#36] $(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$

divide by x^2; $\left(1 + \frac{3y}{x} + \frac{y^2}{x^2}\right)dx - dy = 0$; $\frac{dy}{dx} = \left(1 + \frac{3y}{x} + \frac{y^2}{x^2}\right) = f\left(\frac{y}{x}\right)$;

$f(v) = 1 + 3v + v^2$

a) So is homogeneous.

b) Subs $y = vx$, $\frac{dy}{dx} = v'x + v$;

$v'x + v = 1 + 3v + v^2 \longrightarrow v'x = (1+v)^2$; this is separable:

$(1+v)^2 dv = x\,dx \longrightarrow -\frac{1}{1+v} = \ln|x| + c$

$\longrightarrow y = \frac{x}{c - \ln|x|} - x$

c) see webpage. Is symmetric with respect to the origin (ie, change of variable $x \to -x$, $y \to -y$)

[2.3] #5] $Q(t)$ = amount of salt (in oz) in container. Incoming rate of salt is $2 \cdot \frac{1}{4}\left(1 + \frac{1}{2}\sin t\right) = \frac{1}{2} + \frac{1}{4}\sin t$ oz/min.

Outgoing rate of salt is $2Q/100$ oz/min

$\rightarrow \frac{dQ}{dt} = \frac{1}{2} + \frac{1}{4}\sin t - \frac{Q}{50}$, $Q_0 = Q(0) = 50$.

Integrating factor $\mu(t) = e^{t/50}$.

$\longrightarrow \left(e^{t/50} Q\right)' = e^{t/50}\left(\frac{1}{2} + \frac{1}{4}\sin t\right)$

a) Soln: $Q(t) = 25 + \left[12.5 \sin t - 625 \cos t + 63150\, e^{-t/50}\right]/2501$ oz.

b) See webpage.

c) Amount of salt approaches a steady state, which is an oscillation of amplitude 1/4 about the level 25oz.