#17 $\dfrac{du}{dt} = -\alpha(u^4 - T^4) \rightarrow \boxed{\dfrac{du}{dt} = -\alpha u^4}$ if $u \gg T$

$\dfrac{du}{u^4} = -\alpha\, dt \rightarrow -\dfrac{1}{3u^3} = -\alpha t - \dfrac{1}{3u_0^3}$

a) $\rightarrow u^3 = \dfrac{u_0^3}{3\alpha u_0^3 t + 1}$;

initial data ; $u(t) = \dfrac{2000}{\sqrt[3]{6t/125 + 1}}$

b) see webpage

c) By evaluating $u(t)$ at various times until we find $u(\tau) \approx 600$, we get that $\tau \approx 750.7$ sec. So our model is certainly reliable for this duration.

$\boxed{2.6}$ #10 $\left(\dfrac{y}{x} + 6x\right)dx + (\ln x - 2)\,dy = 0$, $x > 0$

Here $M(x,y) = \dfrac{y}{6} + 6x$, $N(x,y) = \ln x - 2$.
$M_y = N_x = \dfrac{1}{x}$ so equation is exact. Integrating N with respect to y,
$\rightarrow \psi(x,y) = y \ln x - 2y + h(x)$. Differentiating with respect to x,
$\psi_x = \dfrac{y}{x} + h'(x) = M \Rightarrow h'(x) = 6x \rightarrow h(x) = 3x^2$.
Therefore, implicit solution is $\boxed{3x^2 + y\ln x - 2y = c}$

#11 $M(x,y) = x\ln y + xy$, $N(x,y) = 2y - x$. Here $M_y = \dfrac{x}{y} + x$ and $N_x = -1$, so $M_y \neq N_x$ and equation is NOT exact.