Classical Mechanics
Lecture 18

Today's Concepts:

a) Static Equilibrium

b) Potential Energy & Stability

Clicker Question

A (static) mobile hangs as shown below. The rods are massless and have lengths as indicated. The mass of the ball at the bottom right is 1 kg.

What is the total mass of the mobile?

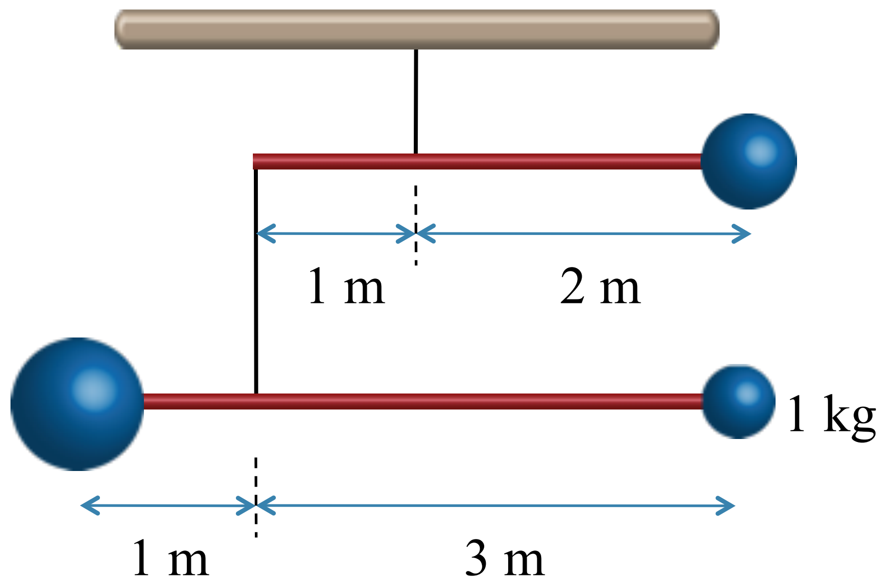
A) 4 kg

B) 5 kg

C) 6 kg

D) 7 kg

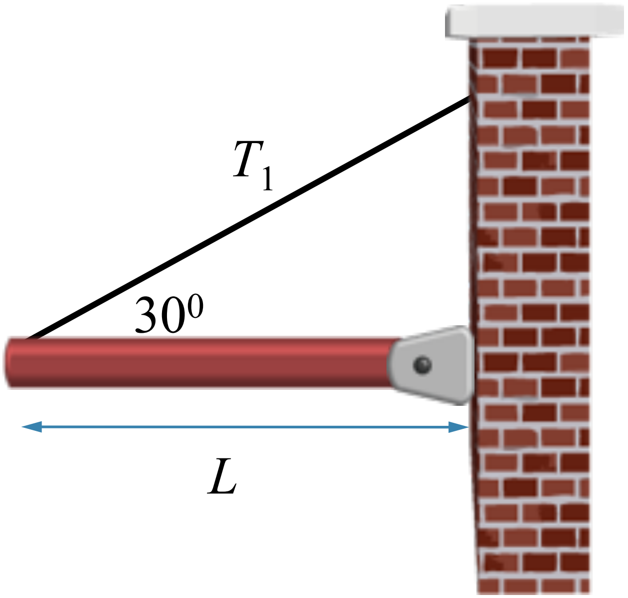
E) 8 kg



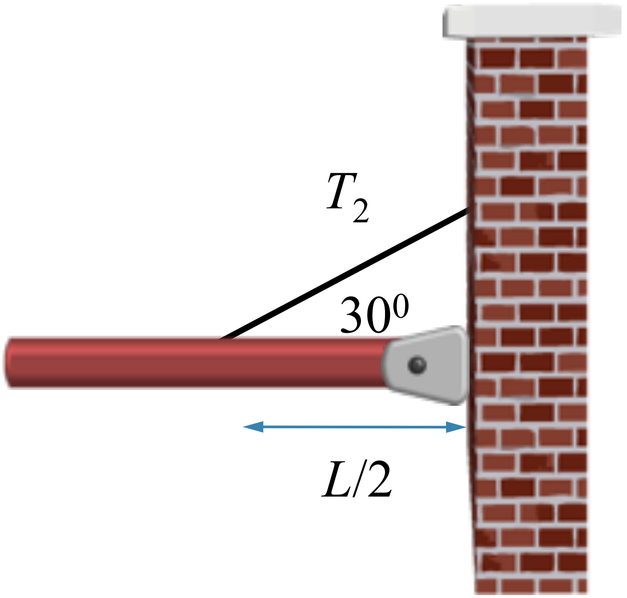
CheckPoint

In which of the static cases shown below is the tension in the supporting wire bigger? In both cases the red strut has the same mass and length.

A) Case 1 B) Case 2 C) Same



Case 1



Case 2

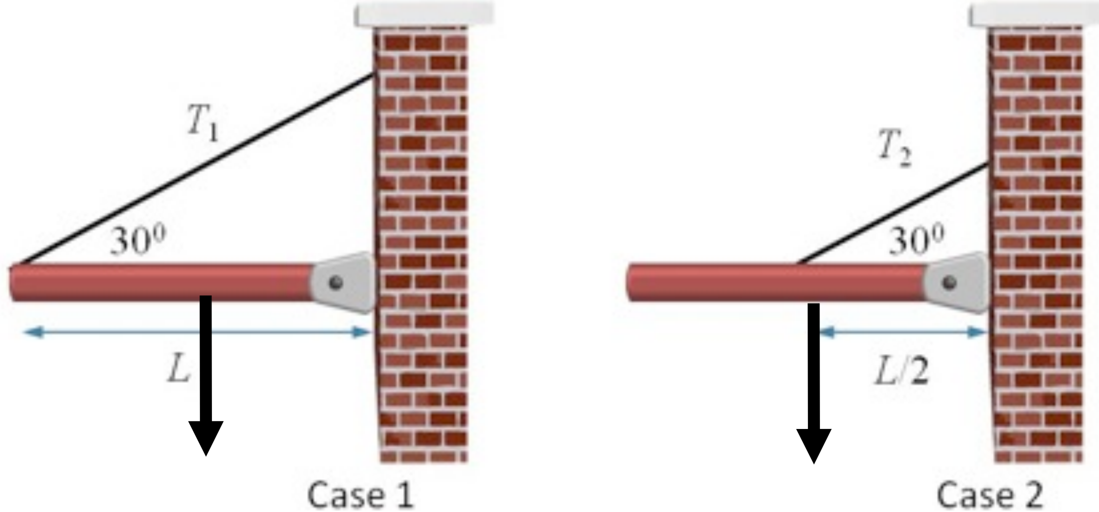
CheckPoint

In which of the static cases shown below is the tension in the supporting wire bigger? In both cases the red strut has the same mass and length.

A) Case 1

B) Case 2

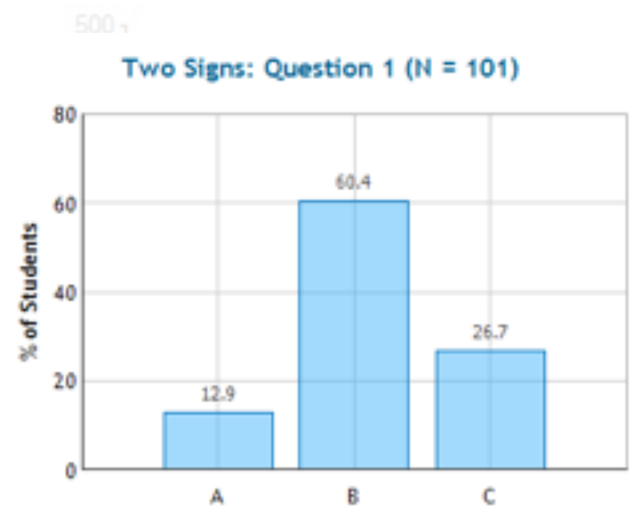
C) Same



A) Case 1 because the lever arm distance is the largest there

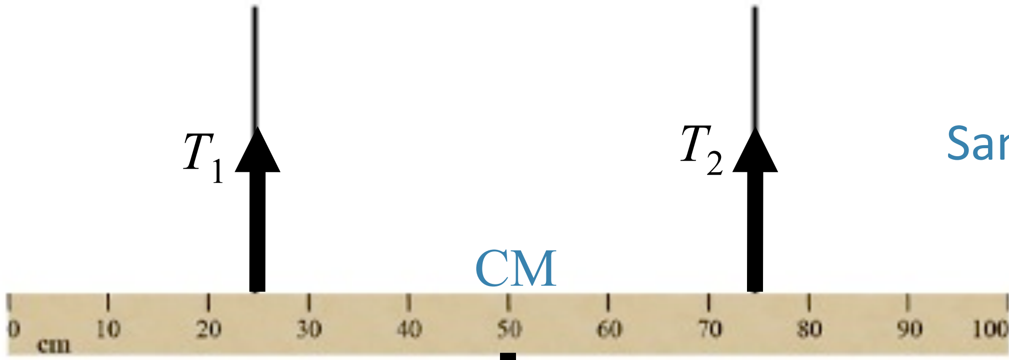
B) The torque caused by gravity is equal to the torque caused by the tension. Since in Case 1, the lever arm is longer, the force does not need to be as long to equalize the torque caused by gravity.

C) The angles are the same so the tensions are the same the lengths do not matter.



Homework Problem

Meterstick



Balance forces: $T_1 + T_2 = Mg$

Same distance from CM: $T_1 = T_2 = T$

So: $T = Mg/2$

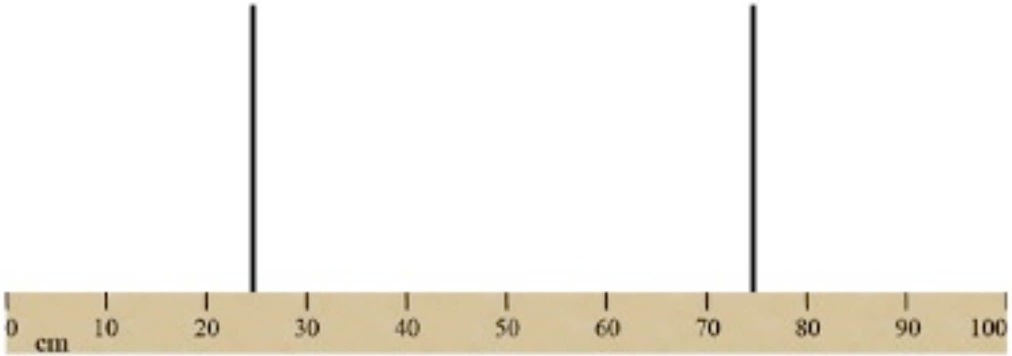
A meterstick (L = 1 m) has a mass of m = 0.133 kg. Initially it hangs from two strings: one at the 25 cm mark and one at the 75 cm mark.

1) What is the tension in the left string?

.6524 N Submit OK ✓ Correct

Homework Problem

Meterstick



A meterstick (L = 1 m) has a mass of m = 0.133 kg. Initially it hangs from two strings: one at the 25 cm mark and one at the 75 cm mark.

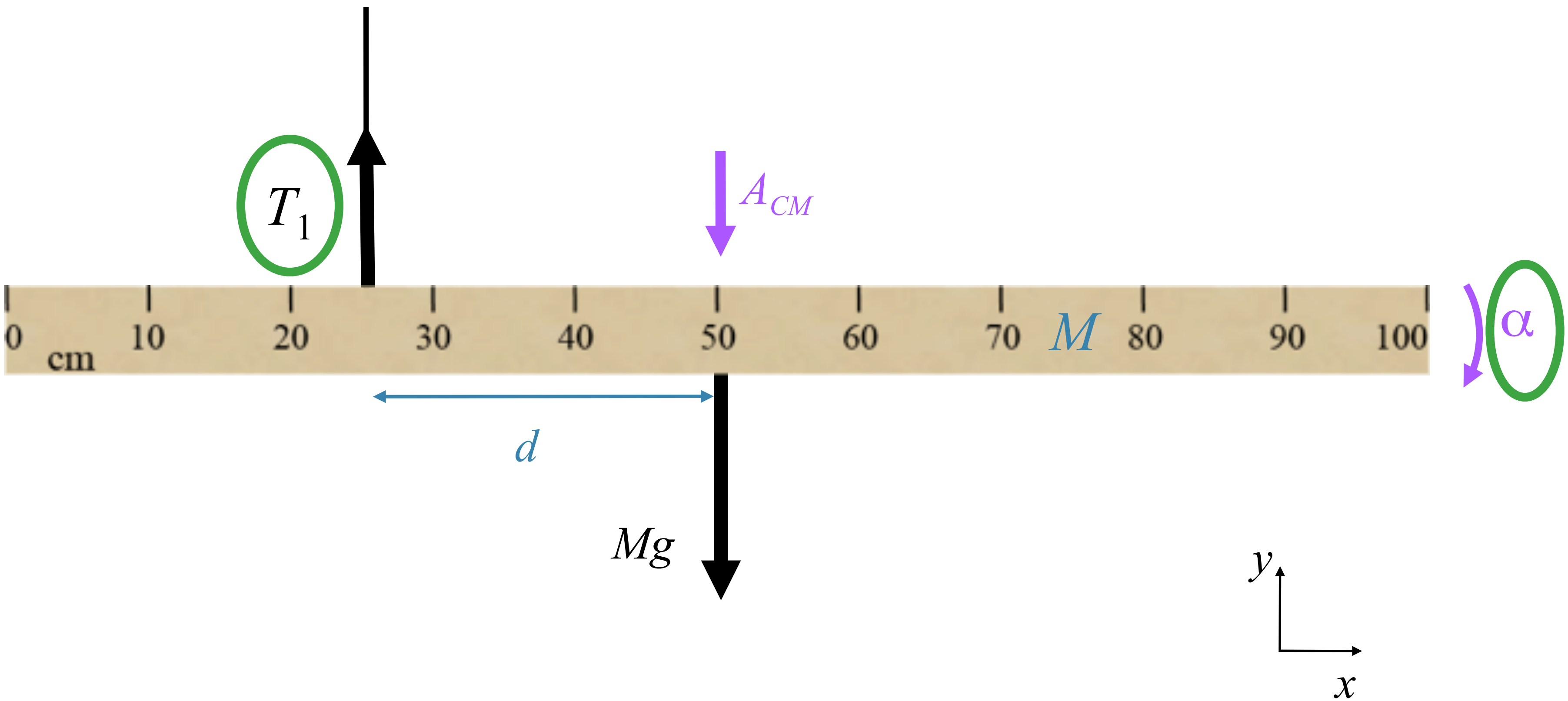
2) Now the right string is cut! What is the initial angular acceleration of the meterstick about its pivot point?

[ ] rad/s^2 Submit

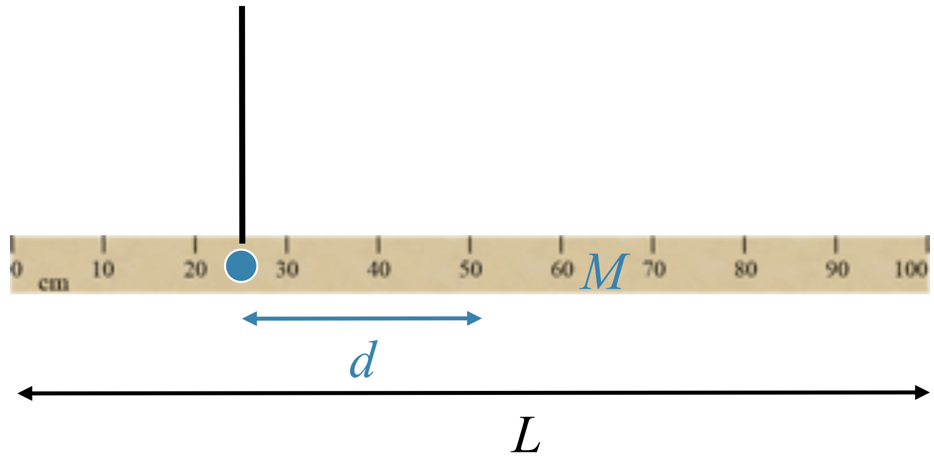
3) What is the tension in the left string right after the right string is cut?

[ ] N Submit

These are the quantities we want to find:



What is the moment of inertia of the beam about the rotation axis shown by the blue dot?



A) $I = \frac{1}{12}ML^2$

B) $I = Md^2$

C) $I = \frac{1}{12}ML^2 + Md^2$

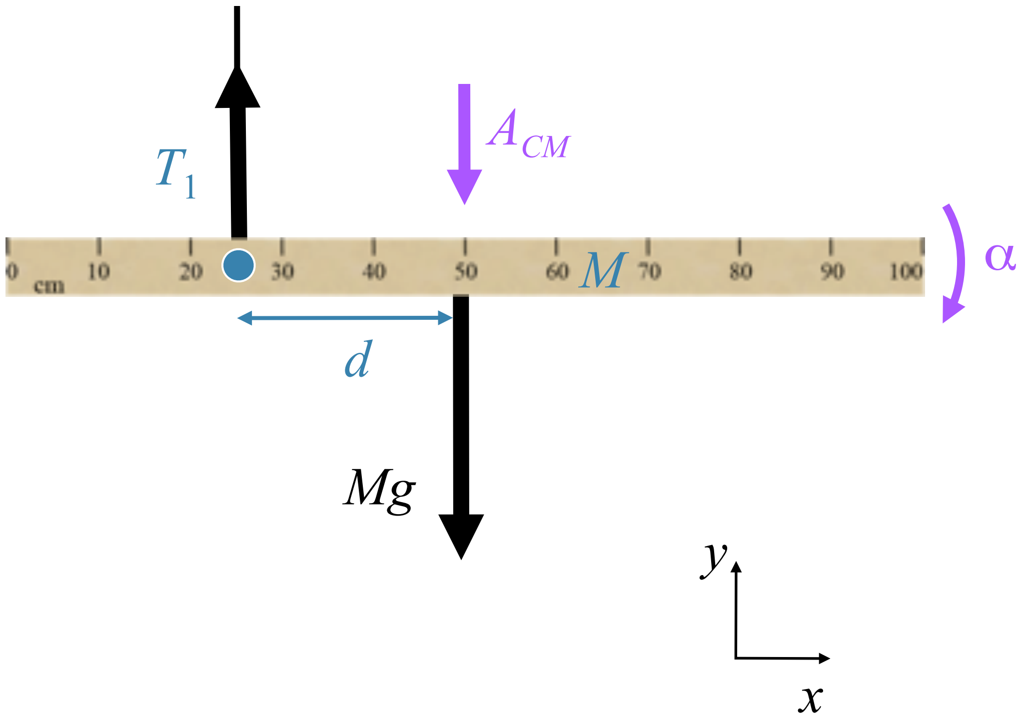
Clicker Question

The center of mass of the beam accelerates downward. Use this fact to figure out how T_1 compares to weight of the beam?

A) $T_1 = Mg$

B) $T_1 > Mg$

C) $T_1 < Mg$



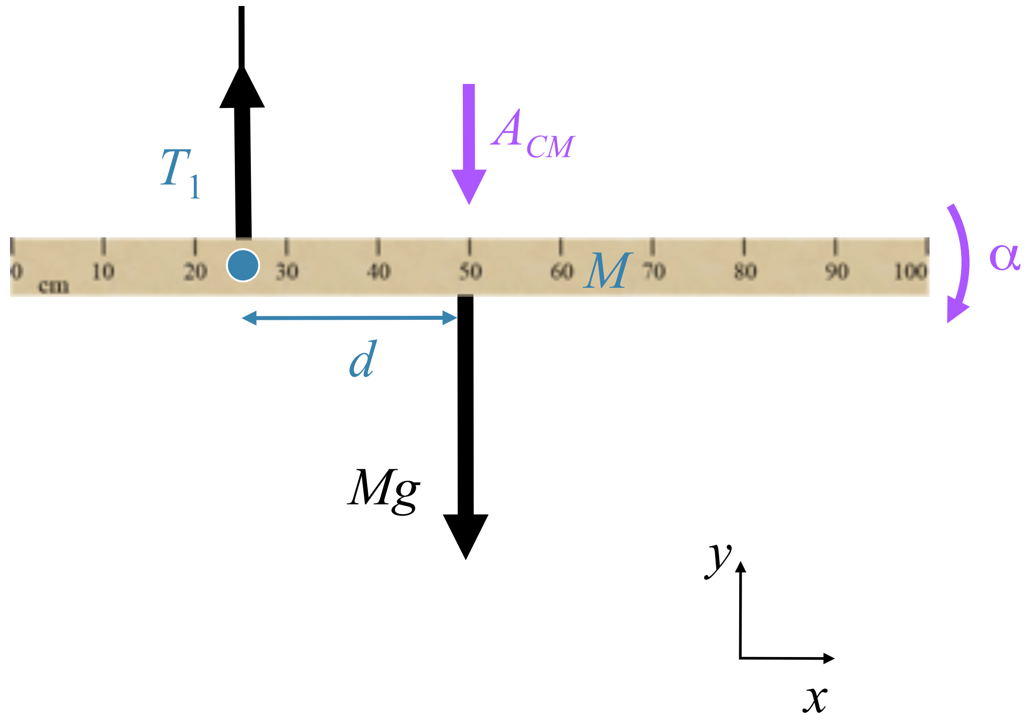
Clicker Question

The center of mass of the beam accelerates downward. How is this acceleration related to the angular acceleration of the beam?

A) $A_{CM} = d\alpha$

B) $A_{CM} = d / \alpha$

C) $A_{CM} = \alpha / d$

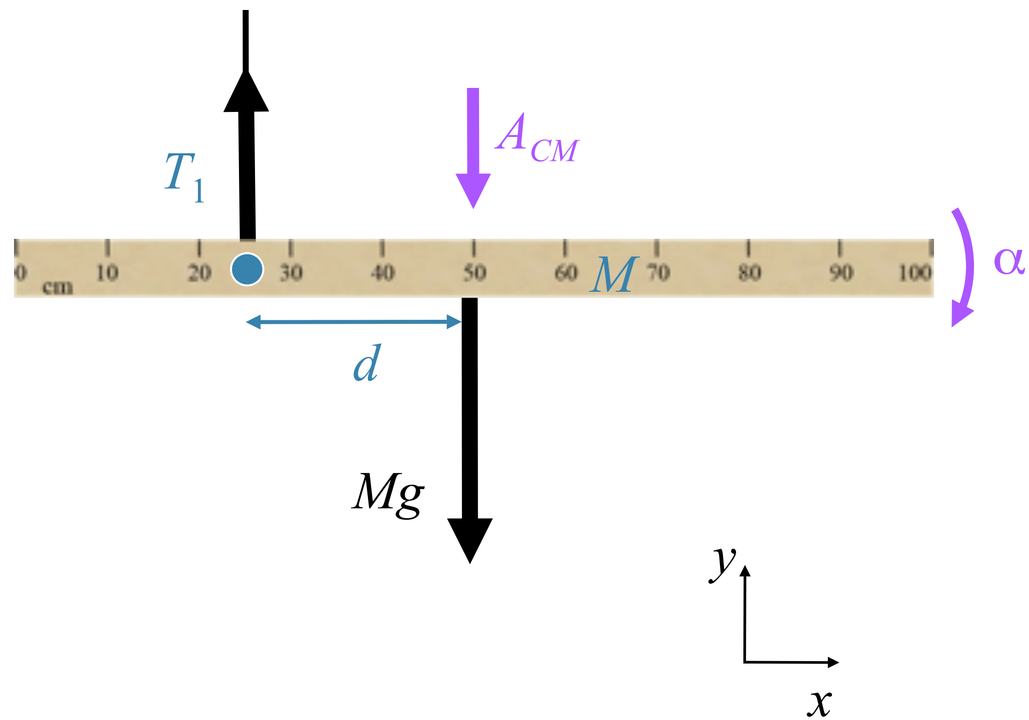


$$A_{CM} = d\alpha$$

Apply $\sum F_{ext} = MA_{CM}$

$$Mg - T_1 = MA_{CM}$$

$$T_1 = Mg - MA_{CM}$$



Apply $\sum \tau_{ext} = I\alpha$

$$Mgd = I\alpha = I\frac{A_{CM}}{d}$$

$$A_{CM} = g\frac{Md^2}{I}$$

Use $A_{CM} = d\alpha$ to find α

Plug this into the expression for T_1

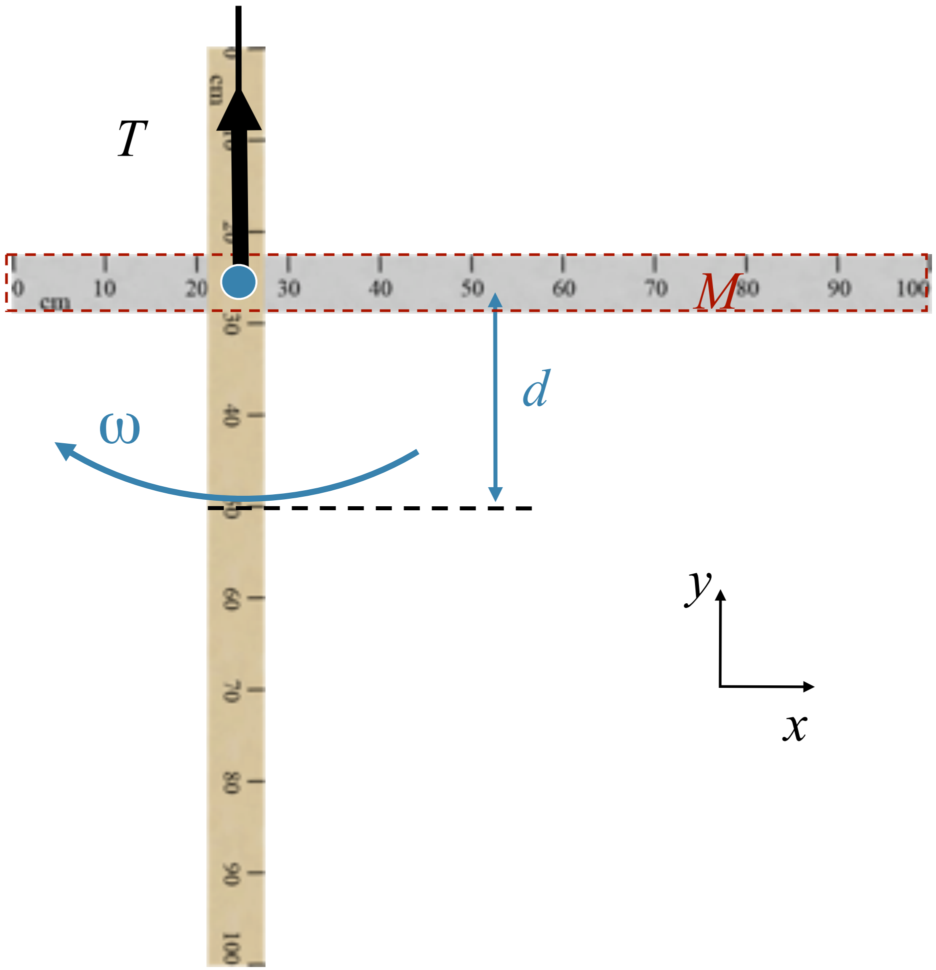
After the right string is cut, the meterstick swings down to where it is vertical for an instant before it swings back up in the other direction.

What is the angular speed when the meter stick is vertical?

Conserve energy:

$$Mgd = \frac{1}{2} I\omega^2$$

$$\omega = \sqrt{\frac{2Mgd}{I}}$$



CM demos

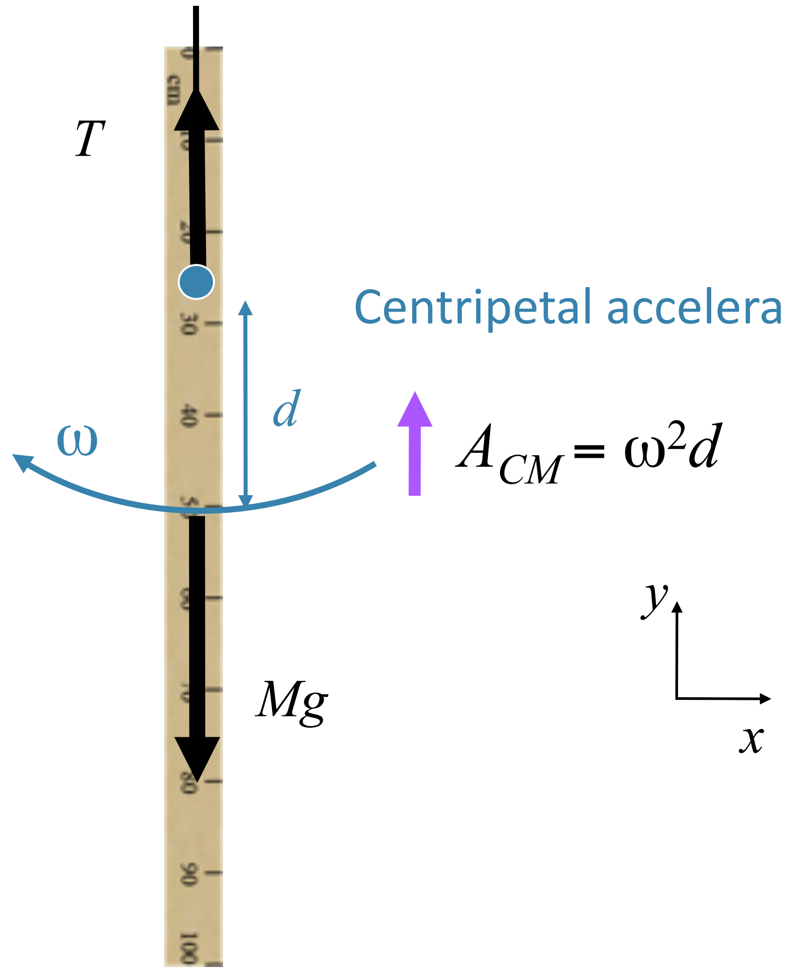
5) What is the acceleration of the center of mass of the meterstick when it is vertical?

6) What is the tension in the string when the meterstick is vertical?

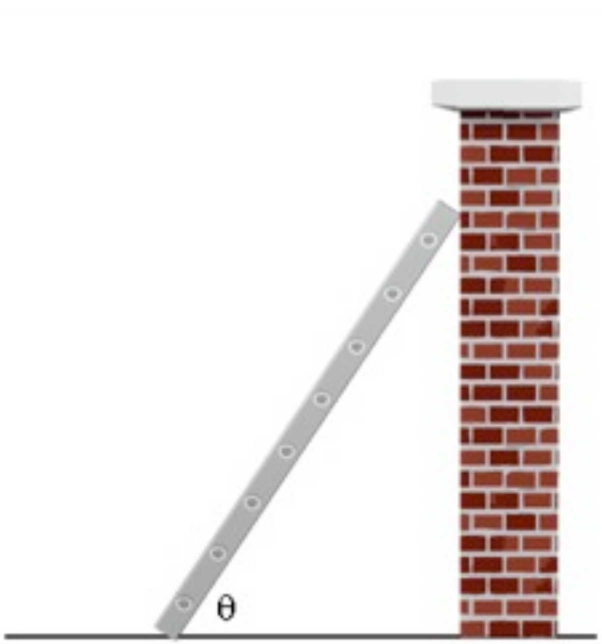
Applying $\sum F_{ext} = MA_{CM}$

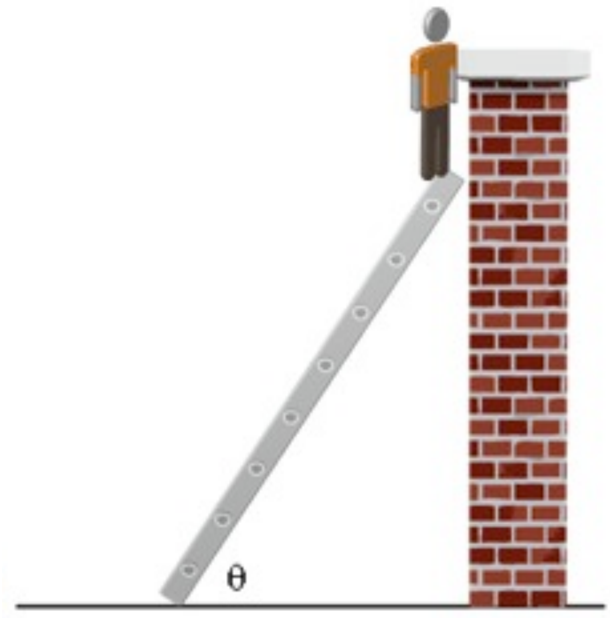
$$T - Mg = M\omega^2 d$$

$$T = Mg + M\omega^2 d$$



Another HW problem:





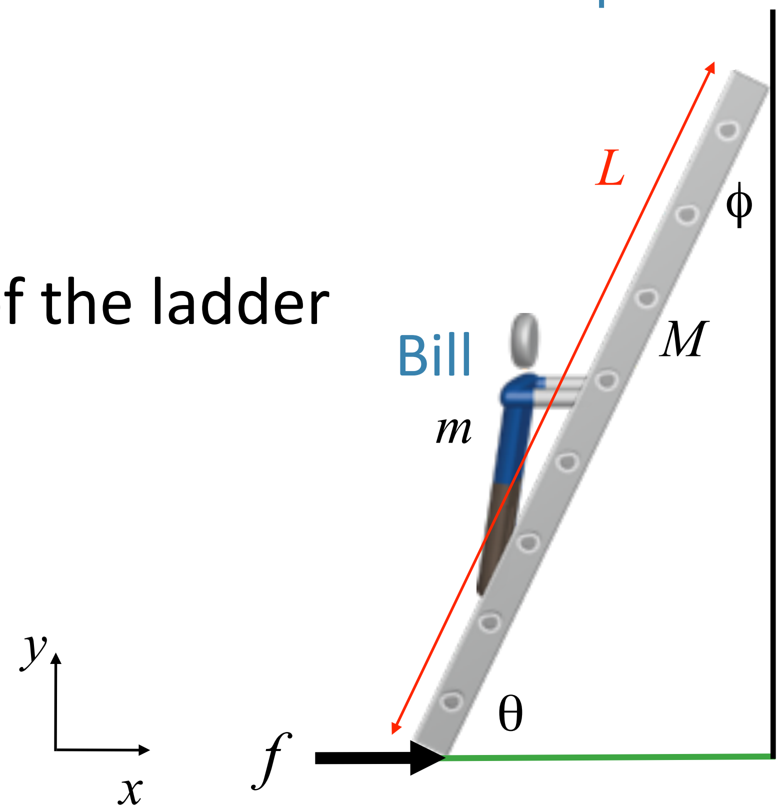
We will now work out the general case

General Case of a Person on a Ladder

Bill (mass m) is climbing a ladder (length L, mass M) that leans against a smooth wall (no friction between wall and ladder). A frictional force f between the ladder and the floor keeps it from slipping. The angle between the ladder and the wall is ϕ.

(The wall is frictionless.)

How does f depend on the angle of the ladder and Bill's distance up the ladder?



Balance forces:

x: $$F_{wall} = f$$

y: $$N = Mg + mg$$

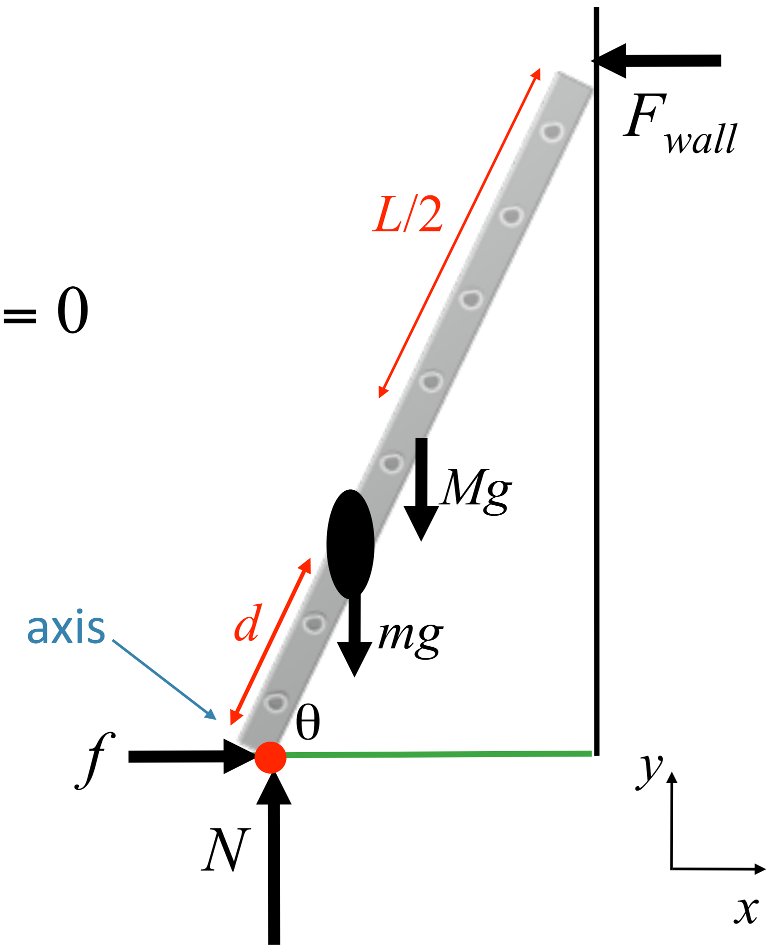
Balance torques:

$$mgd\cos\theta + Mg\frac{L}{2}\cos\theta - F_{wall}\,L\sin\theta = 0$$

$$F_{wall} = \left(mg\frac{d}{L} + \frac{Mg}{2}\right)\cot\theta$$

$$F_{wall} = f$$

$$f = \left(mg\frac{d}{L} + \frac{Mg}{2}\right)\cot\theta$$



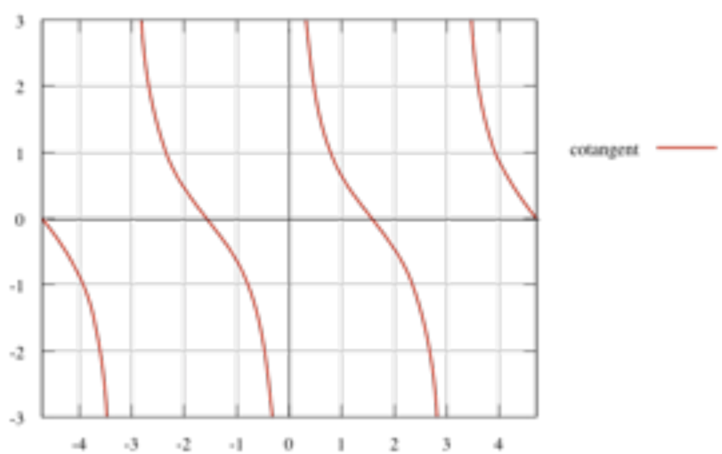
This is the General Expression:

$$f = \left(mg\frac{d}{L} + \frac{Mg}{2} \right) \cot\theta$$

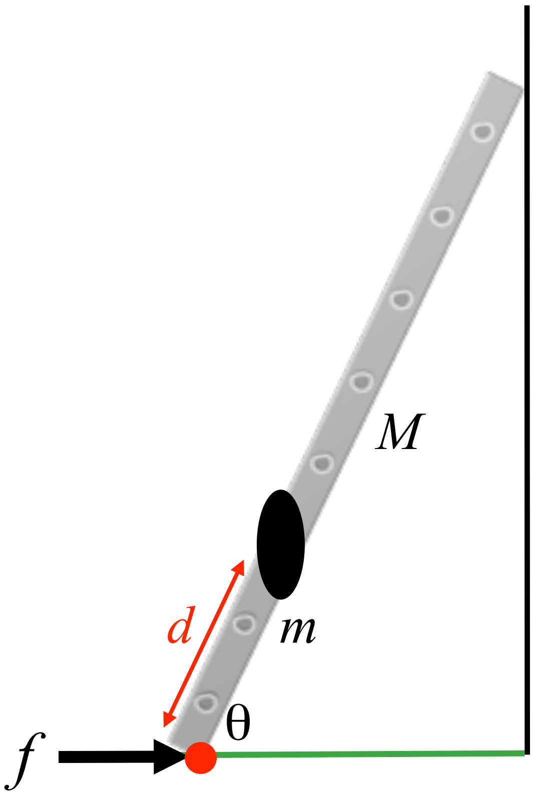
Climbing further up the ladder makes it more likely to slip:

Making the ladder more vertical makes it less likely to slip:

cotangent θ



θ (radians)

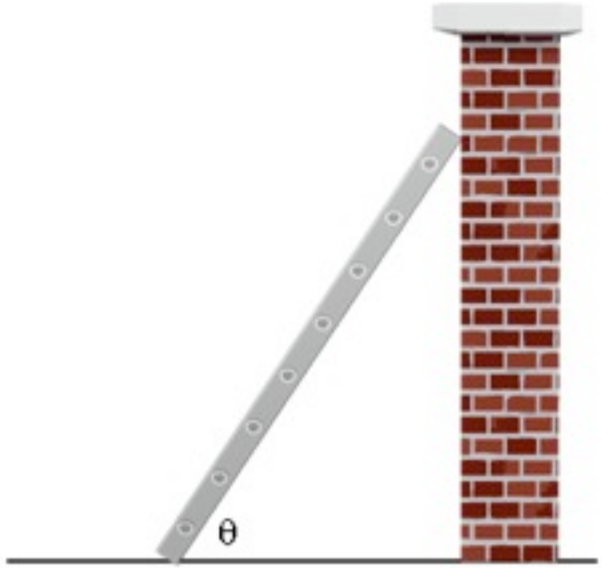


Lets try it out

If its just a ladder…

$$f = \left(mg\frac{d}{L} + \frac{Mg}{2}\right)\cot\theta \rightarrow$$

$$f = \frac{Mg}{2}\cot\theta$$



Moving the bottom of the ladder further from the wall makes it more likely to slip:

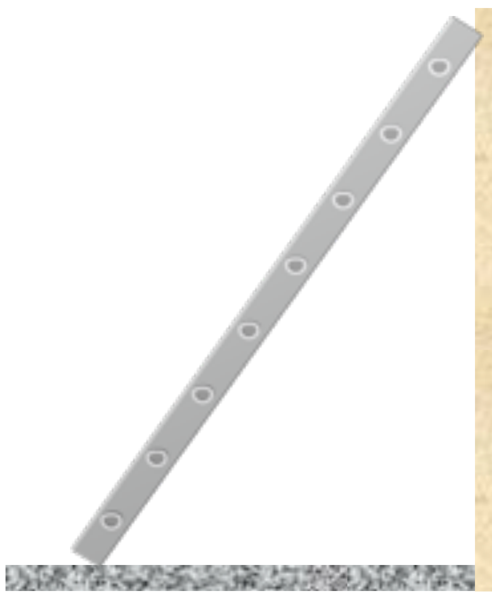
CheckPoint

In the two cases shown below identical ladders are leaning against frictionless walls. In which case is the force of friction between the ladder and the ground the biggest?

A) Case 1

B) Case 2

C) Same



Case 1

Case 2

In the two cases shown below identical ladders are leaning against frictionless walls. In which case is the force of friction between the ladder and the ground the biggest?

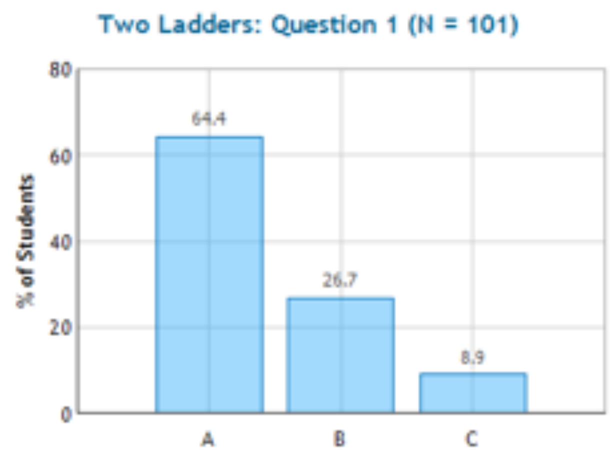
A) Case 1   B) Case 2   C) Same

Case 1   Case 2

A) Because the bottom of the ladder is further away from the wall.

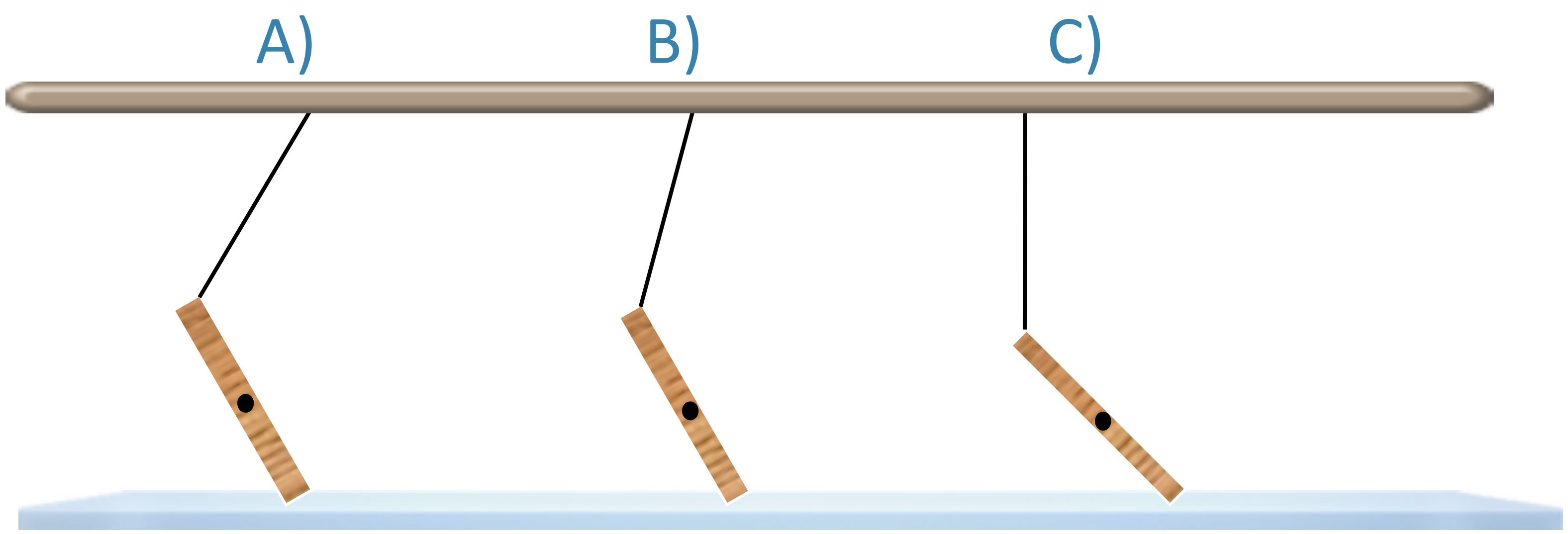
B) The angle is steeper, which means there is more normal force and thus more friction.

C) Both have same mass.



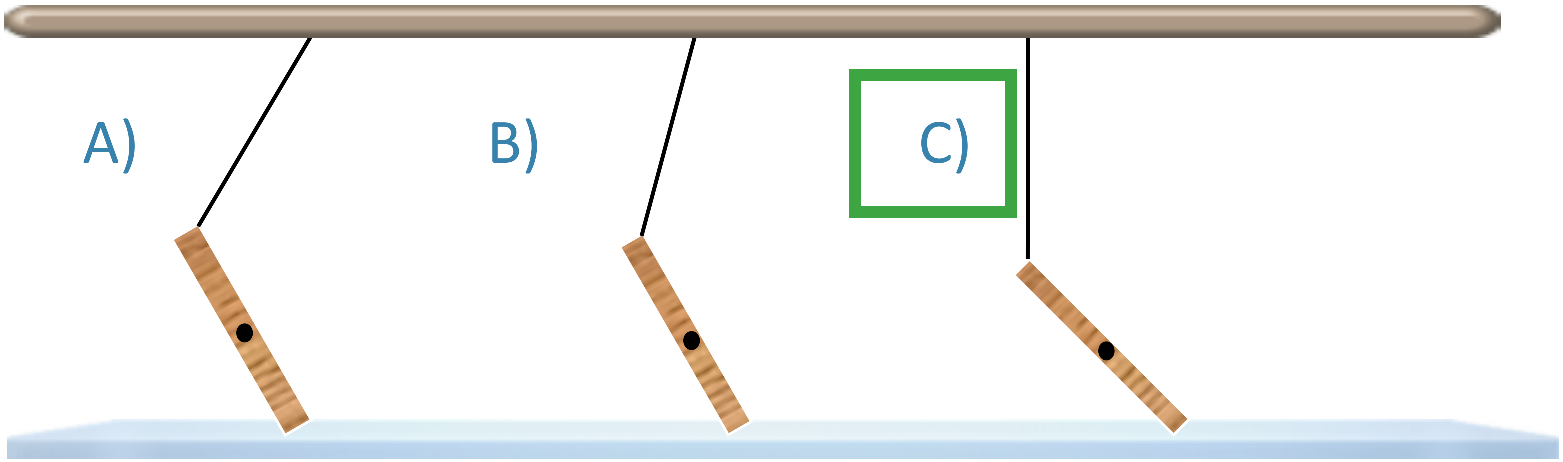
CheckPoint

Suppose you hang one end of a beam from the ceiling by a rope and the bottom of the beam rests on a frictionless sheet of ice. The center of mass of the beam is marked with an black spot. Which of the following configurations best represents the equilibrium condition of this setup?



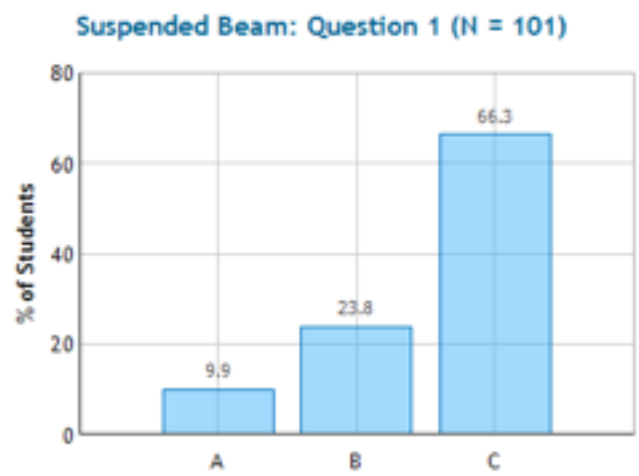
CheckPoint

Which of the following configurations best represents the equilibrium condition of this setup?

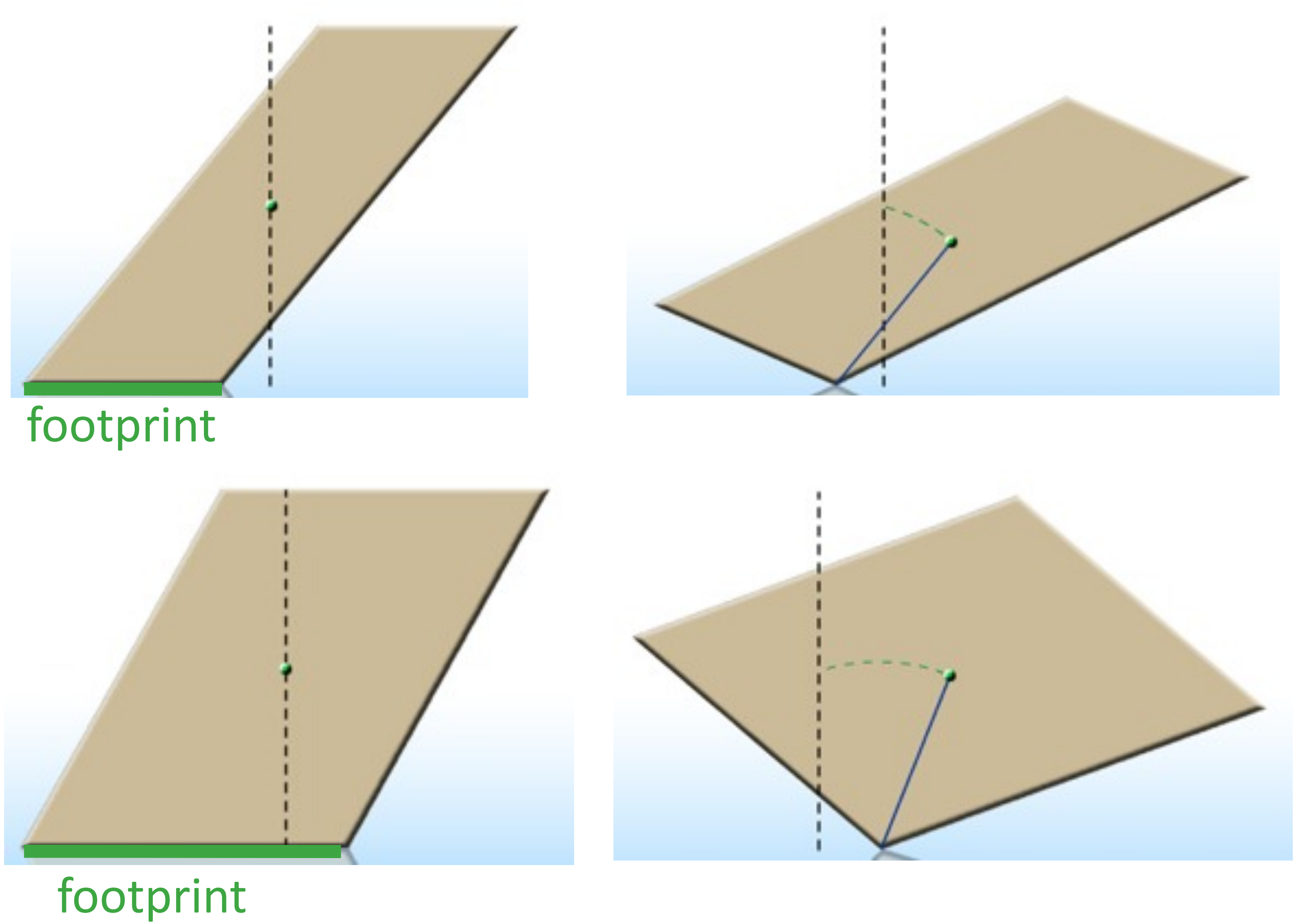


If the tension has any horizontal component, the beam will accelerate in the horizontal direction.

Objects tend to attempt to minimize their potential energy as much as possible. In Case C, the center of mass is lowest.



Stability & Potential Energy



A Question of Balance

Picking up $20

Everyone loves to pick up extra money. We bet you can't stand with your heels touching both a wall, the floor and *each other*, and then bend over (without bending your knees!) and pick up a $20 dollar bill that's lying in front of you without moving your heels away from the floor and the wall. (No fair using a wall with a baseboard either!) You must be able to resume your upright position again without having moved your heels. We're sure enough that this task is very difficult to stake money on it! All of you taking calculus-based introductory physics this semester who can perform this task before the end of the class period under the sharp eye of a bona fide instructor can share the $20 with any others taking the course who can also do the "pickup" job!