Math 310 Practice problems

$\boxed{3.1}$ #21 $y'' - y - 2y = 0, \quad y(0) = \alpha, \; y'(0) = 2$

$r^2 - r - 2 = 0 \;\rightarrow\; r_1 = 2 \quad r_1 = -1$

$y = c_1 e^{2t} + c_2 e^{-t}$ $\quad \boxed{\text{We want } c_1 = 0 \quad \text{so } y(t) \to 0 \text{ as } t \to \infty}$ *

$$\left.\begin{aligned} y(0) &= c_1 + c_2 = \alpha \\ y'(0) &= 2c_1 - c_2 = 2 \end{aligned}\right\} \quad c_1 = \frac{2+\alpha}{3} \qquad c_2 = \frac{4\alpha - 2}{3}$$

So, $c_1 = 0 \Leftrightarrow \underline{\underline{\alpha = -2}}$

#22 $4y'' - y = 0, \quad y(0) = 2, \; y'(0) = \beta.$

$4r^2 - 1 = 0 \;;\; r_1 = \frac{1}{2}, \; r_2 = -\frac{1}{2}$

$y = c_1 e^{\frac{1}{2}t} + c_2 e^{-\frac{1}{2}t} \;;$ $\boxed{\text{We want } c_1 = 0}$

Initial data: $\left.\begin{aligned} c_1 + c_2 &= 2 \\ \tfrac{1}{4}c_1 - \tfrac{1}{4}c_2 &= \beta \end{aligned}\right\} \quad \begin{aligned} c_1 &= 1 + \beta \\ c_2 &= 1 - \beta \end{aligned} \quad \Rightarrow \underline{\beta = -1}$

#23 $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0$

$r^2 - (2\alpha - 1)r + \alpha(\alpha - 1) = 0 \;;\; r = \alpha, \alpha - 1$

$y(t) = c_1 e^{\alpha t} + c_2 e^{(\alpha - 1)t} \;;$

All solutions $\to 0$ as $t \to \infty$ if $\alpha < 0$

All solutions $\to \infty$ as $t \to \infty$ if $\alpha - 1 > 0$ i.e. $\underline{\alpha > 1}$

} All solutions; need to consider both terms

$\boxed{3.2}$ #5 $W(e^t \sin t, e^t \cos t) = \begin{vmatrix} e^t \sin t & e^t \cos t \\ e^t(\sin t + \cos t) & e^t(\cos t - \sin t) \end{vmatrix} = -e^{2t}$

#7 $ty'' + 3y = t, \quad y(1) = 1, \; y'(1) = 2.$

$\rightarrow y'' + \frac{3}{t}y = 1, \quad p(t) = \frac{3}{t}$ is continuous for all $t > 0$. Since $t_0 = 1 > 0$, the IVP has a unique solution for all $t > 0$ (Theorem 3.2.1)

#9 $t(t-4)y'' + 3ty' + 4y = 2, \; y(3) = 0, \; y'(3) = -1.$

$\rightarrow y'' + \frac{3}{t-4}y' + \frac{4t}{t-4} = \frac{2}{t(t-4)}$. $p(t)$, $q(t)$ not continuous at $t = 0, 4$.

Since $t_0 = 3 \in (0, 4)$, the longest interval is $0 < t < 4$.