R_3 and R_4 are connected in series (R_{34}) which is connected in parallel with R_2

D) R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

E) R_2 and R_4 are connected in parallel (R_{24}) which is connected in parallel with R_3



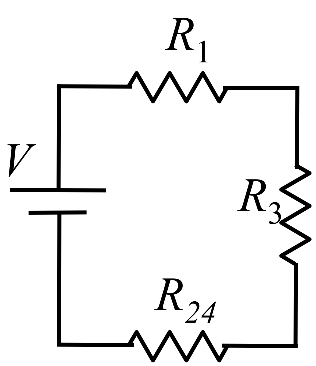
In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

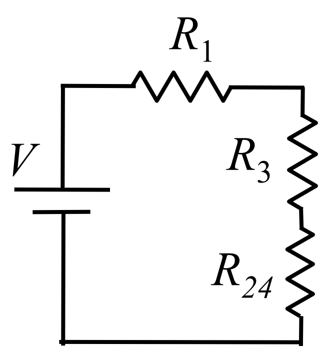
What is V_2, the voltage across R_2?

R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

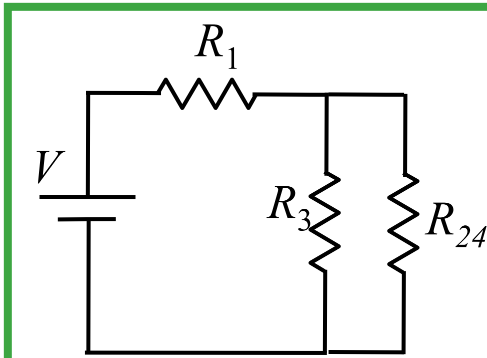
Redraw the circuit using the equivalent resistor R_{24} = series combination of R_2 and R_4.



(A)

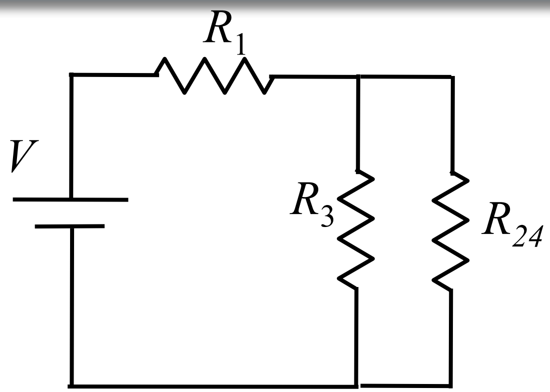


(B)



(C)

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

R_2 and R_4 are connected in series = R_{24}

R_3 and R_{24} are connected in parallel = R_{234}

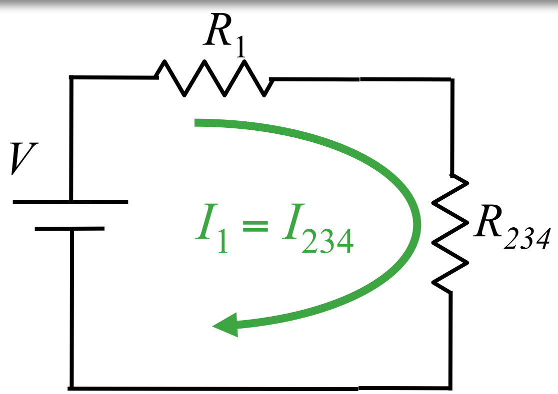
What is the value of R_{234}?

A) $R_{234} = 1\ \Omega$ **B) $R_{234} = 2\ \Omega$** C) $R_{234} = 4\ \Omega$ D) $R_{234} = 6\ \Omega$

R_2 and R_4 in series → $R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$ → $1/R_{234} = (1/3) + (1/6) = (3/6)\ \Omega^{-1}$ → $R_{234} = 2\ \Omega$

Calculation



In the circuit shown: $V = 18V$,

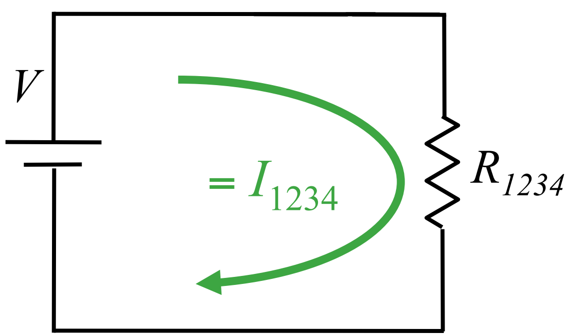
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$

What is V_2, the voltage across R_2?

R_1 and R_{234} are in series. $R_{1234} = 1 + 2 = 3\ \Omega$

Our next task is to calculate the total current in the circuit

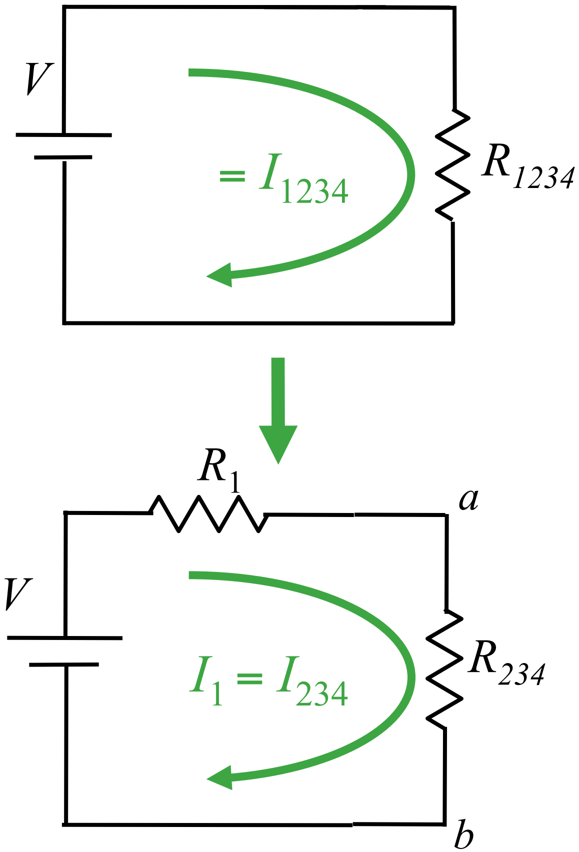


Ohm's Law tells us: $I_{1234} = V/R_{1234}$

$= 18 / 3$

$= 6$ Amps

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$ $I_{1234} = 6\ A$

What is V_2, the voltage across R_2?

$I_{234} = I_{1234}$ Since R_1 in series with R_{234}

$V_{234} = I_{234}\ R_{234}$

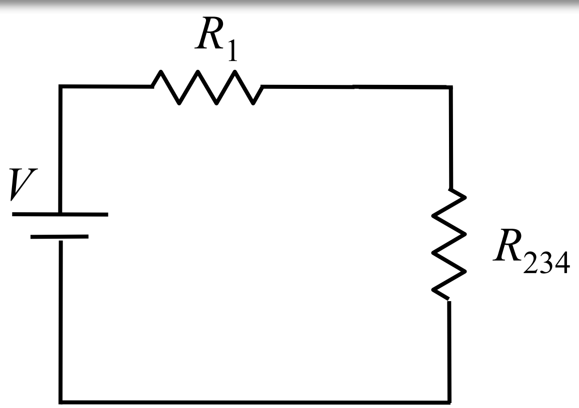
$= 6 \times 2$

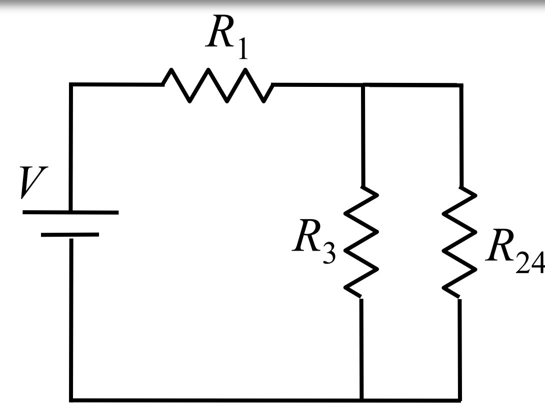
$= 12$ Volts

What is V_{ab}, the voltage across R_{234} ?

A) $V_{ab} = 1\ V$ B) $V_{ab} = 2\ V$ C) $V_{ab} = 9\ V$ D) $V_{ab} = 12\ V$ E) $V_{ab} = 16\ V$

Calculation





$V = 18V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 3\Omega$

$R_4 = 4\Omega$

$R_{24} = 6\Omega$

$R_{234} = 2\Omega$

$I_{1234} = 6$ Amps

$I_{234} = 6$ Amps

$V_{234} = 12V$

What is V_2?

Which of the following are true?

A) $V_{234} = V_{24}$ B) $I_{234} = I_{24}$ C) Both A+B D) None

R_3 and R_{24} were combined in parallel to get R_{234} → Voltages are same!

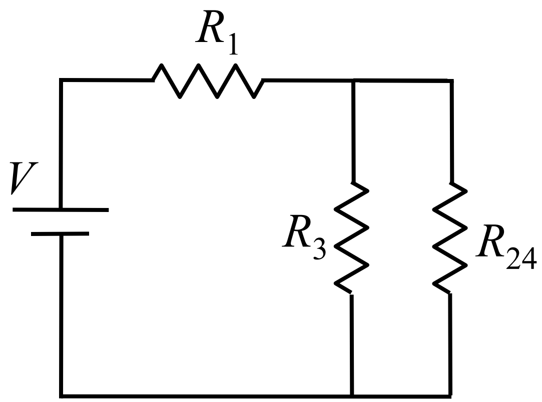
Ohm's Law

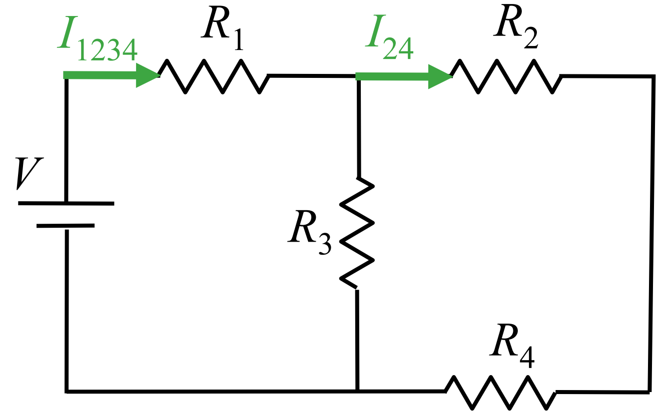
$$I_{24} = V_{24} / R_{24}$$

$$= 12 / 6$$

$$= 2 \text{ Amps}$$

Calculation





$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$.
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6$ Amps
$I_{234} = 6$ Amps
$V_{234} = 12V$
$V_{24} = 12V$
$I_{24} = 2$ Amps

What is V_2?

Which of the following are true?

A) $V_{24} = V_2$ B) $I_{24} = I_2$ C) Both A+B   D) None

R_2 and R_4 where combined in series to get R_{24} → Currents are same!

The Problem Can Now Be Solved!

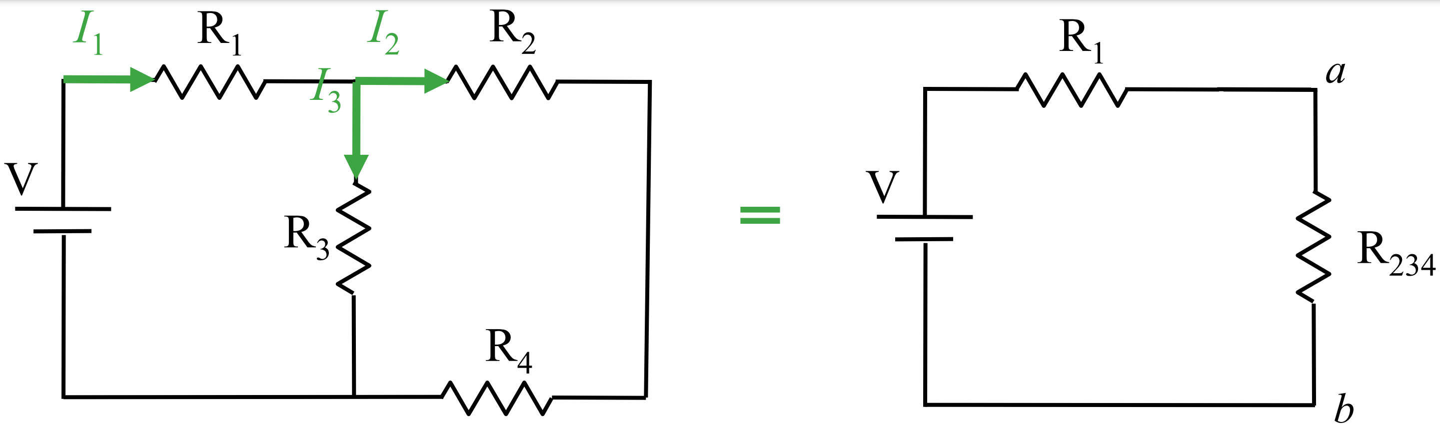
Ohm's Law

$V_2 = I_2 \; R_2$

$= \; 2 \times 2$

$= 4$ Volts!

Quick Follow-Ups



$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$V_{234} = 12V$
$V_2 = 4V$
$I_{1234} = 6$ Amps

What is I_3 ?

A) $I_3 = 2\ A$ B) $I_3 = 3\ A$ **C) $I_3 = 4\ A$**

$V_3 = V_{234} = 12V$ → $I_3 = V_3/R_3 = 12V/3\Omega = 4A$

What is I_1 ?

We know $I_1 = I_{1234} = 6\ A$

NOTE: $I_2 = V_2/R_2 = 4/2 = 2\ A$ → $I_1 = I_2 + I_3$ Make Sense?