

3.1 #13] $y'' + 5y' + 3y = 0$, $y(0) = 1$, $y'(0) = 0$

Characteristic equation: $r^2 + 5r + 3 = 0 \rightarrow r = \frac{-5 \pm \sqrt{13}}{2} = r_1, r_2$

General solution $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$, $y' = r_1 c_1 e^{r_1 t} + r_2 c_2 e^{r_2 t}$ $\left\{ \begin{array}{l} c_1 + c_2 = 1 \\ r_1 c_1 + r_2 c_2 = 0 \end{array} \right.$

Using initial conditions we find $c_1 = \frac{1 - \frac{5}{\sqrt{13}}}{2}$, $c_2 = \frac{1 + \frac{5}{\sqrt{13}}}{2}$.

#14] $2y'' + y' - 4y = 0$, $y(0) = 0$, $y'(0) = 1$

Characteristic eq: $2r^2 + r - 4 = 0 \rightarrow r = \frac{-1 \pm \sqrt{33}}{4} = r_1, r_2$

General solution: $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$
 $y' = r_1 c_1 e^{r_1 t} + r_2 c_2 e^{r_2 t}$

Using initial conditions; $c_1 = \frac{-2}{\sqrt{33}}$, $c_2 = \frac{2}{\sqrt{33}}$

In both plots we see the solution $\rightarrow +\infty$ as $t \rightarrow +\infty$. This is because the largest positive root (r_1 or r_2) dominates.