

$$*17] \quad W(e^{2t}, g) = 3e^{4t} \\ = e^{2t} g'(t) - 2e^{2t} g(t) \quad \left. \vphantom{W(e^{2t}, g)} \right\} g' - 2g = 3e^{2t} \rightarrow g(t) = 3te^{2t} + ce^{2t}$$

*22] $y'' + 4y' + 3y = 0, t_0 = 1$.
 General solution $y = c_1 e^{-3t} + c_2 e^{-t}$. $W(e^{-3t}, e^{-t}) = 2e^{-4t}$, hence are linearly independent. Now we look for a combination c_1, c_2 to produce

$$y_1(1) = 1, y_1'(1) = 0 \rightarrow c_1 = -e^{3/2}, c_2 = 3e^{1/2}$$

$$y_2(1) = 0, y_2'(1) = 1 \rightarrow c_1 = -e^{3/2}, c_2 = e^{1/2}$$

So the fundamental set is $-\frac{1}{2}e^{-3(t-1)} + \frac{3}{2}e^{-(t-1)}, -\frac{1}{2}e^{-3(t-1)} + \frac{1}{2}e^{-(t-1)}$

*24] $y'' - 2y' + y = 0, y_1(t) = e^t, y_2(t) = te^t$
 Check that both y_1 and y_2 satisfy the ODE.
 Then, $W(y_1, y_2) = e^{2t} \neq 0$. Hence, form a fundamental set.

3.3] #3] $f(t) = e^{\lambda t} \cos \mu t, g(t) = e^{\lambda t} \sin \mu t, \mu \neq 0$
 Suppose that $e^{\lambda t} \cos \mu t = Ae^{\lambda t} \sin \mu t$ for some $A \neq 0$, on an interval I .
 Since $\sin \mu t \neq 0$ on some subinterval $I_0 \subset I$, $\tan \mu t = A$ on I_0 .
 But $\tan x$ is not constant on any interval. This contradiction means that an assumption $e^{\lambda t} \cos \mu t = Ae^{\lambda t} \sin \mu t$ is false.

*9] $W(t) = t \sin^2 t$ has only isolated zeros (only points), so $W(t)$ cannot vanish identically on any open interval. Hence, functions are independent.

*11] $c_1 y_1 + c_2 y_2$ are also solutions. $W(c_1 y_1, c_2 y_2) = (c_1 y_1)(c_2 y_2)' - (c_2 y_2)(c_1 y_1)' = c_1 c_2 W(y_1, y_2)$.
 Since $W(y_1, y_2)$ is not identically zero, neither is $W(c_1 y_1, c_2 y_2)$.

*16] $(\cos t)y'' + (\sin t)y' - ty = 0, p(t) = \sin t / \cos t \rightarrow W(t) = c \exp\left(-\int \frac{\sin t}{\cos t} dt\right)$
 $= c \exp(\ln |\cos t|) = c \cos t$.

*25] y_1 and y_2 are differentiable. So, $y_1'(t_0) = y_2'(t_0) = 0$ at some t_0 in the interval of definition. This implies $W(y_1, y_2)(t_0) = 0$. But $W(y_1, y_2)(t) = c \exp\left(-\int p(t) dt\right)$ which cannot be equal to zero unless $c = 0$.
 Hence, $W(y_1, y_2) \equiv 0$, which cannot happen for a fundamental set.