Electricity & Magnetism
Lecture 4: Gauss' Law

Today's Concepts:

A) Conductors

B) Using Gauss' Law

Stuff you asked about:

- "My unrequited love for physics has finally taken dominion over the entirety of the monstrous depths of my soul. Weep, oh weep, for the innocence of the old days hath been lost."
- "i don't understand how to pick a gaussian surface or even when to pick it really :("
- if the conductor allows charges to move freely then why $E_{inside} = 0$?
- ¨Will we have do any integrals?"

  Yes, sorry about that.
- "Easy Stuff"
- "I have no idea what this chapter is freaking talking about. Just like reading Chinese ???"

  sorry too
- Can we recap ???

ok. here we go


Alpha

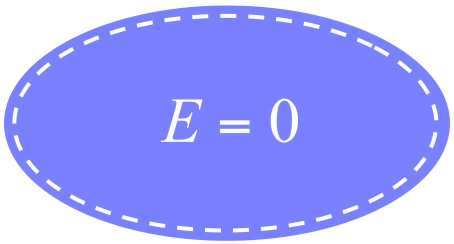
Conductors = Charges Free to Move

Claim: $E = 0$ inside any conductor at equilibrium

Charges in conductor move to make E field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge feels force and moves!

Claim: Excess charge on conductor only on surface at equilibrium



Why?

- Apply Gauss' Law
  - Take Gaussian surface to be just inside conductor surface
  - $E = 0$ everywhere inside conductor $\rightarrow \oint_{\text{surface}} \vec{E} \cdot \vec{A} = 0$
  - Gauss' Law: $\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \rightarrow \boxed{Q_{enc} = 0}$

Gauss' Law + Conductors + Induced Charges

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

If choose a Gaussian surface that is entirely in metal, then $E = 0$ so $Q_{enclosed}$ must also be zero!

$$E = \frac{Q_{enc}}{A\varepsilon_o}$$

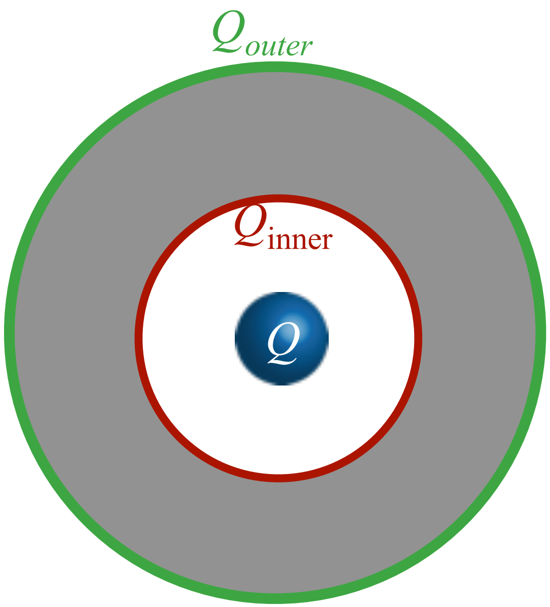
How Does This Work?

Charges in conductor move to surfaces to make $Q_{enclosed} = 0$.

We say charge is induced on the surfaces of conductors

Clicker Question: Charge in Cavity of Conductor

A particle with charge $+Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?



A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$, outer = $+Q/2$

C) inner = 0, outer = 0

D) inner = $+Q/2$, outer = $-Q/2$

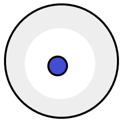
E) inner = $+Q$, outer = $-Q$

Since $E = 0$ in conductor

- Gauss' Law: $\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \longrightarrow Q_{enc} = 0$

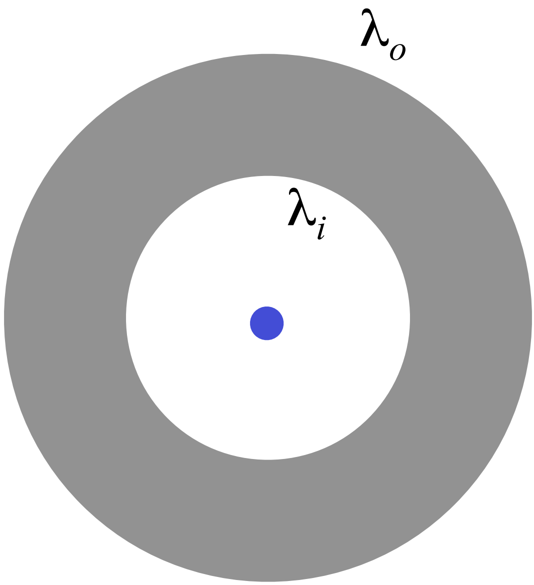
Clicker Question: Infinite Cylinders

A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of –4 C/m.



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder (λ_i) and on the outer surface (λ_o)?

λ_i:	+2.5 C/m	–4 C/m	–2.5 C/m	–2.5 C/m	0
λ_o:	–6.5 C/m	0	+2.5 C/m	–1.5 C/m	–4 C/m
	A	B	C	D	E



Gauss' Law

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $$E = \frac{Q_{enc}}{A\varepsilon_0}$$

In General, integral to calculate flux is difficult…. and not useful!

To use Gauss' Law to calculate E, need to choose surface carefully!

1) Want E to be constant and equal to value at location of interest

OR

2) Want E dot $A = 0$ so doesn't add to integral

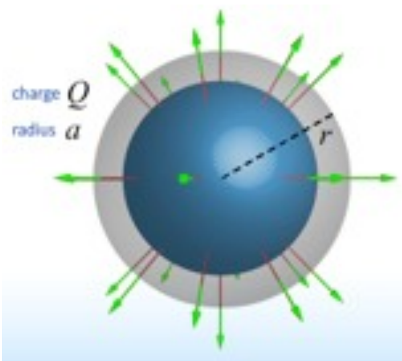
Gauss' Law Symmetries

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \dfrac{Q_{enc}}{A\varepsilon_0}$

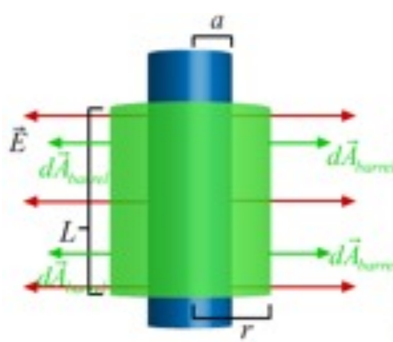
Spherical



$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0}$$

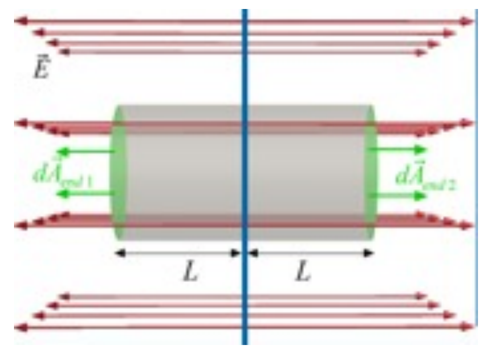
Cylindrical



$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

Planar

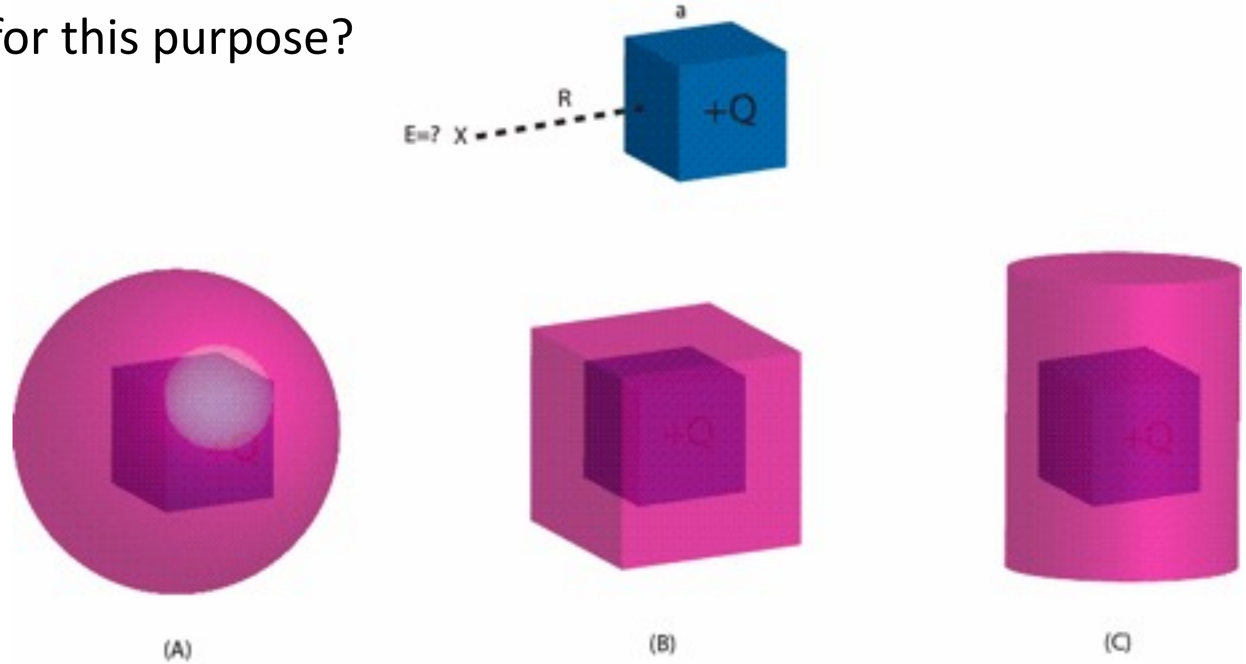


$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

CheckPoint: Gaussian Surface Choice

You are told to use Gauss' Law to calculate the electric field at a distance **R** away from a charged cube of dimension **a**. Which of the following Gaussian surfaces is best suited for this purpose?



A. a sphere of radius R+½a
B. a cube of dimension R+½a
C. a cylinder with cross sectional radius of R+½a
   and arbitrary length
D. This field cannot be calculated using Gauss' law
E. None of the above

CheckPoint Results: Gaussian Surface Choice

You are told to use Gauss' Law to calculate the electric field at a distance **R** away from a charged cube of dimension **a**. Which of the following Gau

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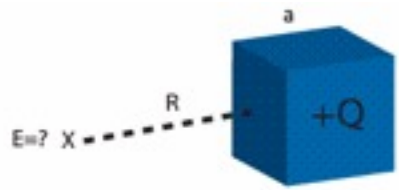
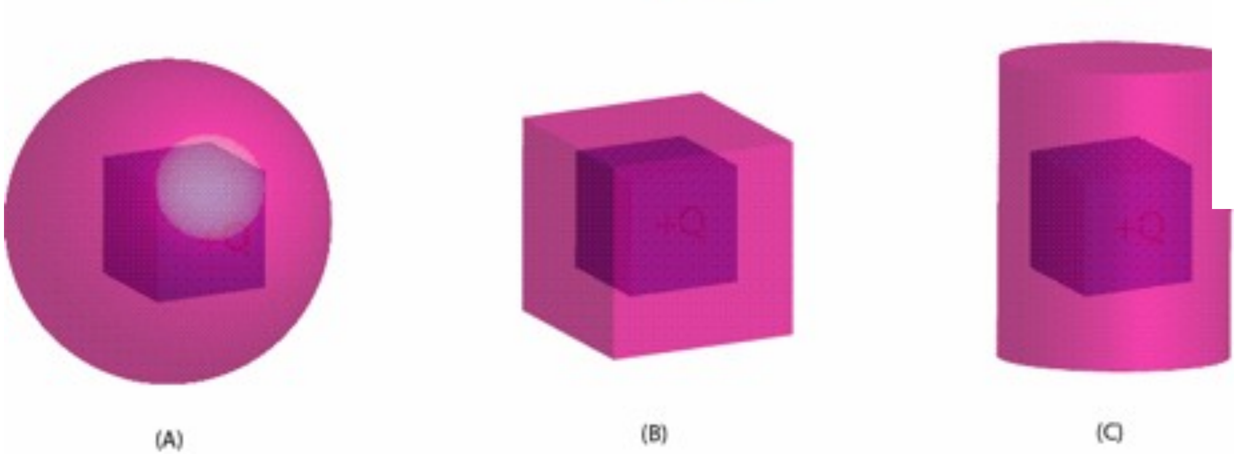
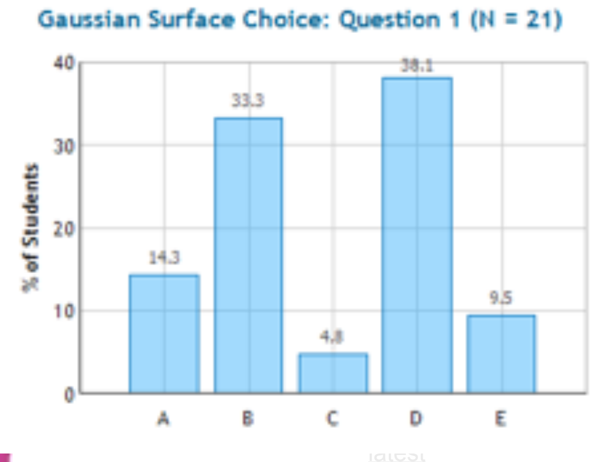






A. a sphere of radius R+½a
B. a cube of dimension R+½a
C. a cylinder with cross sectional radius of R+½a and arbitrary length
D. This field cannot be calculated using Gauss' law
E. None of the above

THE CUBE HAS NO GLOBAL SYMMETRY!

THE FIELD AT THE FACE OF THE CUBE

IS NOT

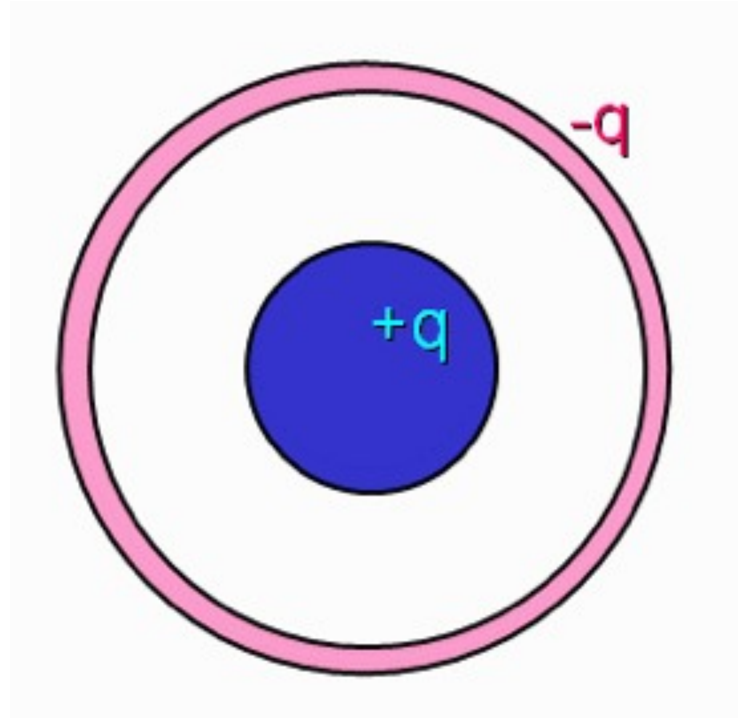
PERPENDICULAR OR PARALLEL

3D	POINT	®	SPHERICAL
2D	LINE	®	CYLINDRICAL
1D	PLANE	®	PLANAR

CheckPoint: Charged Conducting Sphere & Shell 1

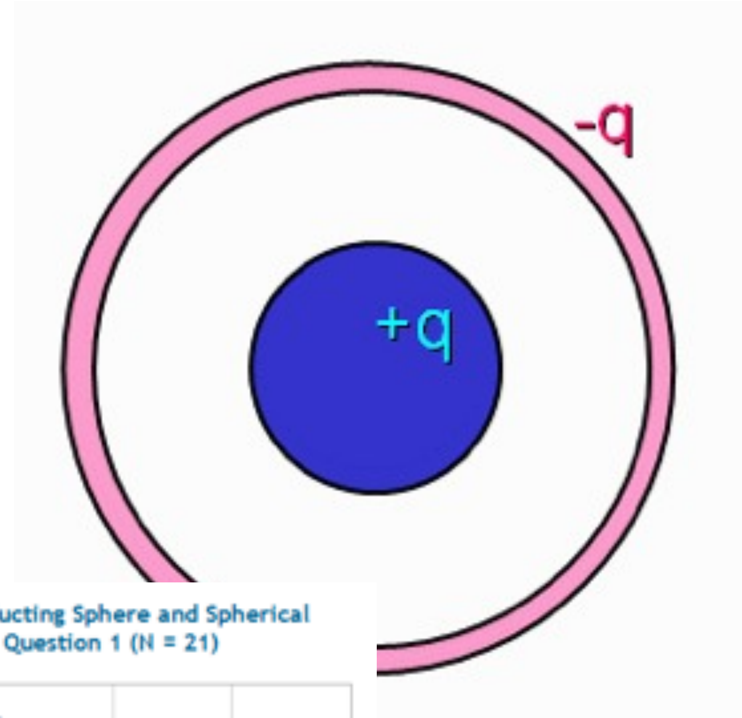
A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same. Which of the following statements best describes the electric field in the region between the spheres?

A. The field points radially outward
B. The field points radially inward
C. The field is zero



CheckPoint: Charged Conducting Sphere & Shell 2

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same. Which of the following statements best describes the electric field in the region outside the red sphere?

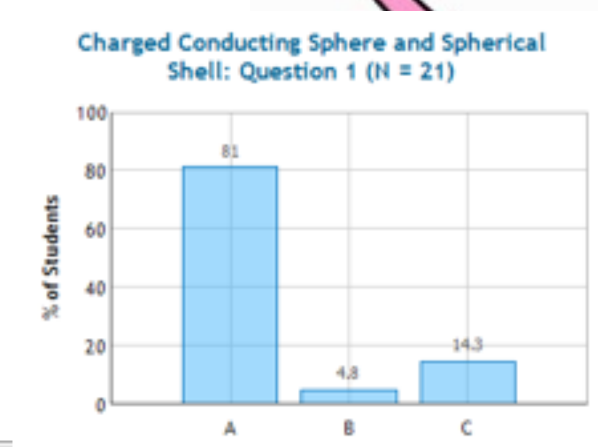


A. The field points radially outward
B. The field points radially inward
$\boxed{\text{C. The field is zero}}$

"Since they have the same charge, the efield from the red sphere is larger than the efield from the blue sphere. So the red field points inwards, the blue sphere points outwards so the resultant is outward"

" closest influence is inwards"

"if +q = –q, the field is zero, because the enclosed charge inside the Gaussian surface is +q + (-q) = 0, since E = total q / A, and q is 0, E is also 0"



CheckPoint: Charged Sphericlal Shell

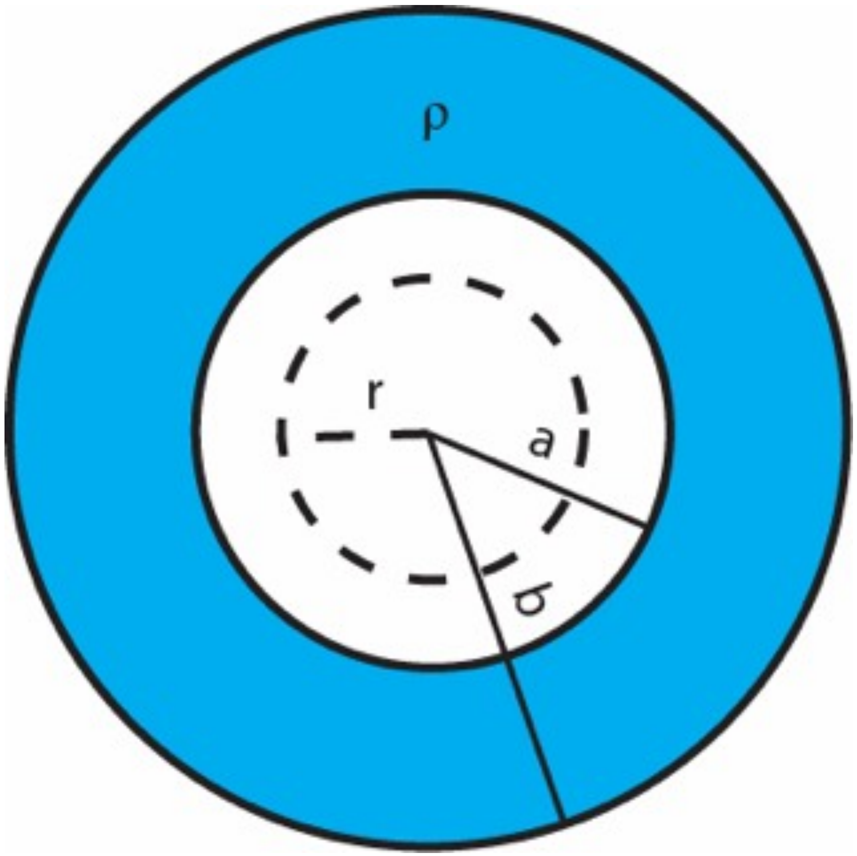
A charged spherical insulating shell has inner radius a and outer radius b. The charge density on the shell is **ρ**. What is the magnitude of the E-field at a distance r away from the center of the shell where r < a?

A. ρ/ε_o

B. zero

C. $\rho(b^3\text{-}a^3)/(3\varepsilon_o r^2)$

D. none of the above



CheckPoint Results: Charged Sphericlal Shell

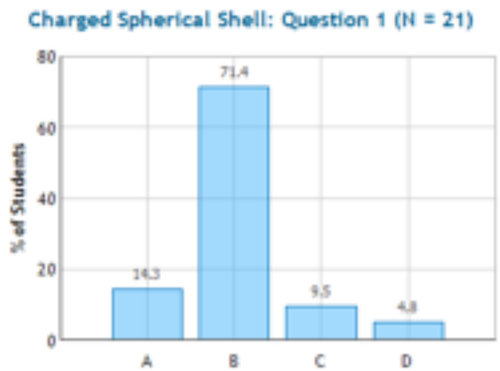
A charged spherical insulating shell has inner radius a and outer radius b. The charge density on the shell is **ρ**. What is the magnitude of the E-field at a distance r away from the center of the shell where r < a?

A. ρ/ε_o

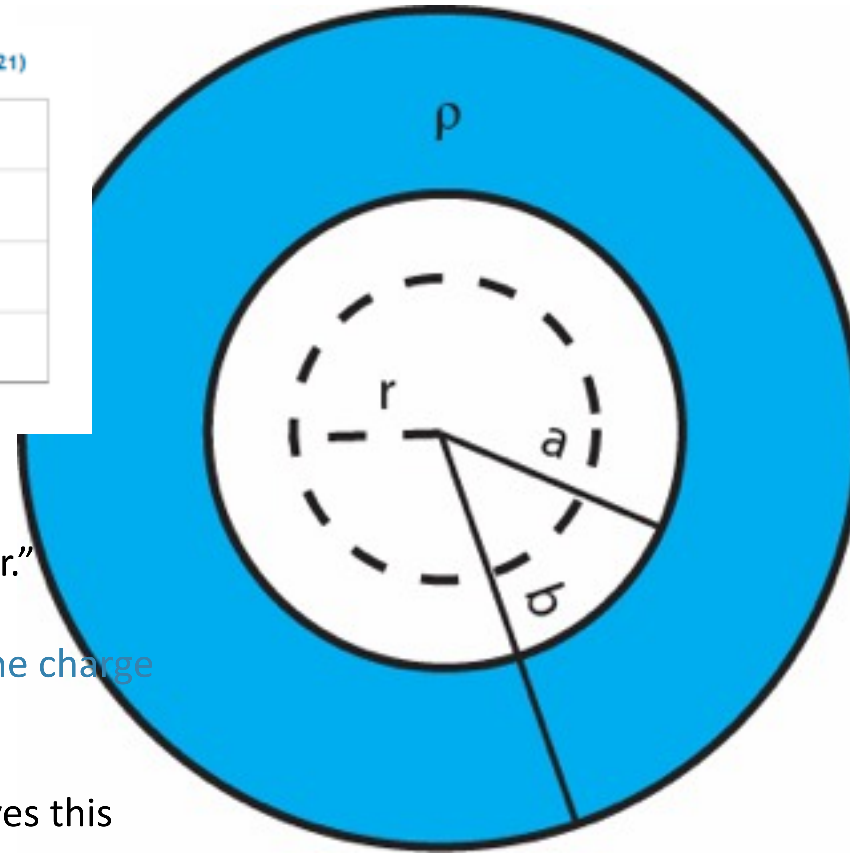
B. zero

C. $\rho(b^3-a^3)/(3\varepsilon_o r^2)$

D. none of the above



Latest



“I'm not actually sure but it seems like the right answer.”

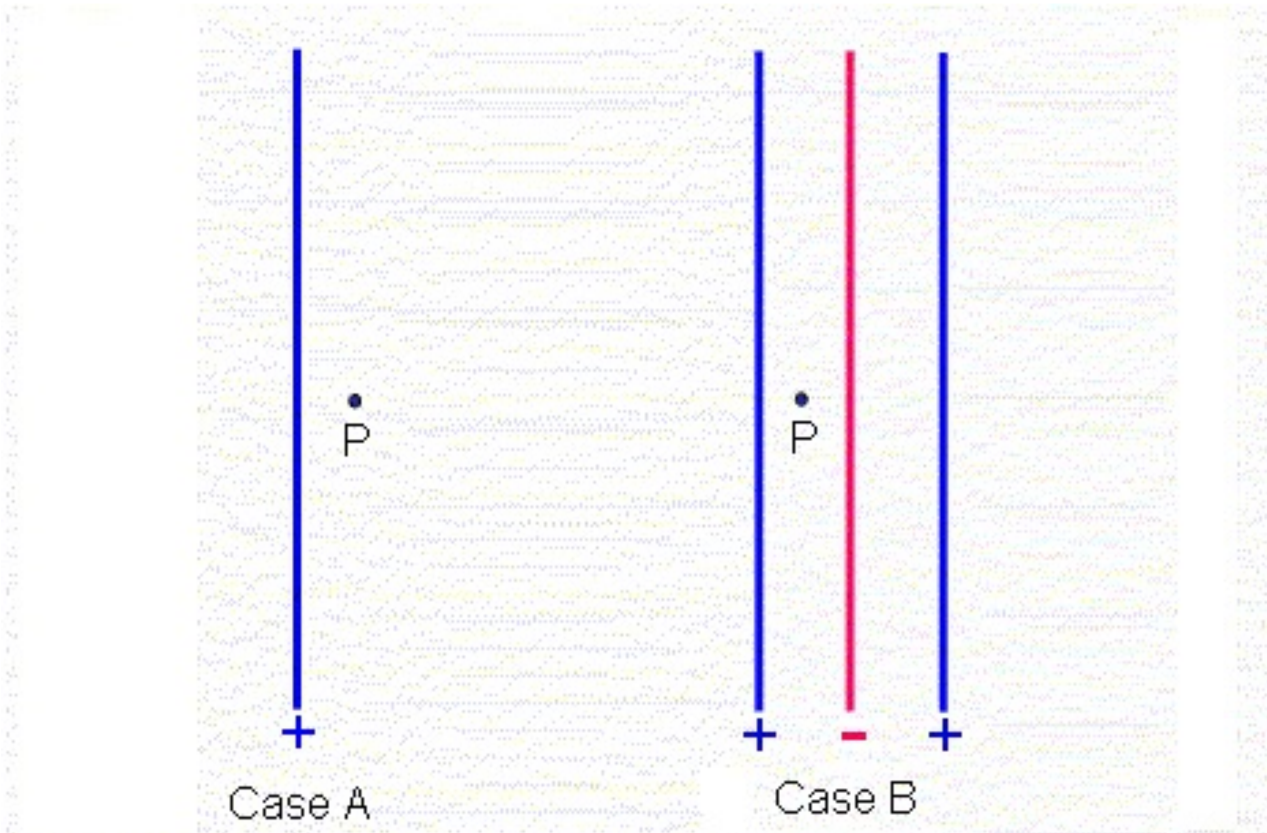
“All of the Electric field of the shell is on the surface, the charge and thus the Electric field inside the shell is 0. “

“ The formula of E-field is $\rho r/\varepsilon_0$. the above formula gives this result when worked out. “

CheckPoint: Infinite Sheets of Charge

In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same. In which case is the magnitude of the electric field at point P bigger?

A. Case A
B. Case B
C. They are the same



CheckPoint Results: Infinite Sheets of Charge

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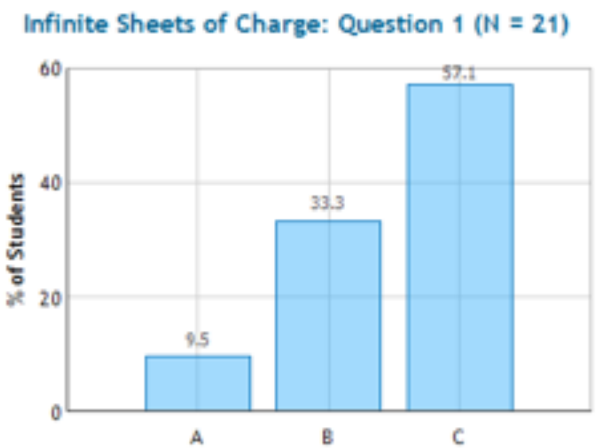
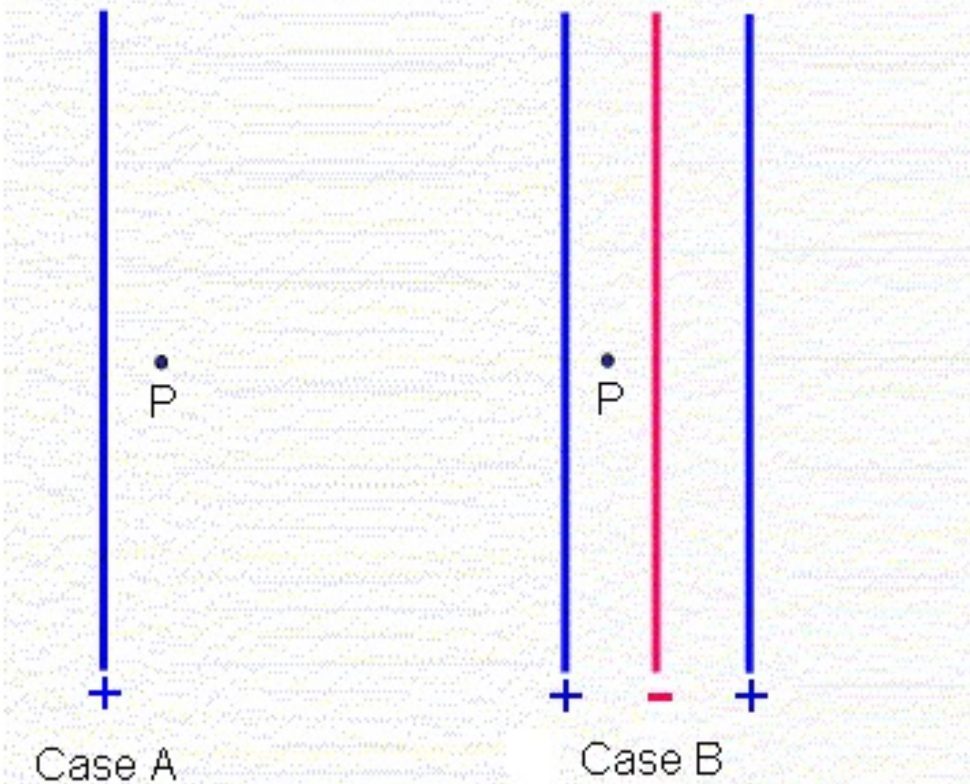
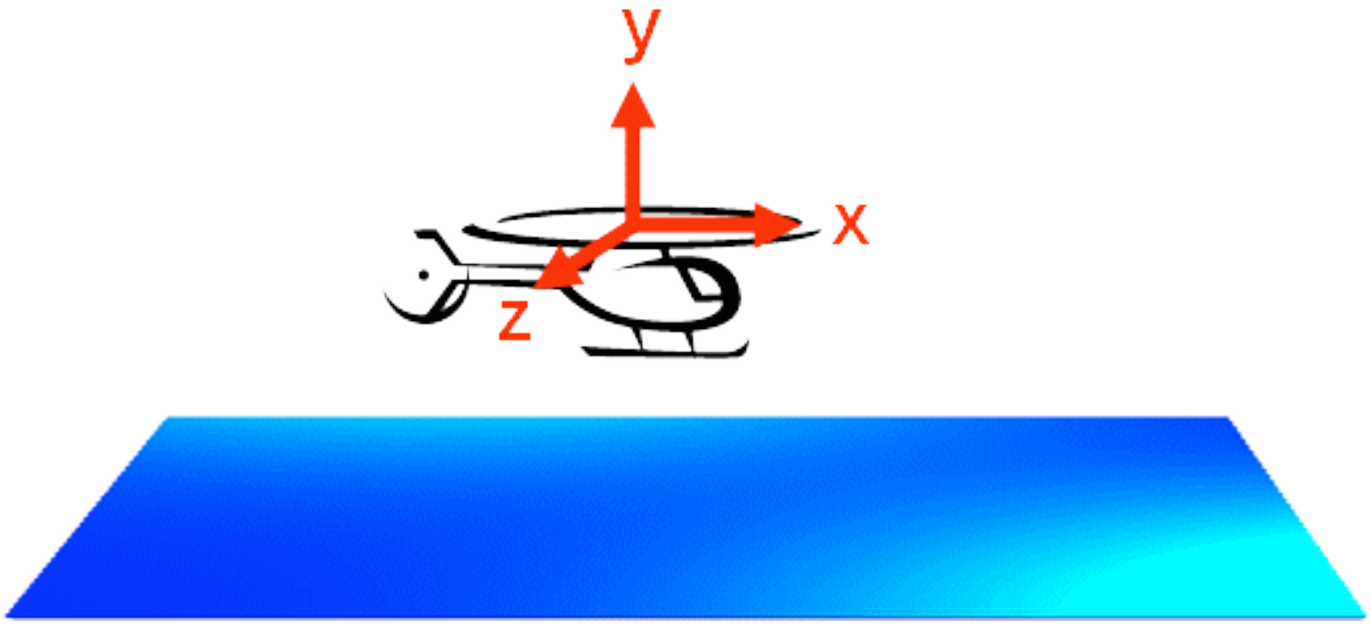
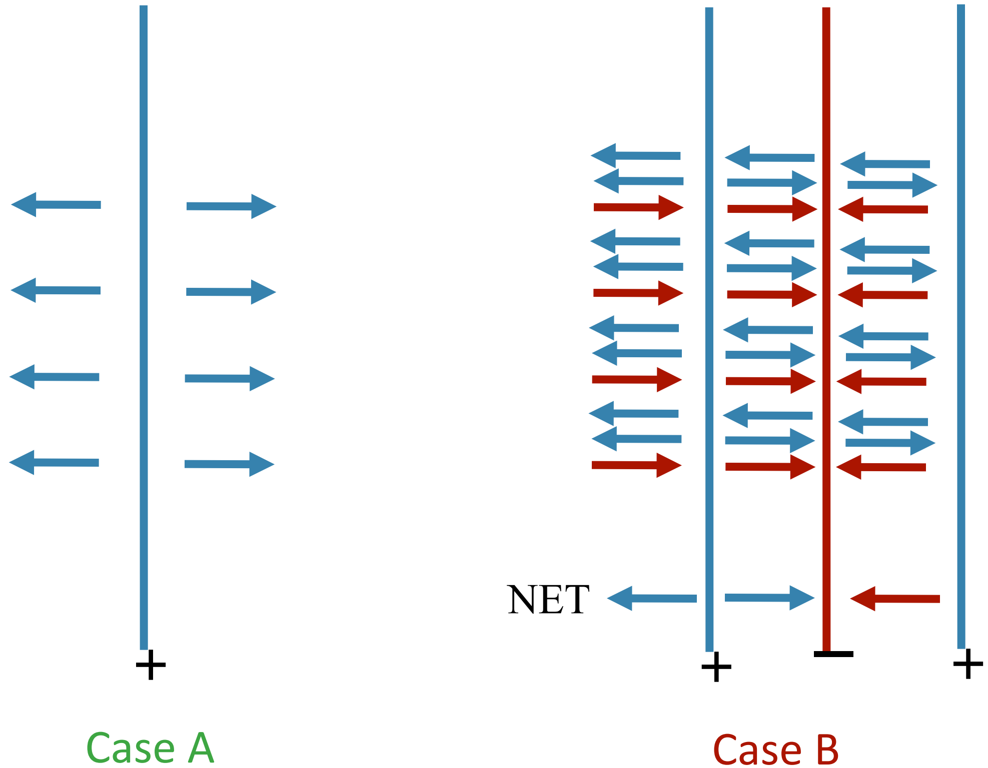


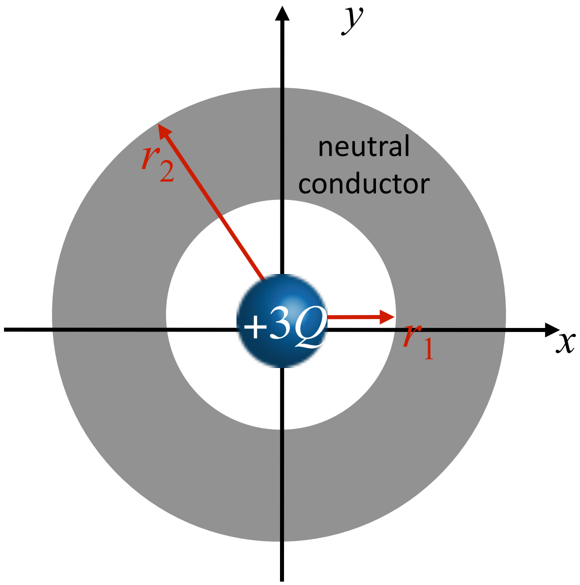



y
x
z

Superposition:



Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

a) What is E everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine E?

Yes, Spherical Symmetry (what does this mean???)

A) Magnitude of E is *fcn* of r

B) Magnitude of E is *fcn* of $(r\text{-}r_1)$

C) Magnitude of E is *fcn* of $(r\text{-}r_2)$

D) None of the above

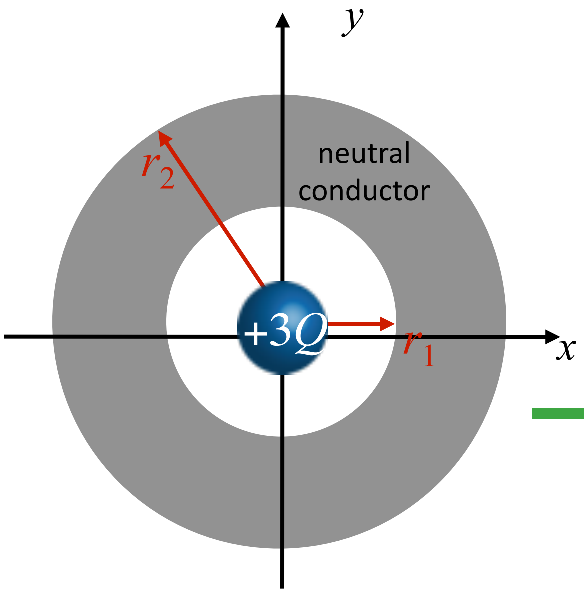
A) Direction of E is along $\hat{x}$

B) Direction of E is along $\hat{y}$

C) Direction of E is along $\hat{r}$

D) None of the above

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

A) What is E everywhere?

We know:

magnitude of E is *fcn* of r

direction of E is along $\hat{r}$

We can use Gauss' Law to determine E

Use Gaussian surface = sphere centered on origin

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$r < r_1$

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = E 4\pi r^2$$

$$Q_{enc} = +3Q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

$r_1 < r < r_2$

A) $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r_1^2}$

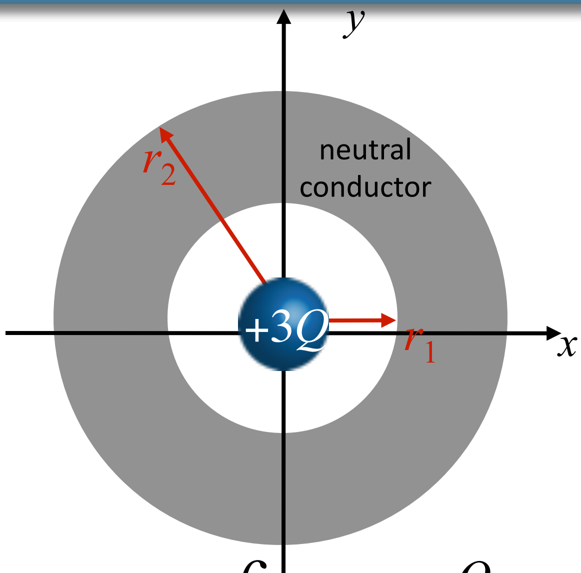
C) $\boxed{E = 0}$

$r > r_2$

A) $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}}$

B) $E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{(r - r_2)^2}$

C) $E = 0$



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

A) What is E everywhere?

We know:

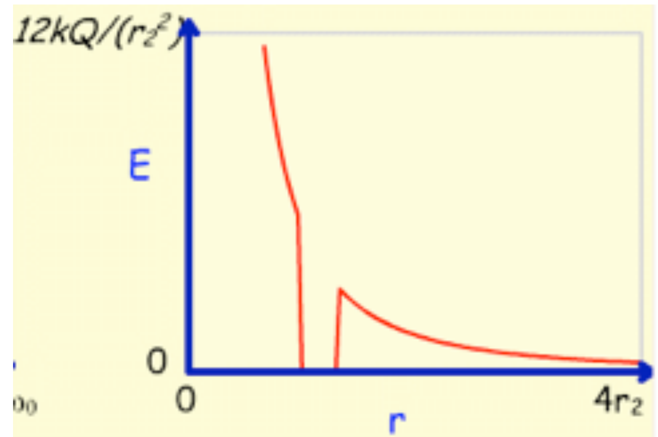
$r < r_1$

$r > r_2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

$r_1 < r < r_2$ $\quad E = 0$

$$\oint_{\text{surface}} \vec{E}\cdot\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

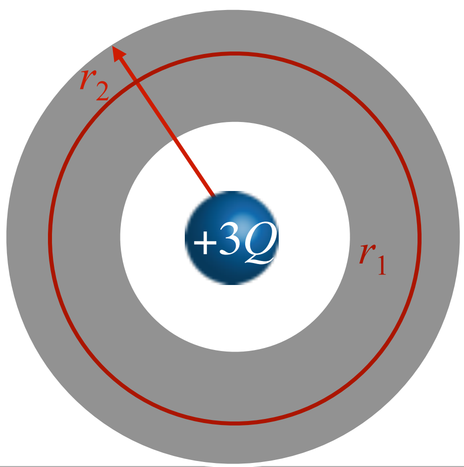


B) What is charge distribution at r_1?

$\boxed{\text{A)} \quad \sigma < 0}$

B) $\sigma = 0$

C) $\sigma > 0$



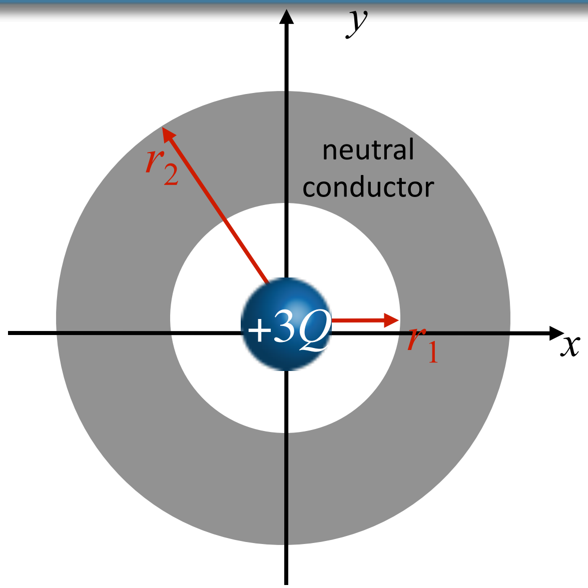
Gauss' Law:

$$E = 0 \rightarrow Q_{enc} = 0 \rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

Calculation

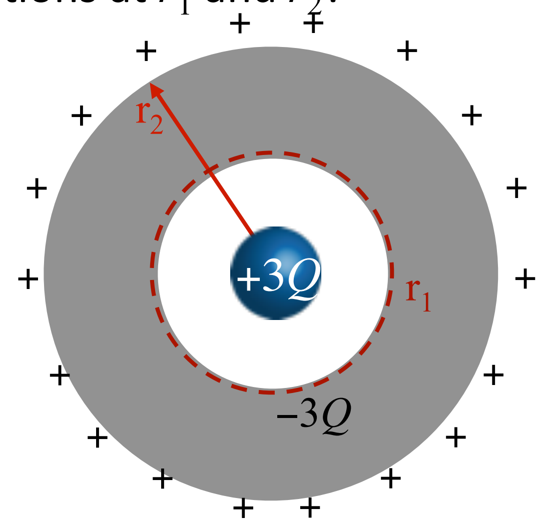


Suppose give conductor a charge of $-Q$

A) What is E everywhere?

B) What are charge distributions at r_1 and r_2?

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$



$r < r_1$

A) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$ $\boxed{}$

B) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

C) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

$r > r_2$

A) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

B) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

C) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

$r_1 < r < r_2$

$$E = 0$$