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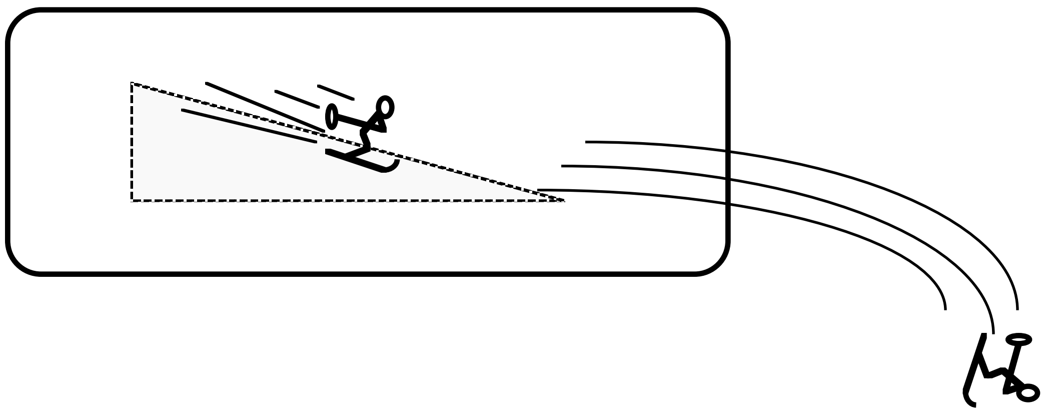
UNIT 7: APPLICATIONS OF NEWTON'S LAWS[1]



The essential fact is that all the pictures which science now draws of nature . . . are mathematical pictures.

Sir James Jeans

OBJECTIVES

1. To explore the phenomenon of uniform circular motion and the accelerations and forces needed to maintain it.

2. To explore the use of Newton's laws to discover the effects of gravitational forces in two dimensions.

3. To explore the characteristics of three different types of passive forces: tension (in strings, ropes, springs, and chains), normal forces (which support objects affected by gravity), and friction.

4. To learn to use free body diagrams to make predictions about the behaviour of systems which undergo multiple forces in two and three dimensions.

1 Some of the materials and exercises on the use of vector diagrams found in this unit were adapted from Active Learning Problem Sheets (ALPS) developed by Alan Van Heuvelen at New Mexico State University and materials on passive forces were adapted from the Tools for Scientific Thinking Curriculum on passive forces by Ronald Thornton at Tufts University and David Sokoloff at the University of Oregon.

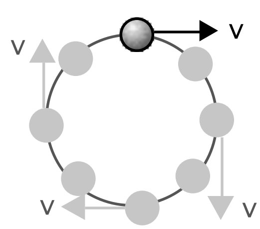
OVERVIEW

In the last unit you began the study of the application of Newton's laws to *projectile motion*. In this unit we are going to consider the application of Newton's laws to several other phenomena in one and two dimensions. Since Newton's laws can be used to predict types of motion or the conditions for no motion, their applications are useful in many endeavours including human body motion, astrophysics, and engineering.

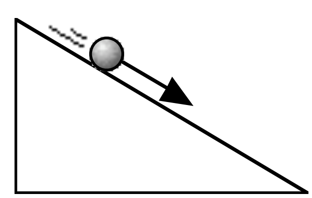
You will begin this unit by exploring *uniform circular motion,* in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep a massive object moving in a circle.

In Session Two you will consider the characteristics of several "invisible" forces that must be taken into account during the comprehensive application of Newton's laws to problems of real interest. These forces, *friction, tension in strings, and normal forces* are known as *passive forces* because they only act in response to other forces.

Finally, in Session Three you will learn techniques for drawing free body diagrams to help you predict motions or find the conditions for equilibrium (i.e., no motion or constant motion) in some relatively complicated situations. One of these situations includes the study of motions up and down ramps or inclined planes in which vector components of forces must be considered. In inclined plane motion the path of the object is constrained to move along a straight line. Thus, the path along an incline is not parabolic like that of a projectile.



(a)



(b)

Figure 7-1: Two types of motion to be explored in this unit.
(a) Uniform circular motion with constant speed, and
(b) frictionless "falling" motion along a straight line that lies along the surface of a ramp.

SESSION ONE: CIRCULAR MOTION AND CENTRIPETAL FORCE

Moving in a Circle at a Constant Speed
When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo *uniform circular motion.* Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction. We will then use Newton's laws to see how the circular motions of the planets can be used to help astronomers discover the laws of gravitation.

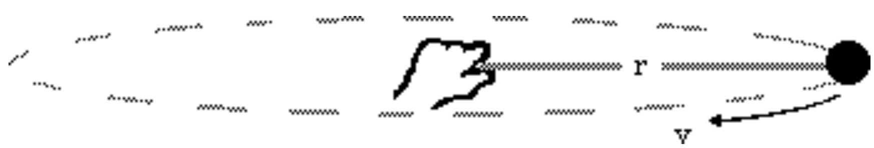


Figure 7-2: Uniform circular motion. A ball moving at a constant speed in a circle of radius *r*.

Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant *speed*.

✍ Activity 7-1: Uniform Circular Motion

(a) Consider Fig. 7-2. What is the speed of a ball that moves in a circle of radius r = 2.5 m if it takes 0.50 s to complete one revolution?

(b) The *speed* of the ball is constant! Would you say that this is accelerated motion?

(c) What is the *definition* of acceleration? (Remember that acceleration is a vector!)

(d) Are *velocity* and *speed* the same thing? Is the *velocity* of the ball constant? (**Hint:** Velocity is a vector quantity!)

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

Using Vectors to Diagram How Velocity Changes

By now you should have concluded that since the *direction* of the motion of the ball is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the *direction* of the acceleration and its magnitude as a function of the speed v of the ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.

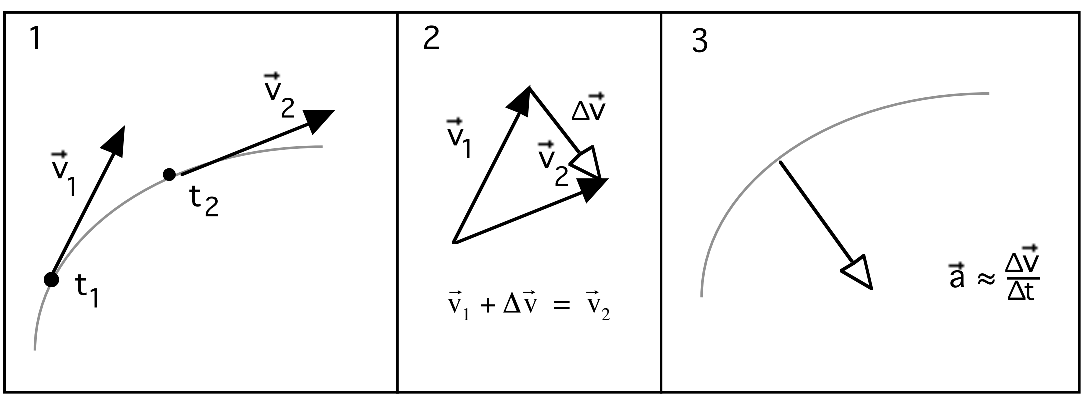


Figure 7-3

1. **Velocities:** Draw an arrow representing the velocity, $\vec{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\vec{v}_2$, of the object at time t_2.

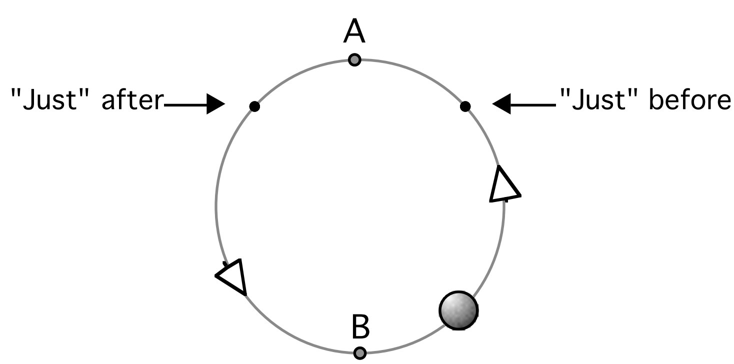
2. **Velocity Change:** Find the change in the velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ during the time interval $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector which points from the head of the first velocity vector to the head of the second velocity vector.

3. **Acceleration:** The acceleration equals the velocity change $\Delta\vec{v}$ divided by the time interval Δt needed for the change. Thus, is in the same direction as Δv but is a different length unless (Δt = 1). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\vec{v}$.

The acceleration associated with uniform circular motion is known as *centripetal acceleration*. You should use the vector diagram technique to find its direction.

✍ Activity 7-2: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot *just before* it reaches point A. Label this vector $\vec{v}_1$.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot *just after* it passes point A. Label this vector $\vec{v}_2$.

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. (See Figure 7-3, diagram 2.) Next, draw the vector that must be added to vector $\vec{v}_1$ to add up to vector $\vec{v}_2$; label this vector $\Delta\vec{v}$. *Be sure that vectors $\vec{v}_1$ and $\vec{v}_2$ have the same magnitude and direction in this drawing that they had in your drawing in part (a)!* (Again, see Figure 7-3.)

(d) Now, draw an exact copy of $\Delta\vec{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\vec{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\vec{a}_c$?

(f) If you redid the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\vec{a}_c$, would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

(h) Use Newton's second law in vector form ($\Sigma\vec{F} = m\vec{a}$) to describe the direction of the net *force* on the ball as it moves around the circle.

(i) If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed.
You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use an understanding of Newton's second law to get a feel for what the mathematical relationships might be. You can then use the rules of mathematics and the definition of acceleration to *derive* the relationship between speed, radius, and magnitude of centripetal acceleration.

✍ Activity 7-3: How Does a_c Depend on v and r?

(a) Consider an object moving in a circle at a constant speed. Do you expect you would need more or less centripetal acceleration to cause the object to move in a smaller circle, without changing its speed? In other words, would the magnitude, a_c, have to increase or decrease as r decreases, an v stays the same and it continues to move in a circle? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_c, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Let's use the definition of acceleration in two-dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

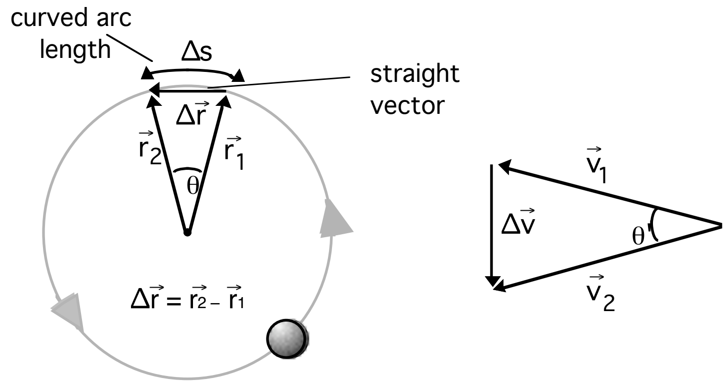
$$a_c = \frac{v^2}{r} \qquad \text{[Eq. 7-1]}$$

In order to do this derivation you will want to use the following definition for acceleration

$$< \vec{a} > \equiv \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \qquad \text{[Eq. 7-2]}$$

✍ Activity 7-4: Finding the Equation for a_c

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the displacement vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. **Hint**: In circular motion, velocity vectors are always perpendicular to their displacement vectors.



(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e., $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e., $v = v_1 = v_2$), use the properties of similar triangles to explain why

$$\frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{r}$$

(c) Now use the equation in part (b) and the definition of $< a >$ to

show that $< a_c > = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{|\Delta\vec{r}|}{(\Delta t)}\frac{v}{r}$

(d) The speed of the object as it rotates around the circle is given by

$$v = \frac{\Delta s}{\Delta t}$$

Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when Δt is very small (so that the angle θ becomes very small) i.e., why is $\Delta s \approx \Delta r$?

(e) If $\Delta s \approx \Delta r$, then what is the equation for the speed in terms of Δr and Δt?

(f) Using the equation in part (c), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by the equation

$$a_c = \frac{v^2}{r}$$

(g) If the object has a mass *m*, what is the equation for the magnitude of the centripetal force needed to keep the object rotating in a circle (in terms of *v, r*, and *m*)? In what direction does this force point as the object rotates in its circular orbit?

Experimental Verification of the Centripetal Force Equation

The theoretical considerations in the last activity should have led you to the conclusion that, whenever you see an object of mass m moving in a circle of radius *r* at a constant speed *v*, it must at all times be experiencing a net centripetal force directed toward the centre of the circle which has a magnitude of

$$F_c = ma_c = m\frac{v^2}{r} \qquad \text{[Eq. 7-3]}$$

Let's check this out. Does this rather odd equation really work for an external force?

To do this experiment you will need the following equipment:

- A smooth, level area of *r* about 2 m.
- A post in the centre of the area
- A cart with bearings to support 2D motion
- A person to ride on the cart

- A belt with rings on the sides for the rider
- 2 smooth nylon ropes
- A spring scale (approx. 150 N max scale or 15 kg)
- A person to maintain the constant orbital speed in a tangential direction
- A stop watch
- A bathroom scale (for determining cart & rider mass)

You should attach the spring scale between the centre post and the rope. The end of the rope should be attached to the cart rider's belt or held by the rider. The second rope should be attached to the cart in a direction which is perpendicular to the original rope. See the diagram below for details.

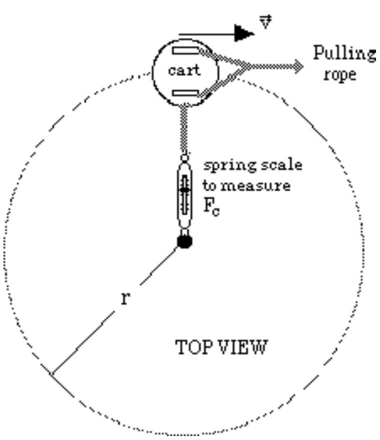


Figure 7-4: a) Top view of the centripetal force experiment.

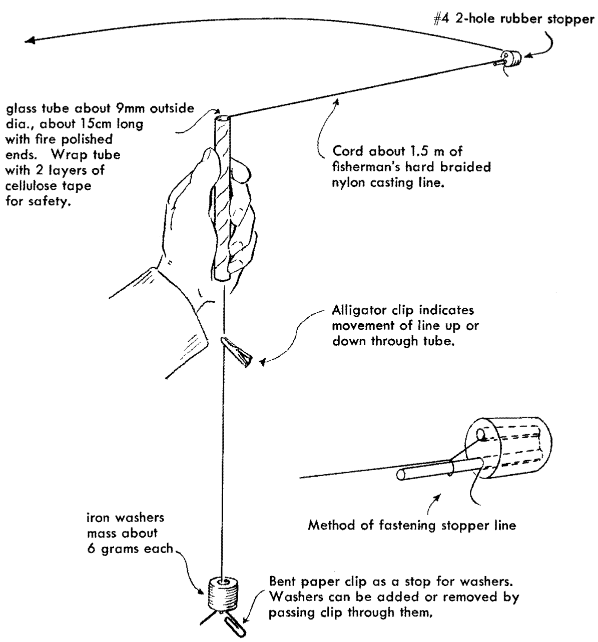
✍ Activity 7-5a: Verifying the F_c Equation Qualitatively

(a) If a puller applies a force on the cart which is always tangent to the circle and which is just sufficient to overcome friction in the cart and maintain the cart's motion at a constant speed, what is the *net* force in a direction perpendicular to the circle? Remember Newton's first law!!??

(b) You and members of your group should practice pulling each of your members around at a constant speed. The rider doesn't need the belt for this exercise. Instead, the rider should hold on to the rope and close his or her eyes and concentrate on feeling the centripetal force.

(c) When you hold on to the centre rope while riding on the cart, what is the direction of the *net* force you feel on you (include consideration of the forces in the radial direction and in the tangential direction.) Does the force you feel seem to increase as you rotate faster? As you are rotate in a smaller circle? Explain.

✍ Activity 7-6b: Verifying the F_c Equation Experimentally



(From *College Physics Laboratory Guide* by Uri Haber-Schaim)

- Rubber Stopper, #4, 2 holes
- Glass rod, 9 mm outside diameter, 15 cm long, ends fire polished and covered with tape for safety
- Fishing line, about 1.5 m
- Iron Washers, about 6 grams each
- Paper clip, bent
- Stop watch and ruler

a)When the glass tube is swung in a small circle above your head, the rubber stopper moves around in a horizontal circle at the end of a string which is threaded through the tube and fastened to some washers hanging below. The force of gravity on these washers, acting along the thread, provides the horizontal force needed to keep the stopper moving in a circle. This horizontal force is called the centripetal force.

b)With only one washer on the end of the string to keep the stopper from getting away, whirl the stopper over your head while holding onto the string below the tube. Do you have to increase the pull on the string when you increase the speed of the stopper? What happens if you let go of the string?

c) Now quantitatively investigate the dependence of the accelerating force on the speed, the mass, and the radius. First find out how the force depends on the speed, keeping the mass and the radius constant.

d) Pull enough string through the tube so that the stopper will whirl in a circle of about 100 cm radius. Attach an alligator clip to the string just below the tube to serve as a marker so that you can keep the radius constant while whirling the stopper. Hang six or more washers on the end of the string.

e) To find the rate of revolution of the stopper, have a partner use a stopwatch to measure the time while you swing the stopper around and count the number of revolutions. From the time and number of revolutions calculate the period (time/rev) and the speed. Repeat the experiment with larger numbers of washers.

BASIC DATA	
Mass of Stopper (kg)	
Radius of Circular Path (m)	
Circumference of Path (m)	

Time/rev t **(s)**	**Avg. Speed (m/s)**	**# Washers**	**Meas. Avg.** F_c **(N)**	**≈ Uncertainty** $\pm\Delta F_c$ **(N)**	**Calculated** F_c**(N)**
∞	0.0	0.0	0.0	0.0	0.0

(e) Plot a graph of F vs. v using a polynomial fit.
Hint: According to Equation 7-3, what order should the polynomial relationship be?
Sketch your graph below.

(f) Draw vertical error bars over each of your data points indicating the estimated uncertainty in your force measurements. (See Appendix D for information about what error bars look like.) Then, look at the visual data fit. Within the limit of experimental uncertainty, how well does your experimental data support the hypothesis that F_c is a function of v^2?

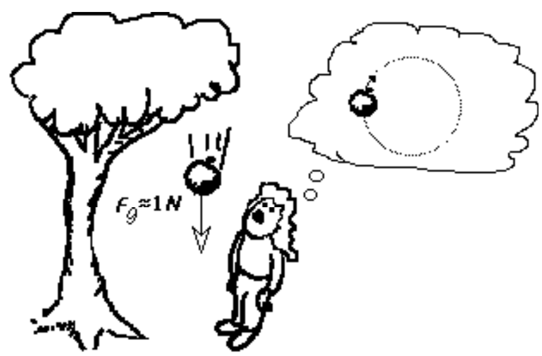
(g) Let's look at agreement between experiment and theory another way. How well does each measured force agree with the corresponding calculated force shown in your data table?

(h) Discuss the major sources of uncertainty in this experiment. There are plenty!

Newton's Apple

Newton used the idea of centripetal acceleration and the observation that the earth moves in an approximately circular orbit around the sun to conclude that the earth's orbit is a result of a *gravitational force* between the sun and the earth.

His biographer, Richard Westfall, commented that even if Newton had died in 1684, before completing the *Principia*, the world would have recognized him as a genius. However, many intellectual historians consider Newton's most profound contribution to science to be his recognition that the laws that govern the orbital motion of planets are the same laws that govern the falling of objects near the earth and the linear acceleration of objects that are pushed or pulled with constant forces.



. . .there is clearly a dynamic identity between uniform circular motion and constantly accelerated motion. . .

Figure 7-6: What Newton might have said

The image of Newton having this idea as a sudden insight while sitting under an apple tree is probably mythical. However, it leads to some interesting modern facts. Nabisco now distributes both Fig and Apple Newtons. Near the surface of the earth the force on an average sized apple is about one newton. Think of this the next time you eat Apple Newtons!

SESSION TWO: NEWTON'S THIRD LAW AND PASSIVE FORCES

Review of Homework Assignment
Come prepared to ask questions about the homework assignments on centripetal forces and acceleration.

An Introduction to Newton's Third Law
In order to apply Newton's laws to complex situations with strings, pulleys, inclined planes and so forth, we need to consider a third force law formulated by Newton having to do with the forces of interaction between two objects. In order to "discover" some simple aspects of the third law, you should make some straightforward observations using the following equipment:

- 2 spring balances
- a cart

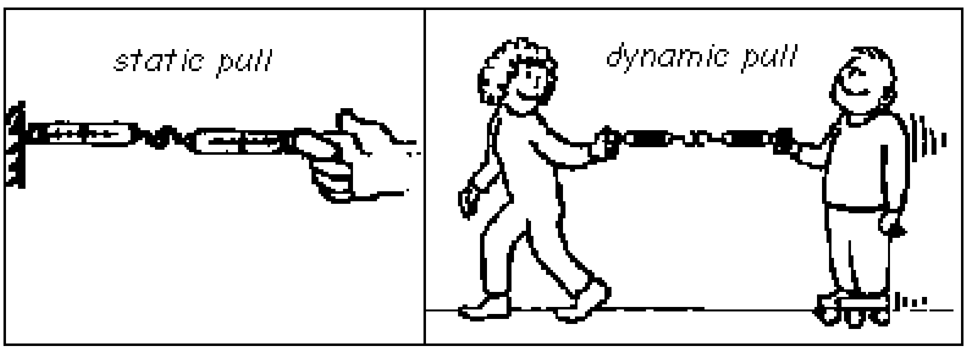


Figure 7-7: Ways to test Newton's third law

✍ Activity 7-7: Newton's 3rd Law–Forces of Interaction

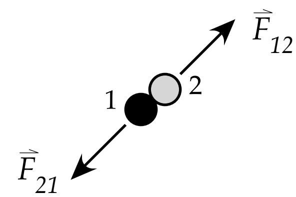
Set up the situations shown in the diagram above and see if there are any circumstances in which the object that is pulling and the object that is being pulled exert *different* forces on each other. Describe your conclusions below. **Note:** You can use a skater, a small cart, a person riding on a large cart, etc. for your dynamic observations.

In contemporary English, Newton's third law can be stated as follows:

Newton's Third Law

If one object exerts a force on a second object, then the second object exerts a force back on the first object which is equal in magnitude and opposite in direction to that exerted on it by the first object.

In mathematical terms, using vector notation, we would say that the forces of interaction of object 1 on object 2 are related to the forces of interaction of object 2 on object 1 as follows:



$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton actually formulated the third law by studying the interactions between objects when they collide. It is difficult to understand the significance of this law fully without first studying collisions. Thus, we will consider this law again in the study of collision processes.

Tension Forces
When you pull on one end of a rope attached to a crate, a force is transmitted down the rope to the crate. If you pull hard enough the crate may begin to slide. *Tension* is the name given to forces transmitted in this way along devices that can stretch such as strings, ropes, rubber bands, springs, and wires.

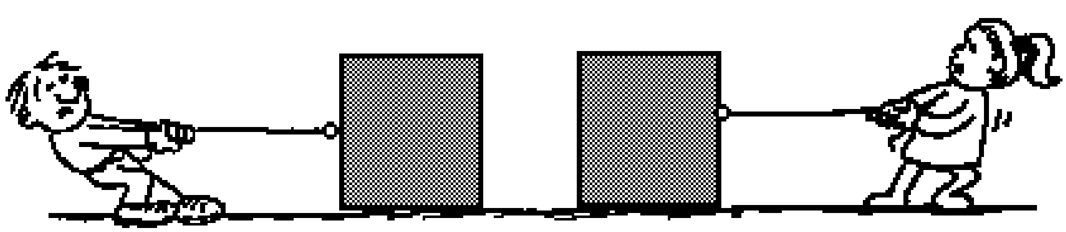


Figure 7-8: Transmitting forces along a string by pulling

The end of the rope which is tied to the crate can apply a force to the crate *only* if you first pull on the other end of the rope. Thus, tension, like friction, is a *passive force* that only acts in reaction to an active force. However, the characteristics of tension are very different than those of friction. In order to analyse situations in which objects are attached by strings, rubber bands, or ropes it is necessary to understand some attributes of tension forces. We need to answer the following related questions:

1) (a) What is the mechanism for creating tension in strings, ropes, and rubber bands?

(b) If a string exerts a tension force on an object at one end, what is the magnitude and direction of the tension force it exerts on another object at its other end?

2) What happens to the magnitude and directions of the tension forces at each end of a string and in the middle of that string when the direction of the string is changed by a post or pulley?

3) Can a flexible force transmitter (like a string) support a lateral (or sideways) force?

In order to investigate the nature of tension forces you will need the following apparatus:

- 4 rubber bands (#16)
- 4 short lengths of string (approx. 6")
  with small loops at each end
- 4 long lengths of string (approx. 24")
  with small loops at each end
- 3 matched spring scales with 5 N max readings
- 3 matched spring scales with 20 N max readings
- A set of small hanging objects such as

(masses are approximate but pairs should be balanced)
  pair - 100 g masses (e.g., three 30 g washers)
  pair - 200 g masses (e.g., six 30 g washers)
  pair -1000g masses (e.g., 1 litre bottles of water)
  one - 10 kg mass, (e.g., a bowling ball in a bag)
- 2 frictionless pulleys
- Clamps, stands, and rods to hold strings pulleys and masses

(1) Mechanisms for Tension and the Direction of Forces
For these observations you should stretch a rubber band and then a string between your hands as shown in the diagram below. First, just feel the directions of the forces. Then add the spring scales and both feel and measure the forces.

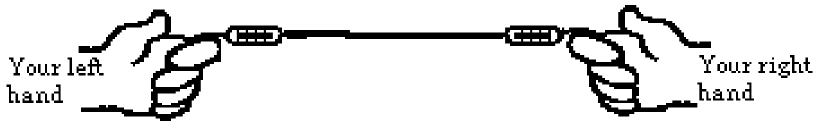


Figure 7-8: Stretching a string

✍ Activity 7-8: Tension Mechanisms & Force Directions

(a) Pull on the two ends of a rubber band. (Forget about the spring scales for now). What is the direction of the force applied by the rubber band on your right hand? On your left hand?

(b) Does the magnitude of the forces applied by the rubber band on each hand feel the same?

(c) Repeat this activity with a string instead of a rubber band. This time, use a spring scale at each end to measure the forces at the ends of the string. Does the string stretch? (Look carefully!)

(d) If you pull by the same amount on the string as you did on the rubber band, does substituting the string for the rubber band change anything about the directions and magnitudes of the tension forces exerted on each hand?

(e) If the forces caused by the string on your left and right hands respectively are given by $\left(\vec{F}_T\right)_1$ and $\left(\vec{F}_T\right)_2$, what is the equation that relates these two forces?

(f) What is a mechanism that might cause a rubber band or a string to develop tension in response to a force that you apply?

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(2) Tension Forces when a String Changes Direction
Suppose you were to hang *equal* masses of *about* 1.0 kg in the various configurations shown below. Predict and measure the tension in the string for each of the situations.

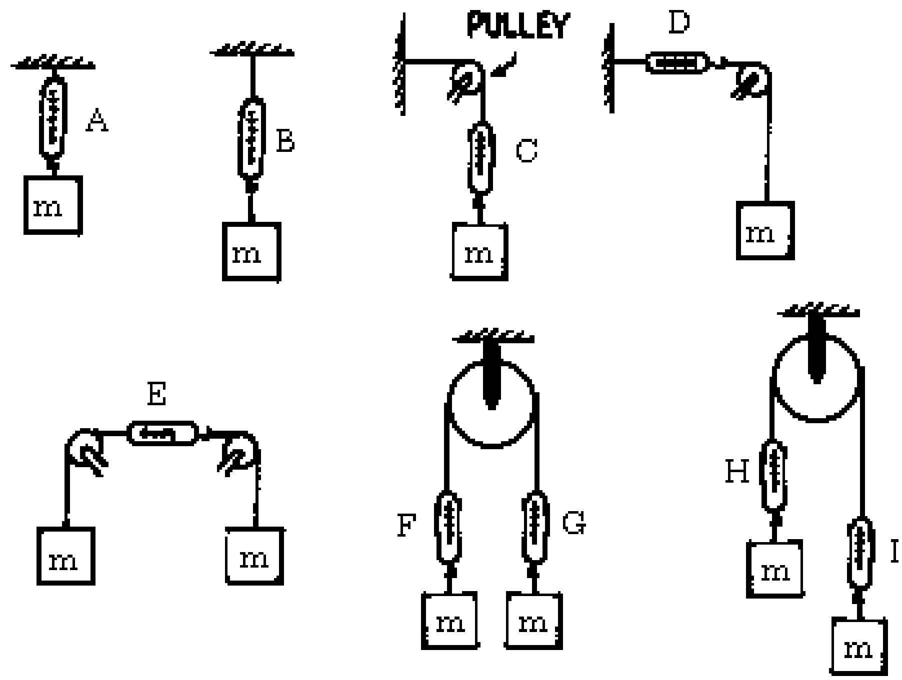


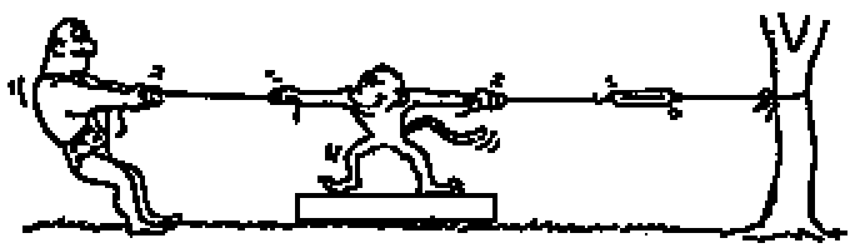
Figure 7-9: Configurations for measuring tension in a string. (You may need to share a pulley with a neighbouring group for the two-pulley test.)

Activity 7-9: Tension and Direction Changes

(a) For each configuration in the Figure 7-9, predict the reading in newtons on each of the spring scales; these readings indicate the forces that are transmitted by the tensions at various places along the string. Then measure all of the forces and record their values. **Note:** Remember that m is the same in all cases..

Predicted Force Magnitudes	**Measured Force Magnitudes**
F_A = ________ N	F_A = ________ N
F_B = ________ N	F_B = ________ N
F_C = ________ N	F_C = ________ N
F_D = ________ N	F_D = ________ N
F_E = ________ N	F_E = ________ N
F_F = ________ N	F_F = ________ N
F_G = ________ N	F_G = ________ N
F_H = ________ N	F_H = ________ N
F_I = ________ N	F_I = ________ N

(b) Based on Newton's third law and the observations you just made, answer the following questions *using vector notation*. If the muscle man in the diagram below is pulling to the left on a rope with a force of $\vec{F} = (-150\text{N})\hat{\imath}$.



(1) What is the magnitude and direction of the force that the rope is exerting on the man?

(2) What force is the left-hand rope exerting on the monkey's left arm?

(3) What force is the spring scale experiencing on its left end?

(4) What force is the spring scale experiencing on its right end?

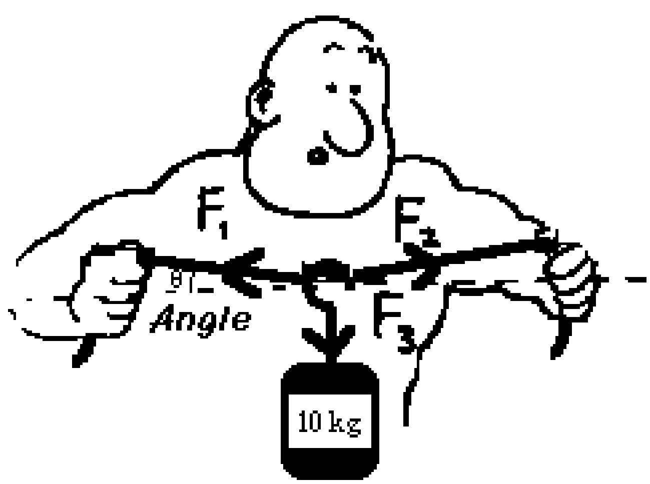
(5) What is the reading on the spring scale?

(6) What force is the rope exerting on the tree?

(7) What force is the tree exerting on the rope?

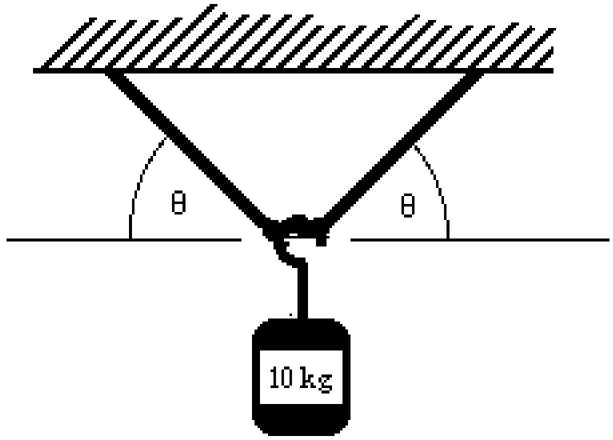
(d) Summarize what your observations reveal about the nature of tension forces everywhere along a string.

(3) Can a String Support Lateral Forces?
Take a look at the diagram below. Can the strongest member of your group stretch a string or rope so that it is perfectly horizontal when a 10 kg mass is hanging from it? In other words, can the string provide a force that just balances the force exerted by the mass?



✍ Activity 7-10: Can a String Support a Lateral Force?

(a) Draw a vector diagram showing the directions of the forces exerted by the strings on the mass hook in the diagram below. What would happen to the direction of the forces as θ goes to zero? Do you think it will be possible to support the mass when θ = 0?

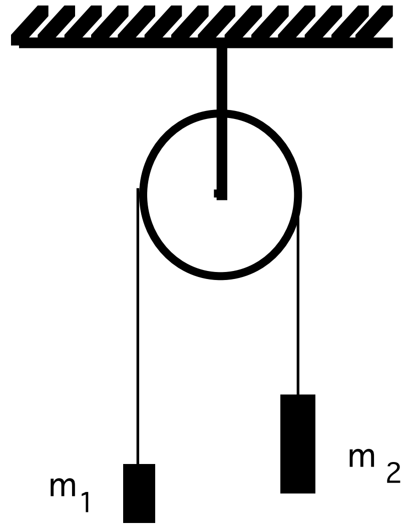


(b) Now, experiment with holding a mass horizontally with a string. What do you conclude about the ability of a string to support a mass having a force which is perpendicular to the direction of the string?

Applying Tension Concepts to the Atwood's Machine

Sometime before 1780, a physicist at Cambridge University named George Atwood devised a marvellous machine for measuring the acceleration of a falling mass without the aid of high speed timers, motion detectors or video cameras. It consists of two masses connected to each other by means of a light string passing over a relatively frictionless light pulley, as shown in the diagram below.[1]

[1] The meaning of "light": In referring to a "light" pulley and string, we mean that the mass of these items is very small compared to the masses of the falling weights. Thus, the masses of the string and pulley can be neglected in any calculations.



Atwood's machine is not only historically important but it allows us to practice applying Newton's laws and the kinematic equations to the analysis of motion. In order to make observations of the motion of the masses on an Atwood's machine you will need:

- A frictionless pulley ("superpulley")
- A smooth length of string
- 2 identical masses of apx. 50 g.
- Clamps and rods to support the pulley

✍ Activity 7-11: Behaviour of the Atwood's Machine

(a) Assume that the two masses are equal. If you pull down gently on one of them, what motion do you predict will result? Explain your reasoning.

(b) Set up the Atwood's Machine with combinations of equal masses, pull on one of them gently, and describe what you observe. How does your observation compare with your prediction?

(c) Suppose that m_2 is greater than m_1. What do you expect to observe and why?

(d) Set up the Atwood's Machine with combinations of unequal masses, and describe what you observe. How do your observations compare with your prediction?

(e) If the tension on the string is denoted by F_T, draw a diagram describing the forces on m_1. Draw another diagram showing the forces on m_2. **Hint**: Include the gravitational forces acting in the negative y-direction on masses 1 and 2 and the tension forces acting upwards. Assume that m_2 is greater than m_1.

(f) Write down an equation for the net force on m_1 in terms of F_T, g, and m_1. Use Newton's second law to relate this net force to the acceleration, a_1, of m_1.

(g) Next, write down an equation for the net force on m_2 in terms of F_T, g, and m_2. Use Newton's second law to relate this net force to the acceleration a_2, of m_2.

(h) Why can we say the acceleration of m_2 is $a_2 = -a$ if the acceleration of m_1 is $a_1 = a$?

(h) Eliminate F_T from the two equations to show that the net acceleration for the Atwood's machine is given by the equation

$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

You have just derived the “famous” Atwood's equation.

$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

This equation may seem pretty arcane at first glance, but it could end up saving your life if you're a bricklayer! Bricklayers have been known to use pulleys to haul bricks to the upper floors of buildings. This led a British humorist, Gerald Hoffnung, to compose the following song.

The Bricklayer's Lament

Dear sir, I write this note to you to tell you of my plight,
For at the time of writing it I'm not a pretty sight;
My body is all black and blue, my face a deathly grey,
And I write this note to say why I am not at work today.

While working on the 14th floor, some bricks I had to clear,
But tossing them down from such a height was not a good idea.
The foreman wasn't very pleased; he is an awkward sort
He said I had to cart them down the ladders in me hod.
Well, clearing all these bricks by hand – it was so very slow,
So I hoisted up a barrel and secured a rope below.
But in me haste to do the job, I was too blind to see
That a barrel full of building bricks was heavier than me.

And so when I untied the rope the barrel fell like lead,
And clinging tightly to the rope, I started up instead.
I shot up like a rocket and to my dismay I found
That halfway up I met the bloody barrel coming down.

Well, the barrel broke me shoulder as to the ground it sped,
And when I reached the top I banged the pulley with me head.
But I clung on tightly, numb with shock, from this almighty blow,
While the barrel spilled out half its bricks, some 14 floors below.

Now when these bricks had fallen from the barrel to the floor
I then outweighed the barrel, and so started down once more.
But I clung on tightly to the rope, me body racked with pain
And half way down I met the bloody barrel once again.

The force of this collision half way down the office block

Caused multiple abrasions and a nasty case of shock,
But I clung on tightly to the rope as I fell towards the ground,
And I landed on the broken bricks the barrel scattered 'round.

Well as I lay there on the floor I thought I'd passed the worst,
But the barrel hit the pulley wheel and then the bottom burst.
A shower of bricks rained down on me; I didn't have a hope,
As I lay there bleeding on the ground I let go the bloody rope.

The barrel now being heavier, it started down once more.
It landed right across me as I lay there on the floor.
It broke three ribs and my left arm, and I can only say
I hope you'll understand why I am not at work today!

Normal Forces

A book resting on a table does not move; neither does a person pushing against a wall. According to Newton's first law the net force on the book and on the person's hand must be zero. We have to invent another type of passive force to explain why books don't fall through tables and hands don't usually punch through walls. The force exerted by any surface always seems to act in a direction perpendicular to that surface; such a force is known as a *normal force*. Normal forces are passive forces because they seem to act in response to active forces like pushing forces and gravitational forces.

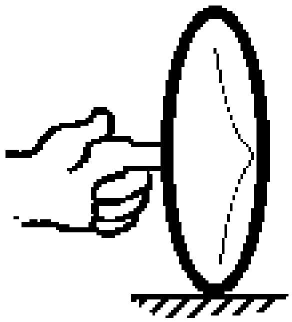
To investigate some attributes of normal forces you will need the following apparatus:

- An embroidery hoop with 1 piece of cloth
- An embroidery hoop with 2 pieces of cloth
- A table top
- A wall
- A 50 g and a 100 g mass

By applying forces perpendicular to a flexible surface with different degrees of stiffness you can discover a mechanism for the passive normal forces which crop up in reaction to active forces.

✍ **Activity 7-12: Normal Forces**

(a) Hold the one-cloth embroidery hoop upright and , with your finger, press the centre of the cloth perpendicular to the cloth surface. What does your finger feel? What happens to the cloth surface?



(b) Press harder in the centre of the cloth surface. Now what does your finger feel? What happens to the cloth surface?

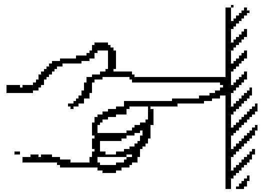
(c) Repeat parts (a) and (b) using the two-cloth hoop. Does the surface bend more or less than it did when one cloth was used?

(d) Do a similar set of investigations using the same hoops held horizontally with 50g, then 100g, and finally 150 g masses placed in the centre of each.

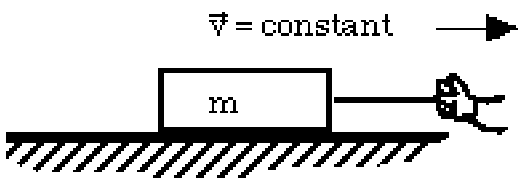
(e) What mechanism do you think might explain the ability of the cloth to react to an active force by applying a normal force? Is there any relationship to the mechanism you used to explain the phenomenon of tension?

(f) Based on your observations with two layers of stretched cloth, what would happen to the bending of the surface if we used hundreds of layers of cloth to resist a person's push or to hold up a book?

(g) Does a table or a wall bend noticeably if an active force is applied to it? What mechanism do you propose to explain how walls and tables can exert normal forces without bending noticeably? Do you think that the surface moves at all?



(h) The diagram below shows a block sliding along a table near the surface of the earth at a constant velocity. According to Newton's first law, what is the net force on the block? In other words, what is the vector sum of all the forces on the block?



(i) The net force is made up of four mutually perpendicular forces. What are the two active forces? What are the two passive forces? In what direction does each one act? Draw a diagram indicating the direction of each of the forces.

Gravitational Force on a Mass on an Incline

Suppose that a block of mass m is perched on an incline of angle θ as shown in the diagram below. Also suppose that you know the angle of the incline and the magnitude and direction of the gravitational force vector. What do you predict the magnitude of the components of the force vector will be parallel to the plane? Perpendicular to the plane?

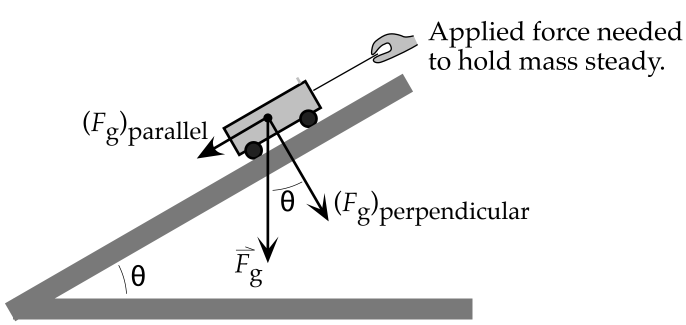


Figure 7-10: A roller on an inclined plane

✍ Activity 7-13: Components of F_g on an Incline

(a) The angle that the incline makes with the horizontal and the angle between $F_{perpendicular}$ and F_g are the same. Explain why.

(b) Choose a co-ordinate system with the x-axis parallel to the plane with the positive direction up the plane. Using normal mathematical techniques for finding the components of a vector, find the values of $F_{parallel}$ and $F_{perpendicular}$ as a function of F_g and the angle of the incline θ.

(c) What is the equation for the magnitude of the normal force exerted on the block by the surface of the incline? **Hint:** Use Newton's first law and the knowledge that the block is not moving in a direction perpendicular to the plane.

SESSION THREE: FRICTION AND APPLYING THE LAWS OF MOTION

Review of Homework Assignment
Come prepared to ask questions about the homework assignments on passive forces and forces on inclined planes.

Predicting and Measuring Friction Factors
If Newton's laws are to be used to describe the sliding of a block in *contact* with a flat surface, we must postulate the existence of a passive friction force that crops up to oppose the applied force. There are two kinds of friction forces: *static friction* and *kinetic or sliding friction*, which is the friction between surfaces in relative motion. We will concentrate on the study of kinetic friction for a sliding block. For this project you will have the following equipment available:

- A block with hook
- A 0 to 5 N. spring scale
- OPTIONAL: a force probe
- four 200 g masses
- A balance
- Several flat surfaces

✍ Activity 7-14: Prediction of Friction Factors

List several parameters that might influence the magnitude of the kinetic friction force.

1.____________________

2.____________________

3.____________________

4.____________________

5.____________________

6.____________________

(b) Pick one of the factors that interests you and describe how you might do an experiment to determine the effect of that factor on the magnitude of the friction force.

Measuring Kinetic Friction Factors
Let's determine what parameters actually influence the friction force. Students at each table can vary one parameter of interest to the class and determine the friction forces associated with it. Each group should analyse its results using a computer graphing routine. Enter the data points directly onto a computer, label each column, and produce an appropriate scatter plot.

✍ Activity 7-15: Friction Data and Analysis

(a) Describe the factor your group studied, create a data table for the friction force as a function of that factor, and then summarize your data and do a sketch of the scatter graph of your sliding friction force as a function of your factor (or place a computer printout of your graph in the space below).

(b) If you didn't study the friction force as a function of mass, summarize the data taken by some of your classmates and include a graph of F_f vs. m below. What is the meaning of the slope of the graph?

(c) Look up kinetic friction in the index of your text. Read about the coefficient of sliding friction, μ_k, and figure out how to determine μ_k from the data you have taken or might take. Using data provided by those groups that studied F_f as a function of m, calculate μ_k for the block sliding on one of the surfaces in the lab. Be sure to specify the two surfaces that were in contact.

Theories of Friction

No material is perfectly "smooth and flat." Any surface when examined under a microscope is full of irregularities. It is usually assumed that sliding friction forces result from the rubbing of rough surfaces, i.e. from the interlocking of surface bumps during the sliding process. How reasonable is this explanation for sliding friction? To make the observations described below you should have following equipment:

- A block with hook
- A 0-5 N spring scale
- OPTIONAL: a force probe
- Several clean surfaces, including at least one made of acetate used for making overhead transparencies

✍ Activity 7-16: What Surfaces Have High Friction?

(a) Which kinds of surfaces do you think will have the most friction – rough ones or smooth ones? Why?

(b) Make some qualitative observations to test your hypothesis by sliding the smooth wooden block on some smooth and rough surfaces. Make sure that at least one of these is a smooth sheet of glass or Plexiglas. Which situation has the most sliding friction with it? How does your observation compare with your prediction? Are you surprised?

TABLE 7-1: Summary of Newton's Laws

First: If the net force acting on an object is zero its acceleration is zero.

If $\Sigma\vec{F} = 0$ then $\vec{a} = 0$ so that $\vec{v}$ = constant or zero

Second: The net force on an object can be calculated by multiplying its force times its acceleration.

$$\Sigma\vec{F} = m\,\vec{a}$$

Third: Any two objects that interact exert forces on each other which are equal in magnitude and opposite in direction.

$$\vec{F}_{12} = -\vec{F}_{21}$$

The fact that smooth surfaces sometimes have more sliding friction associated with them than rough surfaces has led to the modern view that other factors such as *adhesion* (i.e., the attraction between molecules on sliding surfaces) also play a major role in friction. Predicting the coefficient of sliding friction for different types of surfaces is not always possible and there is much yet to be learned about the nature of the forces that govern sliding friction. Most authors of introductory physics texts still tend, incorrectly, to equate smooth surfaces with "frictionless ones" and to claim that the rubbing of rough surfaces is the cause of friction.

Free Body Diagrams — Putting It All Together

You have made a series of observations which hopefully led you to reconstruct Newton's three laws of motion and some of its ramifications for yourself. In summary the laws are summarized in Table 7-1.

These three laws are incredibly powerful because an understanding of them allows you to either: (1) use a complete knowledge of forces on a system of objects to predict motions in the system or (2) identify the forces on a system

TABLE 7-2: Forces Studied and Notation

Active Forces:

- External Forces (pushes or pulls) $\vec{F}_{ext}$
- Gravitational Force $\vec{F}_{\mathrm{g}} = m\,\vec{g}$
- Centripetal Force $\vec{F}_c = m\,\vec{a}_c = m\dfrac{v^2}{r}\hat{r}$

Passive Forces:

- Tension Force $\vec{F}_{\mathrm{T}}$
- Normal Force $\vec{F}_{\mathrm{norm}}$
- Static Friction Force $F_{\mathrm{f,stat}} = \mu_{\mathrm{f,stat}} F_{\mathrm{norm}}$
- Kinetic Friction Force $F_{\mathrm{f,kin}} = \mu_{\mathrm{f,kin}} F_{\mathrm{norm}}$

of objects based on observations of its motions. In fact, you have already used a belief in Newton's laws to identify several active and passive "invisible" forces". The forces identified so far are shown in table 7-1.

There are other forces such as spring forces, air friction, electrical forces, and magnetic forces which we will study, but for now we will only deal with the forces summarized in Table 7–2.

Using Free Body Diagrams to Predict Motions and Calculate Forces

The key to the effective application of Newton's laws is to identify and diagram all the forces acting on each object in a system of interest. The next step is to define a co-ordinate system and break the forces down into components to take advantage of the fact that if

$$\Sigma\vec{F} = m\,\vec{a} \quad \text{then} \quad \Sigma F_x = m\,a_x \quad \text{and} \quad \Sigma F_y = m\,a_y$$

A *free-body diagram* consists of a set of arrows representing all the forces on an object, but NOT the forces that the object exerts on other objects. To create a free-body diagram you should do as follows:

1. Draw arrows to represent all force acting on the object or objects in the system of interest.

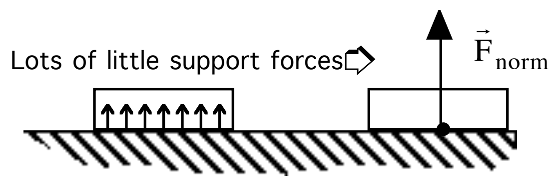
2. Place the tail of each arrow at the point where the force acts on the object.

3. Each arrow should point in the direction of the force it represents.

4. The relative lengths of the arrows should, if possible, be made to correspond to the magnitudes of the forces.

5. A set of co-ordinate axis should be chosen and indicated and all of the arrows should be labelled using standard notation to indicate the type of force involved.

Important Note: The idea of using a single force vector to summarize external forces that act in the same direction is a useful simplification which is not real. For example, when a block rests on a table, we will say that the table exerts a normal force on the block. It is conventional to draw a single upward arrow at the point where the middle of the bottom surface of the block touches the table. This arrow actually represents the sum of all the smaller forces at each point where the block touches the table. This is shown in the diagram below:



An Example of a Free-Body Diagram
Consider a block of mass *m* sliding on a rough inclined plane as shown in the diagram below. It has three forces on it: (1) a gravitational force, (2) a normal force perpendicular to the surface of the plane, and (3) the friction force opposing its motion down the plane.

Since the block is sliding it could either be moving at a constant velocity or with a constant acceleration. Thus, it is possible that the vector sum of forces on it is not equal to zero in some cases. For example, if there is accelerated motion, then:

$$\sum \vec{F} = \vec{F}_{\text{norm}} + \vec{F}_{\text{f,kin}} + \vec{F}_{\text{g}} = m\vec{a}$$

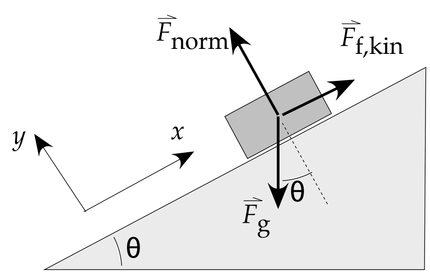
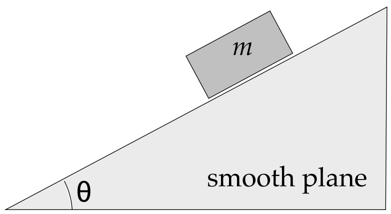


Figure 7-14: An Example of a free-body diagram

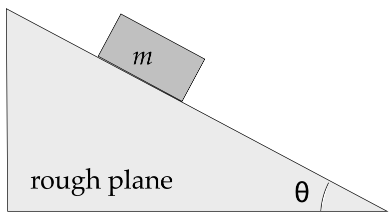
Based on the example in Figure 7-14, try your hand at drawing the free body diagrams for the situations described below. In each case, write the equation for the vector sum of forces.

✍ Activity 7-17: Some Free-Body Diagrams

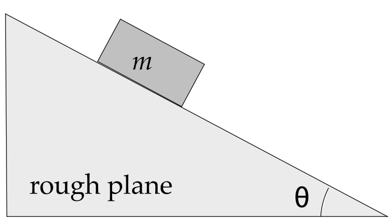
(a) A block slides freely down a smooth inclined plane.



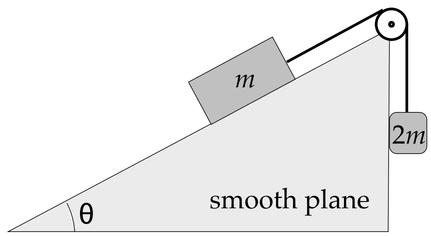
(b) A block is on a rough plane but is not moving due to static friction.



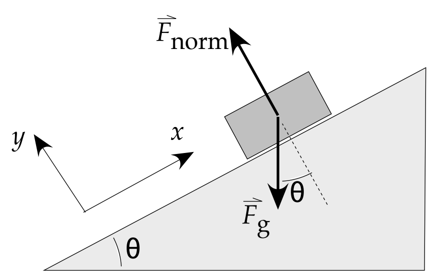
(c) A block is on a rough plane but the coefficient of friction between the block and the plane is small. The block is sliding down the plane at a constant velocity so that kinetic friction is acting.



(d) The block is on a frictionless plane but is attached to a hanging mass of $2m$ by one of our famous "massless" strings over a pulley. Construct free-body diagrams for the forces on the mass m and for the forces on the mass $2m$.



Breaking the Forces into Components – An Example
Consider the case of the block sliding down a "smooth" plane with a negligible amount of friction. The free body diagram and co-ordinate system chosen for analysis are shown in the figure below. [Please note: By convention, the magnitude of a vector is assumed to be positive. In the diagram below, F_g (note the absence of an arrow, which indicates that we are discussing the magnitude of the vector rather than the vector itself) is a positive quantity; when taking components of F_g, it may be necessary to introduce a negative sign.]



Taking components:

$F_{g,x} = -F_g \sin(\theta)$
$F_{g,y} = -F_g \cos(\theta)$

There is no motion perpendicular to the surface so

$\Sigma F_y = 0 = F_{norm} + F_{g,y}$ so that

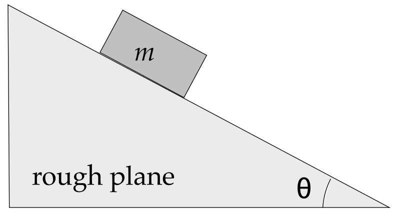
$F_{norm} = - F_{g,y} = F_g \cos(\theta)$

There is no balancing force for the x-component of F_g so according to Newton's second law

$\Sigma F_x = ma_x = - F_g \sin(\theta) = - mg \sin(\theta)$
$a_x = - g \sin(\theta)$

✍ Activity 7-18: Solving an Inclined Plane Problem

(a) Consider a block sliding down a rough inclined plane at a *constant* velocity. What is the net force on it?



(b) Refer to the free-body diagram you created for this situation in the last activity. Break the forces up into components and apply Newton's first law to find the equation for the coefficient of kinetic friction as a function of m, θ, and F_g. You may need to consult a standard textbook for hints on how to tackle this problem.