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SESSION ONE: CIRCULAR MOTION AND CENTRIPETAL FORCE

10 min

Moving in a Circle at a Constant Speed

When a race car speeds around a circular track, or when David twirled a stone at the end of a rope to clobber Goliath, or when a planet like Venus orbits the sun, they undergo *uniform circular motion*. Understanding the forces which govern orbital motion has been vital to astronomers in their quest to understand the laws of gravitation.

But we are getting ahead of ourselves, for as we have done in the case of linear and projectile motion we will begin our study by considering situations involving external applied forces that lead to circular motion in the absence of friction. We will then use our belief in Newton's laws to see how the circular motions of the planets can be used to help astronomers discover the laws of gravitation.

Figure 7-2: Uniform circular motion. A ball moving at a constant speed in a circle of radius *r*.

Let's begin our study with some very simple considerations. Suppose an astronaut goes into outer space, ties a ball to the end of a rope, and spins the ball so that it moves at a constant *speed*.

✍ Activity 7-1: Uniform Circular Motion

(a) Consider Fig. 7-2. What is the speed of a ball that moves in a circle of radius $r = 2.5$ m if it takes 0.50 s to complete one revolution?

(b) The *speed* of the ball is constant! Would you say that this is accelerated motion?

(c) What is the *definition* of acceleration? (Remember that acceleration is a vector!)

(d) Are *velocity* and *speed* the same thing? Is the *velocity* of the ball constant? (**Hint:** Velocity is a vector quantity!)

(e) In light of your answers to (c) and (d), would you like to change your answer to part (b)? Explain.

15 min

Using Vectors to Diagram How Velocity Changes
By now you should have concluded that since the *direction* of the motion of the ball is constantly changing, its velocity is also changing and thus it is accelerating. We would like you to figure out how to calculate the *direction* of the acceleration and its magnitude as a function of the speed v of the ball as it revolves and as a function of the radius of the circle in which it revolves. In order to use vectors to find the direction of velocity change in circular motion, let's review some rules for adding velocity vectors.

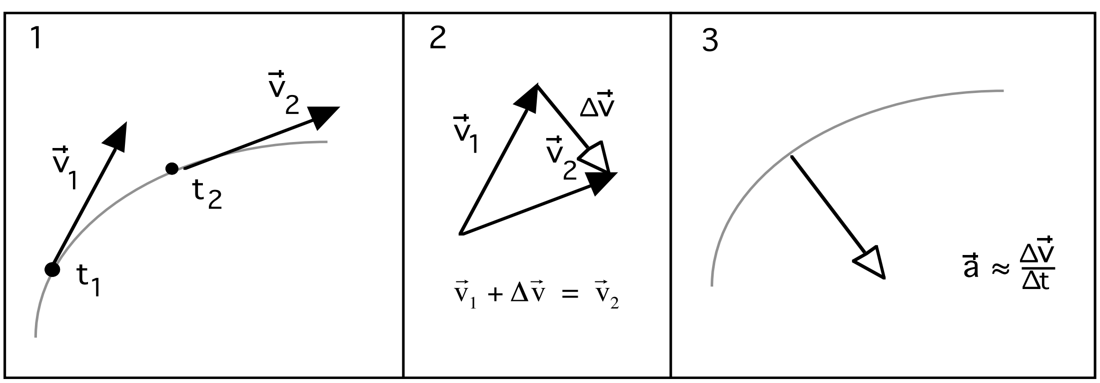


Figure 7-3

1. **Velocities:** Draw an arrow representing the velocity, $\vec{v}_1$, of the object at time t_1. Draw another arrow representing the velocity, $\vec{v}_2$, of the object at time t_2.

2. **Velocity Change:** Find the change in the velocity $\Delta\vec{v} = \vec{v}_2 \pm \vec{v}_1$ during the time interval $\Delta t = t_2 - t_1$. Start by using the rules of vector sums to rearrange the terms so that $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$. Next place the tails of the two velocity vectors together halfway between the original and final location of the object. The change in velocity is the vector which points from the head of the first velocity vector to the head of the second velocity vector. [This method of subtraction is equivalent to the other method we learned earlier.]

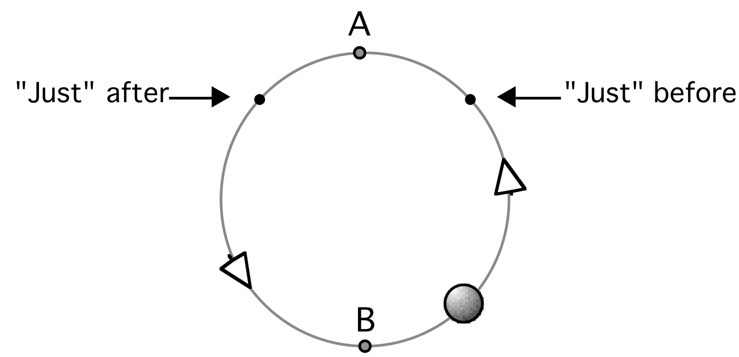
3. **Acceleration:** The acceleration equals the velocity change $\Delta\vec{v}$ divided by the time interval Δt needed for the change. Thus, is in the same direction as Δv but is a different length unless ($\Delta t = 1$). Thus, even if you do not know the time interval, you can still determine the direction of the acceleration because it points in the same direction as $\Delta\vec{v}$.

The acceleration associated with uniform circular motion is known as *centripetal acceleration*. You should use the vector diagram technique to find its direction.

✍ Activity 7-2: The Direction of Centripetal Acceleration

(a) Determine the direction of motion of the ball shown below if it is moving counter-clockwise at a constant speed. Note that the direction of the ball's velocity is always tangential to the circle as it moves around. Draw an arrow representing the direction and magnitude of the ball's velocity as it passes the dot *just before* it reaches point A. Label this vector.

 Supported by FIPSE (U.S. Dept. of Ed.) and NSF. Modified for SFU by N. Alberding, 2005.



(b) Next, use the same diagram to draw the arrow representing the velocity of the ball when it is at the dot *just after* it passes point A. Label this vector .

(c) Find the direction and magnitude of the change in velocity as follows. In the space below, make an exact copy of both vectors, placing the tails of the two vectors together. (See Figure 7-3, diagram 2.) Next, draw the vector that must be added to vector $\vec{v}_1$ to add up to vector $\vec{v}_2$; label this vector $\Delta\vec{v}$. *Be sure that vectors* $\vec{v}_1$ *and* $\vec{v}_2$ *have the same magnitude and direction in this drawing that they had in your drawing in part (a)!* (Again, see Figure 7-3.)

(d) Now, draw an exact copy of $\Delta\vec{v}$ on your sketch in part (a). Place the tail of this copy at point A. Again, make sure that your copy has the exact magnitude and direction as the original $\Delta\vec{v}$ in part (c).

(e) Now that you know the direction of the change in velocity, what is the direction of the centripetal acceleration, $\vec{a}_c$?

(f) If you redid the analysis for point B at the opposite end of the circle, what do you think the direction of the centripetal acceleration, $\vec{a}_c$, would be now?

(g) As the ball moves on around the circle, what is the direction of its acceleration?

(h) Use Newton's second law in vector form ($\Sigma\vec{F} = m\vec{a}$) to describe the direction of the net *force* on the ball as it moves around the circle.

(i) If the ball is being twirled around on a string, what is the source of the net force needed to keep it moving in a circle?

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20 min

Using Mathematics to Derive How Centripetal Acceleration Depends on Radius and Speed.

You haven't done any experiments yet to see how centripetal acceleration depends on the radius of the circle and the speed of the object. You can use an understanding of Newton's second law to get a feel for what the mathematical relationships might be. You can then use the rules of mathematics and the definition of acceleration to *derive* the relationship between speed, radius, and magnitude of centripetal acceleration.

✍ Activity 7-3: How Does a_c Depend on v and r?

(a) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a_c, have to increase or decrease as r decreases if circular motion is to be maintained? Explain.

(b) Do you expect you would need more centripetal acceleration or less centripetal acceleration to cause an object to rotate at a given radius r if the speed v is increased? In other words, would the magnitude, a_c, have to increase or decrease as v increases if circular motion is to be maintained? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Let's use the definition of acceleration in two-dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration should actually be given by the equation

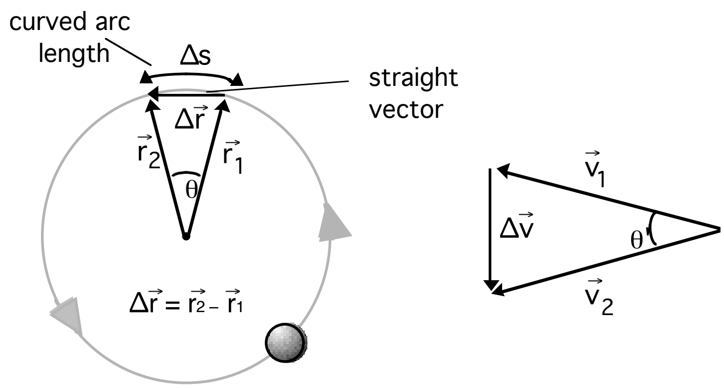
$$a_0 = \frac{v^2}{r} \qquad \text{[Eq. 7-1]}$$

In order to do this derivation you will want to use the following definition for acceleration

$$< \vec{a} > \equiv \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \qquad \text{[Eq. 7-2]}$$

✍ Activity 7-4: Finding the Equation for a_c

(a) Refer to the diagram below. Explain why, at the two points shown on the circle, the angle between the displacement vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. **Hint**: In circular motion, velocity vectors are always perpendicular to their displacement vectors.



(b) Since the angles are the same and since the magnitudes of the displacements never change (i.e. $r = r_1 = r_2$) and the magnitudes of the velocities never change (i.e. $v = v_1 = v_2$), use the properties of similar triangles to explain why

$$\frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{r}$$

(c) Now use the equation in part (b) and the definition of $< a >$ to

show that $< a_c > = \dfrac{|\Delta\vec{v}|}{\Delta t} = \dfrac{|\Delta\vec{r}|}{(\Delta t)}\dfrac{v}{r}$

(d) The speed of the object as it rotates around the circle is given by

$$v = \frac{\Delta s}{\Delta t}$$

Is the change in arc length, Δs, larger or smaller than the magnitude of the change in the position vector, Δr? Explain why the arc length change and the change in the position vector are approximately the same when Δt is very small (so that the angle θ becomes very small) i.e. why is $\Delta s \approx \Delta r$?

(e) If $\Delta s \approx \Delta r$, then what is the equation for the speed in terms of Δr and Δt?

(f) Using the equation in part (c), show that as $\Delta t \to 0$, the instantaneous value of the centripetal acceleration is given by the equation

$$a_0 = \frac{v^2}{r}$$

(g) If the object has a mass m, what is the equation for the magnitude of the centripetal force needed to keep the object rotating in a circle (in terms of v, r, and m)? In what direction does this force point as the object rotates in its circular orbit?